

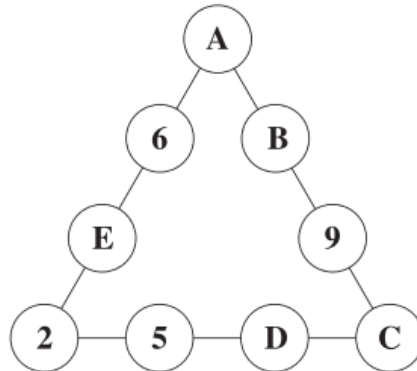
## Math 30-2 Diploma Exam 2019 Solutions

This document contains both the questions and the solutions. If you would like to begin with only the questions, they can be found [here](#).

Topics	Cognitive Levels	Standards
LR – Logical Reasoning	C - Conceptual	Acceptable
PR - Probability	P - Procedural	Excellence
RF – Relations and Functions	PS – Problem Solving	

Use the following information to answer numerical-response question 1.

In a particular puzzle, the digits 1 through 9 are placed in nine circles arranged in a triangle. Each digit can be used only once. When the puzzle is completed, the digits on each side of the triangle must add to 17. A partially completed puzzle is shown below.



### Numerical Response

1. When the puzzle is completed correctly, the value of

B is \_\_\_\_\_ (Record in the **first** column)

D is \_\_\_\_\_ (Record in the **second** column)

E is \_\_\_\_\_ (Record in the **third** column)

(Record your answer in the numerical-response section on the answer sheet.)

## Solution

Read the question carefully to focus on key instructions:

- Only numbers 1-9 can be used with no repetition
- Digits on each side must add to 17

Begin with one of the sides that has two numbers. Try to begin problems where maximum information is provided. We will look at the left side of the triangle that has the numbers 6 and 2. The two missing numbers to fill the spots for A and E must add to 9. We know this because all sides must add to 17 and we already have a sum of 8 (i.e. 6 and 2).

Two numbers that sum to 9 are: (1 and 8), (2 and 7), (3 and 6), and (4 and 5). Because *numbers can only be used once*, (2 and 7), (3 and 6), and (4 and 5) can be eliminated because the numbers 6, 2, and 5 have already been used. Thus, we conclude that A and E must be 1 and 8.

The number 8 cannot go in the A spot because with a 9 already on that side of the triangle, the sum for that side would be over 17. Thus, we conclude that

A = 1 and E = 8.

Looking at the bottom side of the triangle, we are given 2 and 5, which sum to 7. We know that the sum of D and C must be 10. The sums of 10 are (1 and 9), (2 and 8) (3 and 7) and (4 and 6). Since the numbers 1, 9, 2, 8, and 6 have already been used, we know that D and C must be 7 and 3.

C cannot be 7 because the right side of the triangle would then have a sum of (7 + 9 + 1) or 17, while still needing one more number. Thus, we can conclude that

D = 7 and C = 3.

The only number missing now is 4. Thus, B = 4.

**When the puzzle is completed correctly, the value of**

**B is 4**

**D is 7**

**E is 8**

















Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR 1	83.5%	478	LR	1	PS	Acceptable

Use the following information to answer question 1.

Shaye is playing a game where she is trying to determine a four-colour code that has been selected by her opponent. Colours may be repeated in the code. After each of Shaye's guesses, her opponent responds by providing information about how many colours are correct, and whether each colour is in the correct position or not. Shaye's first four guesses are shown below.

	Shaye's Guess	Opponent's Response
1	Red Red Red Red	2 colours are correct and in the right positions
2	Red Red Blue Blue	1 colour is correct and in the right position 1 colour is correct but in the wrong position
3	Red Yellow Red White	3 colours are correct and in the right positions
4	Red Green Red White	2 colours are correct and in the right positions 1 colour is correct but in the wrong position

1. The code selected by Shaye's opponent could be

- A.    
- B.    
- C.    
- D.    

## Solution

Read the question carefully and focus on key instructions:

- Determine a code with 4 correct colors all in the correct position
- After each guess, responses are provided to give information about color and position

All possible answers have red in positions 1 and 3. From statement 1, we know that red cannot be in positions 2 and 4.

Statement 2 states that one color is correct and in the right position. This must mean that the red in position 1 is correct. The red in position 2 is the correct color, but in the wrong position (we know that from statement 1 as it is correct in position 3). We can also deduce that blue cannot be a correct color.

Following statement 3, we know that the two reds are correct and in the correct positions. Either yellow is correct in position 2, or white is correct in position 4.

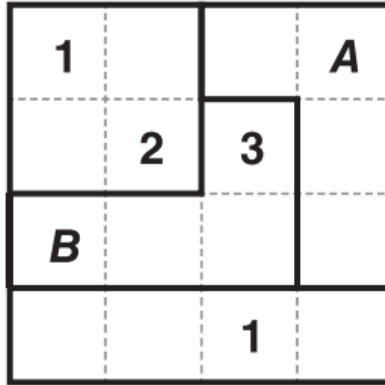
Following statement 4, the two reds are correct and in the right position. If white is the correct color, it would have to be in place of green. But in moving to position 2, it would make statement 3 incorrect. Thus, green is the correct color and it must go in position 4, which in turn means that position 2 is occupied by yellow (validating statement 3).

**The correct answer is D.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC1	77.6%	D	LR	1	PS	Acceptable

Use the following information to answer numerical-response question 2.

A puzzle consists of 16 squares arranged in a four-by-four grid. In the puzzle there are also four shapes that are outlined with bold black lines. The puzzle is completed by entering the digits from 1 through 4 into the squares so that each number appears exactly once in each row, column, and outlined shape. A partially completed puzzle is shown below.



### Numerical Response

2. When the puzzle above is completed correctly, the digit in the box marked

*A* is \_\_\_\_\_ (Record in the **first** column)

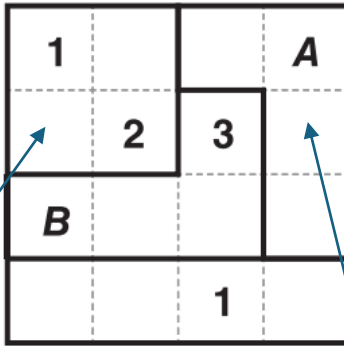
*B* is \_\_\_\_\_ (Record in the **second** column)

(Record your answer in the numerical-response section on the answer sheet.)

