Number of Solutions of a Linear System Practice

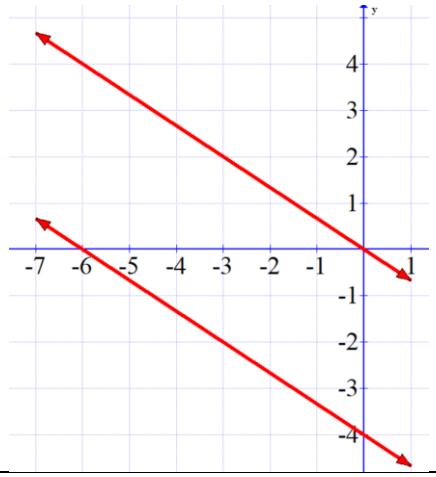
1. The solution to the linear system, ① y = 10x + 6 and ② 2x + 3y = -14, is

   A) (1, 16) B) (-1, -4) C) (0, 6) D) (2, -6)

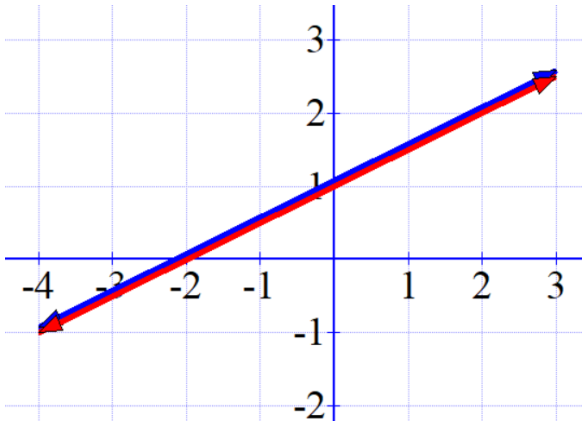
Use the following information to answer the next question.

Consider the following 4 graphs.

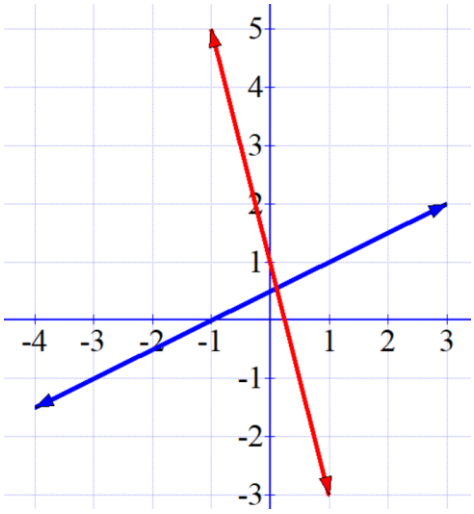
A



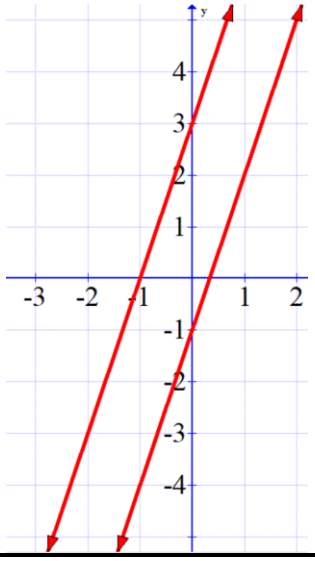
B



C



D



2. The graph that has one solution is

   A) A B) B C) C D) D

3. If the linear system, ① $y = -5x + K$ and ② $3y = -15x + 6$, have an infinite number of solutions, the value for K is ____.

Use the following information to answer the next question.

Consider the following linear system, where m_1 and m_2 are integer values in front of the variable x, and b_1 and b_2 are integer values.

① $y = m_1x + b_1$
② $y = m_2x + b_2$

4. If we are told that there is no solution to this linear system, then we know that

A) $m_1 = m_2$ **and** $b_1 = b_2$
B) $m_1 = m_2$ **and** $b_1 \neq b_2$
C) $m_1 \neq m_2$ **and** $b_1 = b_2$
D) $m_1 \neq m_2$ **and** $b_1 \neq b_2$

Use the following information to answer the next question.