### Solution

Read the question carefully and focus on key instructions:

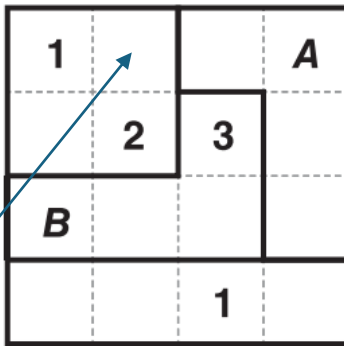
- Only use the digits 1, 2, 3, and 4
- Each row, column and outlined shape must have a unique number



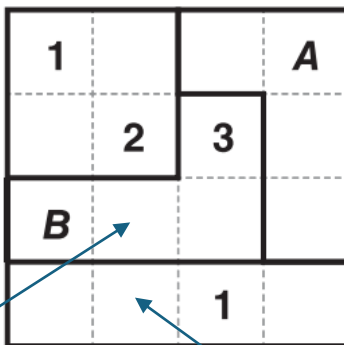
Must be 4

Must be 1

[Thus row 2 has all different numbers]



Must be 3 [completes all numbers for that shape]



Must be 1

Must be 4

[completes all numbers for 2<sup>nd</sup> column]

So far we have:

1	3		
4	2	3	1
	1		
	4	1	

Based on the shape, these two spots need a 2 and a 4

1	3		
4	2	3	1
	1		
	4	1	

1	3		
4	2	3	1
2	1	4	
	4	1	

Fill in the remaining numbers.

1	3	2	<b>4</b>
4	2	3	1
<b>2</b>	1	4	3
3	4	1	2

**A = 4 and B = 2**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR2	80.8%	42	LR	1	PS	Acceptable

Use the following information to answer question 2.

**Three Sets**

$$X = \{BC, SK, ON, YT, NT\}$$

$$Y = \{AB, SK, MB, QC, ON\}$$

$$Z = \{BC, NS, NB, NU, NL\}$$

2. Which of the following set operations will produce an empty set?
- A. Intersection of sets  $Y$  and  $Z$
  - B. Intersection of sets  $X$  and  $Y$
  - C. Union of sets  $Y$  and  $Z$
  - D. Union of sets  $X$  and  $Y$

Solution

Statement A

The intersection of sets  $Y$  and  $Z$  will produce no common elements. There is no element in either set that is also in the other set. With no elements, an empty set is produced.

The correct answer is A.

Statement B

Given the intersection of sets  $X$  and  $Y$ ,  $SK$  and  $ON$  are found in each set. Because there is one or more common elements, the intersection does not produce an empty set.

Statement C

When looking at the union of sets, the result produced is the total number of items in all of the sets. The union of sets  $Y$  and  $Z$  will be a list of the elements  $AB, SK, MB, QC, ON, BC, NS, NB, NU, \text{ and } NL$ . This union does not produce an empty set.

Statement D

As with the above statement, when looking at the union of sets, the result produced is the total number of items in all of the sets. The union of sets  $X$  and  $Y$  will be a list of the elements  $BC, SK, ON, YT, NY, AB, MB \text{ and } QC$ . This union does not produce an empty set.

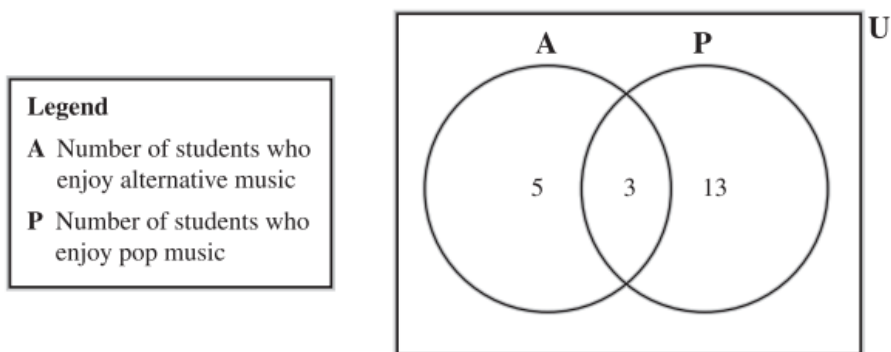


The correct answer is A.

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC 2	76.1%	A	LR	2	C	Acceptable

Use the following information to answer question 3.

A class of 25 students was surveyed to determine the type of music they enjoy—alternative or pop. The results of the survey are illustrated in the partially completed Venn diagram below.



Frieda made the following statements about the sets above.

- Statement 1**  $A \subset P'$
- Statement 2**  $P \cup A = 21$
- Statement 3**  $P \cap A = 21$
- Statement 4**  $(P \cup A)' = 4$

3. The two statements above that are true are numbered

- A. 1 and 2
- B. 1 and 3
- C. 2 and 4
- D. 3 and 4

## Solution

Analyze the numbers in the Venn diagram. The number 3 is positioned in the intersection of the two circles. This means that there are 3 students who like **both** Alternative (A) and Pop (P) music.

The number 5 is positioned in the area of the left circle that represents **only** A. This means that there are 5 students who like **only** A.

The number 13 is positioned in the area of the right circle that represents **only** P. This means that there are 13 students who like **only** P.

The total number of students in these three positions is  $3 + 5 + 13$ , or 21. With a total of 25 students in the class we can deduce that 4 students do not like either A or P.

### Statement 1

The symbol  $\subset$  means subset. In math, a set K is a subset of a set L if **all the elements** of the set K are elements of the set L. In other words, the set K is contained inside the set L.

$A \subset P'$  means that A is a subset of not P. Are all the elements of A contained in the set that is not P? Not P is represented by the numbers 5 and 4. Set A is represented by the numbers 5 and 3. **This statement is not correct.**

### Statement 2

The symbol  $\cup$  means union. In math, the union of two sets K and L is the set of all elements that are a member of K, or L, or both.

$P \cup A$  means all elements that are a member of P, or A, or both. The numbers representing P, or A, or both are 5, 3, and 13. Their sum is 21. **This statement is correct.**

### Statement 3

The symbol  $\cap$  means intersection. In math, the intersection of sets K and L is the set of elements that are common, or belong, to both sets.

$P \cap A$  means the elements that are in both sets P and A. There are 3 people who like both Alternative and Pop. In other words, 3 people belong to both sets.  $P \cap A = 3$ , not,

$P \cap A = 21$ . **This statement is not correct.**

Statement 4

$(P \cup A)'$  means all the elements that are not in the union of P and A.

$P \cup A = 21$ . Since there is a total of 25 in the class,  $(P \cup A)' = 4$ . **This statement is correct.**

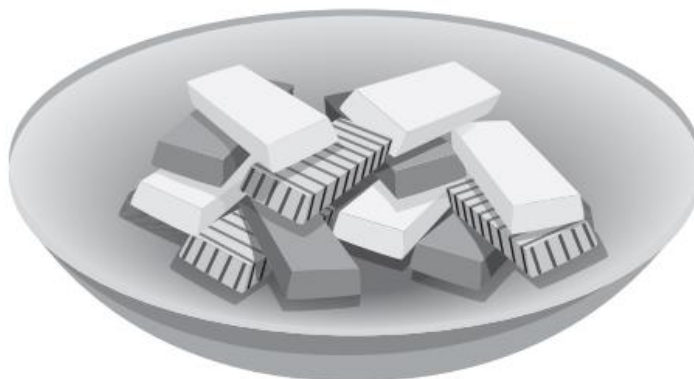
**The correct statements are 2 and 4.**

**The correct answer is C.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC 3	71%	C	LR	2	C	Excellence

*Use the following information to answer question 4.*

A bowl contains 5 white, 4 grey, and 3 patterned erasers, as shown below.



4. If an eraser is randomly selected from the bowl, then the odds against selecting a white eraser are
- A. 7 : 12
  - B. 5 : 12
  - C. 7 : 5
  - D. 5 : 7

## Solution

Remember that we can think of odds in terms of *part:part*. The part we want is shown as the first number. In other words, “how many parts do we want” to “how many parts that we do not want”.

The wording here is important. We are asked for the odds **against** white. So what we want is **not** white. How many parts are **not** white? The 4 grey and 3 patterned are **not** white, and thus we have a total of 7 parts **against** white.

There are 5 remaining parts that are white.

The odds against selecting a white eraser are 7:5.

**The correct answer is C.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC 4	75.9%	C	PR	1	P	Acceptable

*Use the following information to answer numerical-response question 3.*

The probability that Soren watches the evening news on television on any given day is 0.63.

### Numerical Response

3. Out of 365 days, the number of days that Soren is expected **not** to watch the evening news, to the nearest day, is \_\_\_\_\_ days.

(Record your answer in the numerical-response section on the answer sheet.)

## Solution

The probability of watching the evening news and **not** watching the evening news are complimentary. There are no other options; you either watch it or you don't. These two probabilities must add to 1 (or 100%).

$$\begin{array}{rclclcl} \text{Probability of watching} & & + & & \text{Probability of not watching} & = & 1 \\ & 0.63 & & & & & \\ & & + & & \text{Probability of not watching} & = & 1 \end{array}$$

Subtract 0.63 from both sides of this equation to isolate “Probability of **not** watching”.

$$\text{Probability of not watching} = 1 - 0.63$$

$$\text{Probability of not watching} = 0.37$$

To find the number of days that Soren is expected **not** to watch the evening news in a year, multiply the probability by the total number of days in a year.

$$(0.37)(365) = 135.05$$

**Out of 365 days, the number of days that Soren is expected not to watch the evening news, to the nearest day, is 135.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR3	63%	135	PR	2	C	Acceptable

*Use the following information to answer question 5.*

A child is allowed to have 2 treats from a bag containing 21 chocolates, 11 hard candies, and 10 gummy candies. All three types of treat are in packages of similar size and shape. Since he cannot decide what he would like, the child reaches into the bag and randomly selects 2 treats, one at a time.

5. The probability that the child selects 1 chocolate and then 1 gummy candy, to the nearest thousandth, is
- A. 0.744
  - B. 0.738
  - C. 0.122
  - D. 0.119

## Solution

Analyze the problem. It is a probability question that has two events. The child selects 1 chocolate and **then** one gummy. Order is important. With more than 1 probability event, it is often necessary to determine if it is a dependent or an independent situation.

Within the context of this question, it would not make sense for the child to place the first candy back in the bag once it has been selected. When the first draw is **not** replaced, such as what we have here, this is a dependent situation. The second event depends on the first event.

Recall that probability is  $\frac{\text{favorable outcomes}}{\text{total outcomes}}$ .

For the first selection, there are 21 favorable outcomes and 42 total outcomes. The favourable outcomes are the 21 chocolates and the total outcomes is the sum (21+11+10) of the three different types of candy.

$$P(\text{chocolate}) = \frac{21}{42} = \frac{1}{2} = 0.5$$

$$P(\text{gummy}) = \frac{10}{41} = 0.243 \dots$$

*[since there are 10 gummies (favorable) and 1 less in the total as the chocolate was not replaced]*

Multiply these probabilities:

$$(0.5) (0.243\dots) = 0.1219\dots$$

**The correct answer is C.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC5	51.9%	C	PR	3	P	Acceptable

Use the following information to answer question 6.

Sally wants to decorate one wall in her room with movie posters. She will hang 5 of the 8 posters that she owns in a single row across the wall.

6. If Sally places her favourite movie poster in the middle, then the number of different poster arrangements that are possible is
- A. 120
  - B. 840
  - C. 1 680
  - D. 6 720

### Solution

A key word in this question is *arrangements*. It is typically a clue that we are dealing with a permutation. Permutations involves an arrangement of objects in a specific order.

One way to proceed is to think about the Fundamental Counting Principle. If one task can be performed in *a* ways, another task can be performed in *b* ways, another task in *c* ways, and so on, then all these tasks can be performed in *(a) (b) (c) ...* ways.

There is a restriction to consider. The favourite must be in the middle.

Consider 5 tasks, or the positioning of 5 posters.

\_\_\_ X    \_\_\_ X    \_\_\_ X    \_\_\_ X    \_\_\_

There is only 1 option for the middle, i.e. the favourite.

\_\_\_ X    \_\_\_ X    1 X    \_\_\_ X    \_\_\_

Of the total of 8 original posters, there are 7 available for position one. Then 6 available for position two, and 5 available for position four, and finally 4 available for position five.

7 X    6 X    1 X    5 X    4

The product of these five numbers is 840.

The correct answer is B.

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC6	59.7%	B	PR	4	PS	Acceptable

Use the following information to answer question 7.

From 1951 to 1981, the first digit of every three-digit telephone area code in North America was a number from 2 to 9, inclusive. The second digit was either 0 or 1. The last digit depended on the following rules.

- If the second digit was 0, the last digit could be any number from 1 to 9, inclusive.
- If the second digit was 1, the last digit could be any number from 2 to 9, inclusive.

Digits could be used more than once.

7. The number of different telephone area codes that could be formed with these restrictions was
- A. 128
  - B. 136
  - C. 144
  - D. 153

Solution

Repetition of digits is allowed. This is important.

Telephone area codes imply order which leads us to the application of a permutation. Account for the restrictions first as we use the fundamental counting principle. Since the second digit can be 0 or 1, we must consider two separate cases. [cases typically require some addition]

Case 1(second digit was 0) + Case 2(second digit was 1)

In each case, there is only 1 option for the second digit.

\_\_\_ X 1 X \_\_\_ + \_\_\_ X 1 X \_\_\_



Determine the number of options available for each remaining numbered position.

$$\underline{8} \times \underline{1} \times \underline{\quad} + \underline{8} \times \underline{1} \times \underline{\quad}$$

*[the first number can be one of 8 choices, {2,3,4,5,6,7,8,9}]*

$$\underline{8} \times \underline{1} \times \underline{9} + \underline{8} \times \underline{1} \times \underline{8}$$

*[applying restrictions mentioned in the question]*

$$72 + 64$$

$$= 136$$

**The correct answer is B.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC7	55.1%	B	PR	4	PS	Excellence

### Numerical Response

4. A golf shop sells 7 different drivers and 6 different irons. If Susan purchases 3 different drivers and 2 different irons, then the number of different golf club selections she can make is \_\_\_\_\_.

(Record your answer in the numerical-response section on the answer sheet.)

## Solution

There are two different tasks or stages; drivers and irons. To determine the number of options for each stage, we will use combinations.

$$\begin{array}{ccc} \underline{\quad} & X & \underline{\quad} \\ \text{(drivers)} & & \text{(irons)} \end{array}$$

There is a total of 7 drivers and we are requiring 3. ( ${}^7C_3$ )

There is a total of 6 irons and we are requiring 2. ( ${}^6C_2$ )

$$\begin{array}{ccc} \underline{{}^7C_3} & X & \underline{{}^6C_2} \\ \text{(drivers)} & & \text{(irons)} \end{array}$$

$$\begin{array}{ccc} \underline{35} & X & \underline{15} \\ \text{(drivers)} & & \text{(irons)} \end{array}$$

$$(35)(15) = 525$$

**If Susan purchases 3 different drivers and two different irons, then the number of different golf club selections she can make is 525.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR4	45.7%	525	PR	6	PS	Acceptable

Use the following information to answer question 8.

Finley has 8 game pieces that differ only in colour. Each game piece is either all red or all black. When Finley lines up the 8 game pieces in a single row, there are 28 distinguishable arrangements.

8. Based on the information above, Finley could have   *i*   red game pieces and   *ii*   black game pieces.

The statement above can be completed by the information in row

Row	<i>i</i>	<i>ii</i>
A.	4	4
B.	5	3
C.	6	2
D.	7	1

### Solution

If each of the game pieces were distinctly different (for example all different colors), the number of arrangements would be  $8!$ , or 40320.

The fact that some of the pieces can be all of one color means that there is repetition of objects that need to be accounted for.

$$\text{Distinguishable arrangements} = \frac{8!}{(\text{number of red})!(\text{number of black})!}$$

Of the given choices, only 6 red and 2 black will result in 28 distinguishable arrangements.

$$28 = \frac{8!}{(6!)(2!)}$$

**The correct answer is C.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC8	44.5%	C	PR	5	PS	Acceptable

Use the following information to answer question 9.

A group of camp employees, 5 leaders and 8 assistants, are working at a summer camp. A particular activity requires the participation of 5 employees, at least 4 of whom must be leaders.

9. Which of the following expressions can be used to determine the number of possible participant groups?
- A.  ${}_5C_4 \cdot {}_8C_1 + {}_5C_5$
  - B.  ${}_5C_4 \cdot {}_8C_1 \cdot {}_5C_5$
  - C.  ${}_5C_4 + {}_5C_5$
  - D.  ${}_5C_4 \cdot {}_5C_5$

### Solution

A key phrase in this question is “**at least 4**”. In this context, with an activity requiring a maximum of 5 participants, “**at least 4**” means 4 or 5 leaders. Thus, we are dealing with cases that will involve addition.

Determining the number of different ways that leaders and assistants can be selected with these criteria involves using combinations. From a total of 5 leaders, case 1 will select groups of 4, while case 2 will select groups of 5. From a total of 8 assistants, case 1 will select 1 assistant, while case 2 will select no assistants. This will ensure that the total number of people in the activity is 5.

Case 1 – (4 leaders and 1 assistant)

Case 2 – (5 leaders and 0 assistants)

$$({}_5C_4) ({}_8C_1) \quad + \quad ({}_5C_5)$$

[Note: for case 2, we could have shown  $({}_8C_0)$ , but since it is equal to 1, it is not necessary]

**The correct answer is A.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC9	51.0%	A	PR	6	C	Excellence

10. How many distinct 4-letter arrangements can be made using the letters in the word **SCRIPT**?

- A. 720
- B. 360
- C. 24
- D. 15

**Solution**

Questions involving arrangements typically imply the application of a permutation.

Check to see if any of the items (in this case, letters) are repeated. There is no repetition.

With a total of 6 letters, we are required to select 4 at a time.

$${}_6P_4 = 360$$

Alternatively, we can think of the fundamental counting principle where there are 4 stages and we need to determine the number of options for each stage. The final part is to multiply all of the option numbers together.

$$\begin{array}{ccccccccccc} \_ & \times & \_ & \times & \_ & \times & \_ & & & & \\ \underline{6} & \times & \underline{5} & \times & \underline{4} & \times & \underline{3} & & = & & 360 \end{array}$$

**The correct answer is B.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC10	69.2%	B	PR	5	P	Acceptable

11. The simplified form of the expression  $\frac{3x^2 - 3x}{x^2 - 1}$ , and the restriction on  $x$ , are, respectively,
- A.  $3 + 3x$  and  $x \neq 1$
  - B.  $3 + 3x$  and  $x \neq -1, 1$
  - C.  $\frac{3x}{x+1}$  and  $x \neq -1$
  - D.  $\frac{3x}{x+1}$  and  $x \neq -1, 1$

### Solution

When given a rational expression and asked to simplify, often the first task is to factor.

Both the numerator and the denominator have two terms. Always check for a common factor first. The numerator has a common factor of  $(3x)$  in each term. Although the denominator does not have a common factor, it can be factored by difference of squares.

$$\frac{3x(x - 1)}{(x - 1)(x + 1)}$$

Restrictions for rational expressions typically involve the notion that the denominator cannot be equal to zero, as division by zero is undefined. The key question to ask is, what value of the variable will make the denominator equal to zero. In this case,

if  $x = -1, 1$ , the denominator will be equal to zero. Thus, we have to state the restriction that  $x \neq -1, 1$ . Restrictions should be stated **prior** to simplification.

A common binomial of  $(x - 1)$  can now be divided out of the numerator and the denominator to simplify.

$$\frac{3x(x - 1)}{(x - 1)(x + 1)} = \frac{3x}{x + 1}$$

**The correct answer is D.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC11	71.2%	D	RF	1	P	Acceptable

Use the following information to answer numerical-response question 5.

An expression equivalent to  $\frac{3x-8}{x+3}$ ,  $x \neq -3, 0$ , is written in the form  $\frac{Ax^B - 16x}{Cx^2 + Dx}$ , where  $A$ ,  $B$ ,  $C$ , and  $D$  represent single-digit whole numbers.

### Numerical Response

5. The value of

$A$  is \_\_\_\_\_ (Record in the **first** column)

$B$  is \_\_\_\_\_ (Record in the **second** column)

$C$  is \_\_\_\_\_ (Record in the **third** column)

$D$  is \_\_\_\_\_ (Record in the **fourth** column)

(Record your answer in the numerical-response section on the answer sheet.)

### Solution

Most often we are given an expression and asked to simplify. Here, in a sense we must work backwards, as we are provided with the simplification and asked to determine the original expression.

Focus on the term given in each expression. The second term in the numerator of the original expression is  $(-16x)$ , while the second term in the numerator of the simplified expression is  $(-8)$ . This information leads us to believe that  $(2x)$  has been factored out of each term in the numerator.

$$\frac{2x(3x - 8)}{Cx^2 + Dx}$$

Now multiplying the numerator:

$$\frac{6x^2 - 16x}{Cx^2 + Dx}$$

we can determine that  $A = 6$  and  $B = 2$ .

Going back to the factored form of the numerator,  $2x(3x - 8)$ , in order to simplify to just  $(3x - 8)$ , there must have been a  $(2x)$  term to divide out.

In factored form, the denominator is  $2x(x + 3)$ . Now expand to  $2x^2 + 6x$ .

We now know that  $C = 2$  and  $D = 6$

The original expression was  $\frac{6x^2 - 16x}{2x^2 + 6x}$

The factored form of this expression was  $\frac{2x(3x-8)}{2x(x+3)}$

The simplified form of this expression was  $\frac{3x-8}{x+3}$

**The value of**

**A is 6**

**B is 2**

**C is 2**

**D is 6**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR5	69%	6226	RF	1	PS	Acceptable

**12.** The number of distinct non-permissible values in the expression

$$\frac{(x-1)(x+7)}{(x+7)(x-9)} \div \frac{x}{(x-9)(x+8)} \text{ is}$$

**A.** 5

**B.** 4

**C.** 3

**D.** 2



## Solution

When dealing with rational expressions, non-permissible values are those values for the variable that would make the denominator equal to zero (since division by zero is undefined).

Specifically, when dividing rational expressions, we must pay close attention to the **numerator of the divisor**, since division is the same as multiplying by the reciprocal of the divisor. Therefore, the numerator of the divisor cannot be equal to zero, as well.

Given the initial expression of  $\frac{(x-1)(x+7)}{(x+7)(x-9)} \div \frac{x}{(x-9)(x+8)}$ , the non-permissible values for these two denominators would be  $x = -7$ ,  $x = 9$  and  $x = -8$ .

Now writing it in an equivalent multiplication form,  $\frac{(x-1)(x+7)}{(x+7)(x-9)} \div \frac{x}{(x-9)(x+8)} =$   
 $\frac{(x-1)(x+7)}{(x+7)(x-9)} \times \frac{(x-9)(x+8)}{x}$

One more non-permissible value is  $x = 0$  (since it is now in the denominator of the second part of the expression).

There is a total of 4 non-permissible values.

**The correct answer is B.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC12	59.8%	B	RF	2	C	Acceptable

13. Which of the following expressions is equivalent to  $\frac{1}{x+2} - \frac{4}{x}$ , where  $x \neq -2, 0$ ?

A.  $\frac{5x+8}{x(x+2)}$

B.  $\frac{5x+2}{x(x+2)}$

C.  $\frac{-3x+2}{x(x+2)}$

D.  $\frac{-3x-8}{x(x+2)}$

### Solution

This question is testing the ability to subtract rational expressions. Just like rational numbers, a common denominator is required to add or subtract rational expressions.

The first term has a denominator of  $(x+2)$  and the second term has a denominator of  $(x)$ . The smallest expression that each of these denominators will divide evenly into is the product of the two terms, i.e.  $(x+2)(x)$ .

In order to achieve this, the numerator and the denominator of the first term must be multiplied by  $(x)$ , **and**, the numerator and the denominator of the second term must be multiplied by  $(x+2)$ .

$$\left[ \frac{1(x)}{(x+2)(x)} \right] - \left[ \frac{4(x+2)}{x(x+2)} \right]$$

$$\left[ \frac{1(x)}{(x+2)(x)} \right] - \left[ \frac{4x+8}{x(x+2)} \right]$$

$$\frac{x-4x-8}{x(x+2)} = \frac{-3x-8}{x(x+2)}$$

**The correct answer is D.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC13	68.1%	D	RF	2	P	Acceptable

Use the following information to answer numerical-response question 6.

A rational expression in the form  $\frac{A}{B} \cdot \frac{3m^3}{6n^2}$  can be simplified to  $\frac{m}{n}$ , where  $m \neq 0$ ,  $n \neq 0$ . Celine knows that the original rational expression can be formed by selecting expressions for  $A$  and  $B$  from the tables below.

Possible Expression for $A$	Code	Possible Expression for $B$	Code
$n^2$	1	$nm$	5
$n^3$	2	$nm^2$	6
$2n^2$	3	$n^2m^2$	7
$2n^3$	4		

### Numerical Response

6. Record the code numbers that identify the expressions Celine can select for  $A$  and  $B$  to form the original rational expression. (There is more than one correct answer.)

Code Number: \_\_\_\_\_  
 Expression:             $A$                        $B$

(Record **both digits** of your answer in the numerical-response section on the answer sheet.)

### Solution

Begin by focusing on the coefficients. The simplified expression has a coefficient of 1 for  $m$  and 1 for  $n$ . In other words the coefficients that were the products of the two terms would have to be the same. Thus if  $A$  is either  $2n^2$  or  $2n^3$ , the product of the numerators would be 6, and with a coefficient of 6 in the denominator, the coefficients simplify to  $\frac{1}{1}$ .

Now focus on the letters and their respective exponents. With an  $m^3$  in the numerator of the original expression that must simplify to  $m$ , the denominator must have  $m^2$ . This eliminates code 5 and we are left with options of code 6 and 7.

There are two possible answers. If code 3 was matched with code 6 ( $A = 2n^2$  and  $B = nm^2$ ), we would have:

$$\frac{2n^2}{nm^2} \cdot \frac{3m^3}{6n^2} = \frac{6n^2m^3}{6n^3m^2} = \frac{m}{n}$$

If code 4 was matched with code 7, ( $A = 2n^3$  and  $B = n^2m^2$ ), we would have:

$$\frac{2n^3}{n^2m^2} \cdot \frac{3m^3}{6n^2} = \frac{6n^3m^3}{6n^4m^2} = \frac{m}{n}$$

The two possible answers are: 36 or 47.

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR6	58.6%	36,47	RF	2	PS	Acceptable

Use the following information to answer question 14.

When a scuba diver dives below 33 ft, the time,  $t$ , in minutes, that she can remain underwater and then safely surface without a decompression stop can be modelled by the function

$$t = \frac{1700}{d - 33}$$

where  $d$  represents the maximum depth of the dive in feet.

14. During a 50-minute dive, the maximum depth, to the nearest foot, to which the diver can dive and then safely surface without a decompression stop is
- A. 20 ft
  - B. 34 ft
  - C. 67 ft
  - D. 100 ft

**Solution**

The given equation has two variables. Time is represented by the variable,  $t$ , and the maximum depth of the dive is represented by the variable,  $d$ .

Since we are told time, a 50-minute dive, substitute this value into the equation and then solve for  $d$ .

$$t = \frac{1700}{d - 33}$$

$$(50) = \frac{1700}{d - 33}$$

Multiply both sides of the equation by  $(d - 33)$  to clear the fraction.

$$(d - 33)(50) = \frac{1700}{d - 33}(d - 33)$$

$$50d - 1650 = 1700$$

Add 1650 to both sides.

$$50d - 1650 + 1650 = 1700 + 1650$$

$$50d = 3350$$

Divide both sides by 50.

$$d = 67$$

**The correct answer is C.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC14	66.3%	C	RF	3	P	Acceptable

*Use the following information to answer question 15.*

While correctly solving the rational equation  $\frac{3x+6}{5} + \frac{5x}{x+2} = 2x$  algebraically, a student wrote an equivalent quadratic equation of the form  $ax^2 + bx + c = 0$ .

15. The equivalent quadratic equation could have been

- A.  $2x^2 + 6x - 6 = 0$
- B.  $3x^2 + 35x + 12 = 0$
- C.  $7x^2 - 17x - 12 = 0$
- D.  $10x^2 + 12x - 6 = 0$

## Solution

When solving a rational equation, typically the first step is to clear the fractions by multiplying each term by the smallest expression that all the denominators divide evenly into. The two denominator terms are (5) and (x + 2). The smallest expression is the product of these terms.

Multiply each of the three terms in the equation by (5)(x + 2).

$$\left[ (5)(x + 2) \left( \frac{3x + 6}{5} \right) \right] + \left[ (5)(x + 2) \left( \frac{5x}{x + 2} \right) \right] = [(5)(x + 2)(2x)]$$

$$[(x + 2)(3x + 6)] + [(5)(5x)] = [(5x + 10)(2x)]$$

$$[3x^2 + 12x + 12] + [25x] = [10x^2 + 20x]$$

$$3x^2 + 37x + 12 = 10x^2 + 20x$$

Subtract  $3x^2$  from both sides.

$$37x + 12 = 7x^2 + 20x$$

Subtract  $37x$  from both sides.

$$12 = 7x^2 - 17x$$

Subtract 12 from both sides.

$$0 = 7x^2 - 17x - 12, \text{ which is the same as,}$$

$$7x^2 - 17x - 12 = 0.$$

**The correct answer is C.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC15	43.9%	C	RF	3	C	Excellence

Use the following information to answer question 16.

**Four Expressions, where  $R > 1$**

**Expression I**      $3\log_R R$

**Expression II**      $\log_R(R \cdot R)$

**Expression III**      $\log_R\left(\frac{R}{R^2}\right)$

**Expression IV**      $\log_R\left(\frac{R}{R}\right)$

16. The expression with the largest numeric value is numbered   *i*  , and the expression with the smallest numeric value is numbered   *ii*  .

The statement above is completed by the information in row

Row	<i>i</i>	<i>ii</i>
A.	I	III
B.	I	IV
C.	II	III
D.	II	IV

Solution

Expression 1

In a logarithmic expression, such as  $\log_b a$ ,  $b$  is the base and  $a$  is the argument. When the base and the argument are the same,  $\log_b b$ , the value of the logarithmic expression is equal to the exponent on the argument. In this example, the expression  $\log_b b = 1$ . This is true because  $b^? = b^1$ . Since the bases are equal, the exponents must be equal. Thus,  $? = 1$ .

For expression 1 we are given  $3\log_R R$ . This is the same as  $(3)(\log_R R)$ . This is the same as  $(3)(1)$ , which is 3.

Expression 2

$$\log_R(R \cdot R) = \log_R R^2$$

When the base and the argument are the same, the logarithmic expression is equal to the exponent on the argument. In this case, the expression is equal to 2.

### Expression 3

Given  $\log_R \left( \frac{R}{R^2} \right)$ , simplify the part in brackets by applying the quotient exponent law.

When dividing with the same base, subtract the exponents;  $1 - 2 = -1$

$$\log_R \left( \frac{R}{R^2} \right) = \log_R (R^{-1})$$

Since the base and the argument are the same, the expression is equal to the exponent on the argument. In this case, the expression is equal to -1.

### Expression 4

Given  $\log_R \left( \frac{R}{R} \right)$ , simplify the part in brackets by applying the quotient exponent law. When dividing with the same base, subtract the exponents,  $1 - 1 = 0$

$$\log_R \left( \frac{R}{R} \right) = \log_R (R^0)$$

Since the base and the argument are the same, the expression is equal to the exponent on the argument. In this case, the expression is equal to 0.

To summarize,

Expression 1 = 3

Expression 2 = 2

Expression 3 = -1

Expression 4 = 0

**The expression with the largest numeric value is numbered I, and the expression with the small numeric value is numbered III.**

**The correct answer is A.**



Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC16	62.6%	A	RF	4	PS	Acceptable

### Numerical Response

7. When the expression  $\log_b 32 - \log_b 7$ ,  $b > 1$ , is written in the form  $\log_b a$ , the value of  $a$ , to the nearest hundredth, is \_\_\_\_\_.

(Record your answer in the numerical-response section on the answer sheet.)

### Solution

When subtracting logarithms with the same base, the first thought should usually be the quotient law. Can it be used in this situation? The quotient law is:

$$\log_b \left( \frac{m}{n} \right) = \log_b m - \log_b n$$

Combine these two logarithmic terms into a single term using this law.

$$\log_b 32 - \log_b 7 = \log_b \left( \frac{32}{7} \right)$$

$$a = \frac{32}{7}$$

$a = 4.57\dots$

**To the nearest hundredth,  $a = 4.57$ .**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR7	56.6%	4.57	RF	4	P	Acceptable

17. The logarithmic equation  $2 \log_a b = c$ , where  $a > 1$ , expressed in exponential form, could be

A.  $a^c = 2b$

B.  $a^c = b^2$

C.  $a^{2b} = c$

D.  $a^b = c^2$

### Solution

When converting between logarithmic and exponential form, if there is a number in front of the log, first move it to the exponential position. This can be done because of the power law of logarithms. The Power Law states:

$$\log_b(M^n) = n \log_b M$$

In this question,  $2 \log_a b = c$ , is equivalent to  $\log_a b^2 = c$

Re-writing in exponential form,  $a^c = b^2$

**The correct answer is B.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC17	64.6%	B	RF	4	P	Acceptable

Use the following information to answer question 18.

Antoine incorrectly solved the equation  $4^{(2-x)} = 8^{(x+1)}$ . His work is shown below.

**Step I**  $(2^2)^{(2-x)} = (2^3)^{(x+1)}$

**Step II**  $2^{(4-x)} = 2^{(3x+1)}$

**Step III**  $4 - x = 3x + 1$

**Step IV**  $-4x = 3$

**Step V**  $x = -0.75$

18. Antoine's **first** recorded error is shown in

- A. Step I
- B. Step II
- C. Step III
- D. Step IV

### Solution

Step I is correct. The original bases of 4 and 8 are re-written in equivalent form with a base of 2.

$$4 = 2^2$$

$$8 = 2^3$$

There is a problem in step 2. On the left side of the equal sign, when multiplying the exponent of 2 by  $(2 - x)$ , it should be  $4 - 2x$  (which would be applying the distributive property correctly). As well, on the right side of the equal sign, when multiplying the exponent of 3 by  $(x + 1)$ , it should be  $3x + 3$ .

The first recorded error is in step 2.

**The correct answer is B.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC18	70.0%	B	RF	5	C	Acceptable

Use the following information to answer question 19.

Statistics Canada has been tracking the number of farming families in Alberta. In 2005, there were 31 850 farming families. Since that time, the number of farming families has decreased at an average rate of 3.3% annually.

19. Which of the following exponential functions could be used to model the number of farming families,  $F$ , in Alberta,  $t$  years after 2005?
- A.  $31\ 850 = F(1.033)^t$
  - B.  $31\ 850 = F(0.967)^t$
  - C.  $F = 31\ 850(1.033)^t$
  - D.  $F = 31\ 850(0.967)^t$

### Solution

Exponential growth or decay follows a specific format.

Final Amount = Original Amount (Constant)<sup>exponent</sup>

For growth, the constant must be a number greater than 1, and for decay, the constant must be a number less than 1. Since the number of farming families are decreasing, we are dealing with a decay situation. The constant is less than 1. Options A and C are eliminated. The answer is either B or D.

Decreasing by an average rate of 3.3%, is expressed by multiplying previous values by a constant of (100% - 3.3%). Expressed as a decimal, the constant is 0.967.

The original amount is the number of farming families in 2005, which is 31 850. This is the amount that is positioned immediately after the equal sign.

**The correct answer is D.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC19	62.9%	D	RF	6	C	Excellence

Use the following information to answer numerical-response question 8.

A design for a new sports arena is tested to measure the pressure placed on the building's exterior, in pounds of force per square foot ( $\text{lb}/\text{ft}^2$ ), for various wind speeds, in miles per hour (mph). The data are listed in the table below.

Wind Speed (mph)	Pressure ( $\text{lb}/\text{ft}^2$ )
0	0
40	6.7
50	10.4
60	14.9
70	18.6
80	24.2

These data can be modelled by a quadratic regression function of the form

$$y = ax^2 + bx + c$$

where  $x$  is the wind speed, in miles per hour, and  $y$  is the pressure, in pounds per square foot.

### Numerical Response

8. Based on the quadratic regression function, the pressure created by a wind speed of 20 mph, to the nearest tenth of a pound per square foot, is \_\_\_\_\_  $\text{lb}/\text{ft}^2$ .

(Record your answer in the numerical-response section on the answer sheet.)

### Solution

Use the graphing calculator to determine the quadratic regression function of the form  $y = ax^2 + bx + c$

$$y = 0.003x^2 + 0.050x - 0.042$$

Since the variable  $x$  represents the wind speed and given to be 20 mph, we substitute  $x = 20$  into the equation to solve for  $y$ , which is the pressure.

$$y = 0.003(20)^2 + 0.050(20) - 0.042$$

$$y = 1.2 + 1 - 0.042$$

$$y = 2.158$$

Rounded to the nearest tenth, the pressure is **2.2** pounds per square foot.

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
NR8	68.6%	2.2	RF	7	P	Acceptable

20. The graph of a cubic function of the form  $y = ax^3 + bx^2 + cx + d$ , where  $a < 0$  and  $d > 0$ , extends from Quadrant   *i*   to Quadrant   *ii*   and the sign of the y-intercept is   *iii*  .

The statement above is completed by the information in row

Row	<i>i</i>	<i>ii</i>	<i>iii</i>
A.	2	4	positive
B.	2	4	negative
C.	3	1	positive
D.	3	1	negative

### Solution

The first part of the question (answers related to the quadrants) is assessing knowledge of end behaviour. End behaviour is dictated by the sign of the leading coefficient. Since  $a < 0$ , or negative, this 3<sup>rd</sup> degree polynomial function extends from quadrant 2 to quadrant 4.

The last part of the question is assessing knowledge related to the y-intercept. We are told that  $d > 0$ , or positive. The y-intercept is  $d$  because when  $x = 0$  is substituted into the equation, the result is  $y = d$ . The y-intercept is  $(0, d)$ . The sign of the y-intercept is positive.

**The correct answer is A.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC20	42.2%	A	RF	7	C	Acceptable

Use the following information to answer question 21.

A science class is investigating trajectories by launching balls with a catapult. The pathway of one particular ball can be modelled by the quadratic function

$$h = -4.9t^2 + 7t + 0.5$$

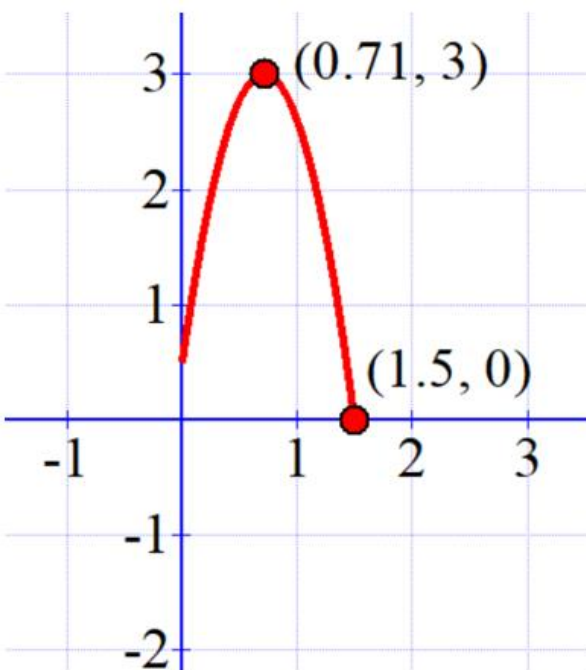
where  $h$  is the height of the ball above the ground, in metres, and  $t$  is the time elapsed since the launch, in seconds.

21. Which of the following rows identifies the domain and range of the function in this context?

Row	Domain	Range
A.	$0 \leq t \leq 0.7$	$0 \leq h \leq 3.0$
B.	$0 \leq t \leq 0.7$	$0 \leq h \leq 0.5$
C.	$0 \leq t \leq 1.5$	$0 \leq h \leq 3.0$
D.	$0 \leq t \leq 1.5$	$0 \leq h \leq 0.5$

### Solution

Use the graphing calculator to show the shape of the graph and to determine key points, such as x-intercept and maximum value.



Time is represented by values on the x-axis. The smallest time unit is zero and the largest time unit is 1.5 seconds. It is at 1.5 seconds when the ball lands on the ground (signified by the x-axis). The domain is  $0 \leq t \leq 1.5$ .

Height is represented by values on the y-axis. The ball reaches a maximum height of 3 metres above the ground, and a minimum height of zero metres when it lands on the ground. The range is  $0 \leq h \leq 3$ .

**The correct answer is C.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC21	57.8%	C	RF	7	C	Excellence

*Use the following information to answer question 22.*

A Ferris wheel at a local fair has a diameter of 68 m and the maximum height above the ground reached by a seat is 74 m.

When discussing the graph of a sinusoidal function that models the height of the seat above the ground during the ride, six students made the following statements.

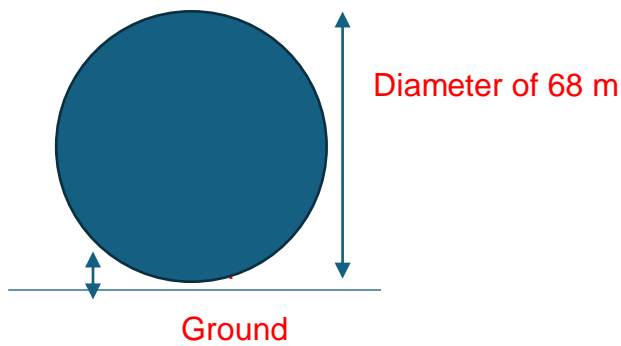
- Statement 1** The median value is 34 m.
- Statement 2** The median value is 40 m.
- Statement 3** The minimum value is 0 m.
- Statement 4** The minimum value is 6 m.
- Statement 5** The amplitude is 34 m.
- Statement 6** The amplitude is 68 m.

22. The three statements above that are true are numbered

- A. 1, 4, and 5
- B. 1, 3, and 6
- C. 2, 4, and 6
- D. 2, 4, and 5



Solution



The maximum height is 74 m ( $68 + 6$ ) and the minimum height is 6 m.

$$\text{amplitude} = \frac{\text{maximum value} - \text{minimum value}}{2}$$

$$\text{amplitude} = \frac{74 - 6}{2}$$

Amplitude = 34

$$\text{median} = \frac{\text{maximum value} + \text{minimum value}}{2}$$

$$\text{median} = \frac{74 + 6}{2}$$

Median = 40

Statements 2, 4, and 5 are correct.

**The correct answer is D.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC22	43.6%	D	RF	8	C	Excellence

Use the following information to answer question 23.

The regular rise and fall of the ocean's water level can limit the times at which ships can enter and exit a harbour. On a given day, the depth of the water in a particular harbour,  $d$ , in metres, can be modelled by the sinusoidal function

$$d = 2.35 \sin(0.51t + 4.11) + 13.13$$

where  $t$  is the time, in hours after midnight.

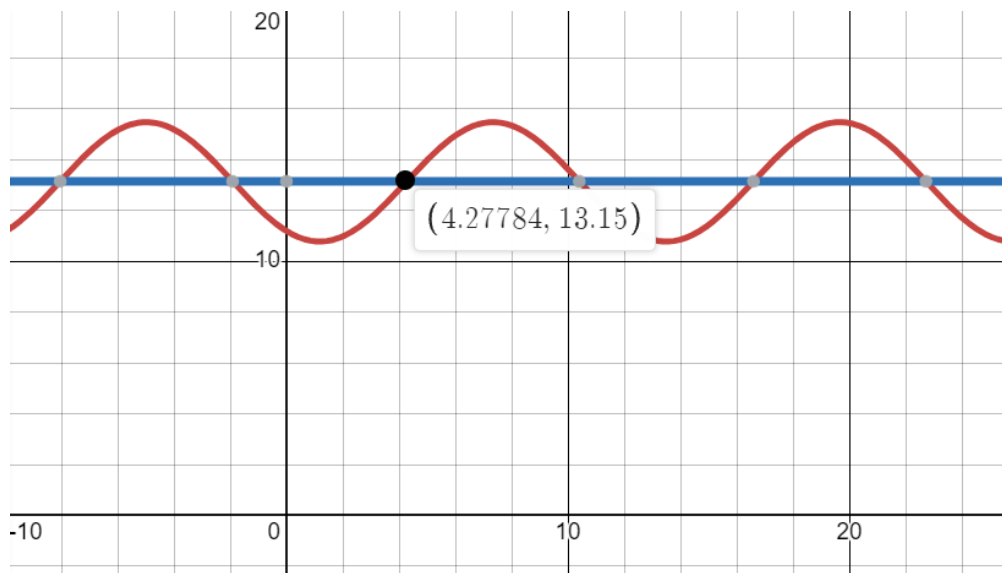
23. If a particular ship requires a minimum water depth of 13.15 m to enter the harbour on the given day, the time at which it can first enter the harbour, to the nearest hundredth of an hour after midnight, is
- A. 1.18 h
  - B. 4.28 h
  - C. 6.13 h
  - D. 10.82 h

**Solution**

Graph the sine function in the graphing calculator. Then graph  $y = 13.15$  as a horizontal line to represent the minimum water depth.

The first intersection point to the right of the origin represents the time at which the ship can safely enter the harbour. The x-coordinate of the intersection point is the answer, or 4.28 hours.

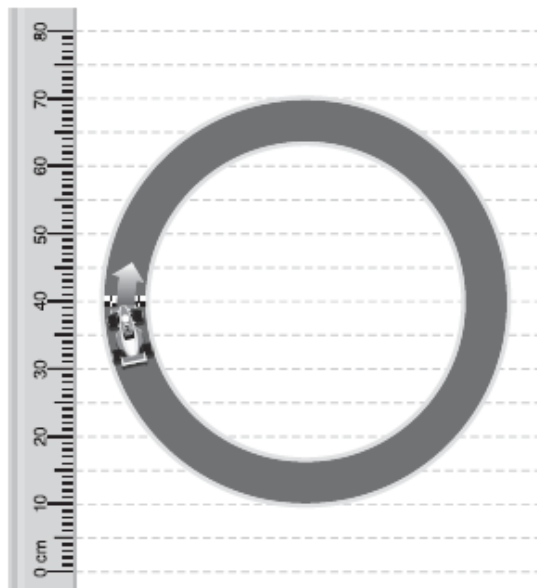
**The correct answer is B.**



Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC23	70.7%	B	RF	8	PS	Acceptable

Use the following information to answer question 24.

An electric toy car is travelling around a circular track at a constant speed. A ruler is positioned beside the track as shown in the diagram below.



The position of the car, measured in centimetres with the ruler, can be modelled by the sinusoidal function

$$p = 30 \sin \frac{\pi}{2} t + 40$$

where  $p$  is the position of the car, in centimetres, and  $t$  is the elapsed time, in seconds.

24. The amplitude of the sinusoidal function is   *i*  , and the time it takes for the car to travel around the track once is   *ii*  .

The statement above is completed by the information in row

Row	<i>i</i>	<i>ii</i>
A.	30 cm	4 s
B.	30 cm	2 s
C.	60 cm	4 s
D.	60 cm	2 s

### Solution

In a sinusoidal equation, the amplitude is represented by the number immediately in front of the function. In this case, the amplitude is 30 cm.

The time it takes for the car to travel around the track once is the period. The period is determined by dividing the 'b' value from the equation into  $2\pi$ . In this equation, the 'b' value is  $\frac{\pi}{2}$ .

$$\text{Thus, } \frac{2\pi}{\frac{\pi}{2}} = 4$$

The time it takes for the car to travel around the track once is 4 seconds.

**The correct answer is A.**

Question	Difficulty	Key	Topic	Outcome	Cognitive Level	Standard
MC24	61.3%	A	RF	8	PS	Acceptable