Four math student were given a task. When presented with the equation, $-6x + y = 3$, the students had to create another linear equation that will form a linear system with no solution. The student submissions are shown below.

Dean	$y = -6x - 1$
Randy	$\frac{1}{2}x + \frac{1}{3}y = 1$
Elaine	$2y - 5 = 12x$
Chelsea	$-18x = -3y + 9$

5. The student having the correct equation is

A) Dean B) Randy C) Elaine D) Chelsea

Use the following information to answer the next question.

Given the linear system, ① $2x - y = -5$ and ② $4x + y = -7$, Julian said that since the slopes are different, there is exactly 1 solution. Julian also said that the solution is (1,7) because he verified it in the equation ① $2x - y = -5$.

$$2(1) - (7) = -5$$
$$-5 = -5$$

6. Do you agree or disagree with Jilian? Explain.

7. The linear system, ① $y = -7x + 1$ and ② $y + kx = 3$, has no solution. The value for K is ____.

8. Usually if a linear system has exactly 1 solution, the slopes and the y-intercepts are different. Is it possible for the slopes to be different, the y-intercepts to be the same and for there to be exactly 1 solution? Explain.

Number of Solutions of a Linear System Practice Solutions

1. The solution to the linear system, ① $y = 10x + 6$ and ② $2x + 3y = -14$, is

   A) (1, 16) B) (-1, -4) C) (0, 6) D) (2, -6)

Solution

A solution satisfies (makes a true statement) both linear equations in a linear system.

Option A

$y = 10x + 6$	$2x + 3y = -14$
$(16) = 10(1) + 6$	$2(1) + 3(16) = -14$
$16 = 16$	$50 \neq -14$

This point does not satisfy both equations.

Option B

$y = 10x + 6$	$2x + 3y = -14$
$(-4) = 10(-1) + 6$	$2(-1) + 3(-4) = -14$
$-4 = -4$	$-14 = -14$

This point satisfies both equations. Option is the solution.

Option C

$y = 10x + 6$	$2x + 3y = -14$
$(6) = 10(0) + 6$	$2(0) + 3(6) = -14$
$6 = 6$	$18 \neq -14$

This point does not satisfy both equations.

Option D

$y = 10x + 6$	$2x + 3y = -14$
$(-6) = 10(2) + 6$	$2(2) + 3(-6) = -14$
$-6 \neq 26$	$-14 = -14$

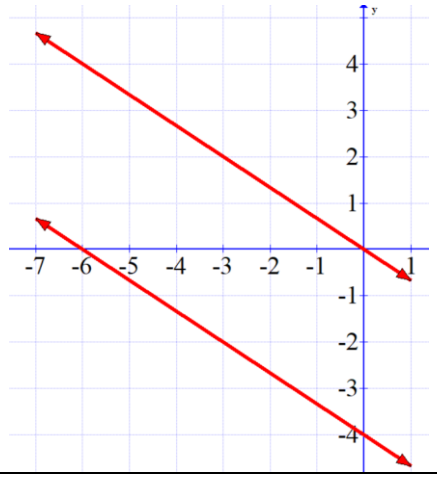
This point does not satisfy both equations.

The correct answer is B.

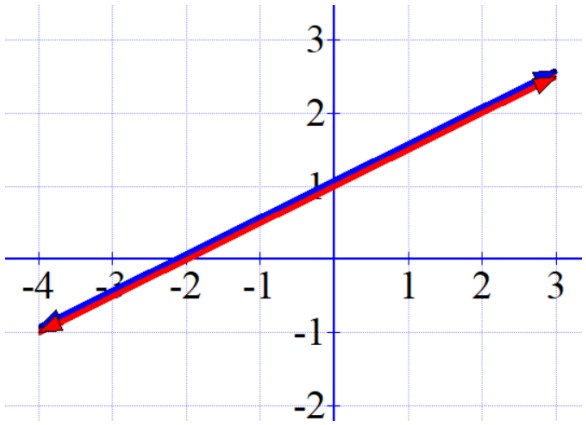
Use the following information to answer the next question.

Consider the following 4 graphs.

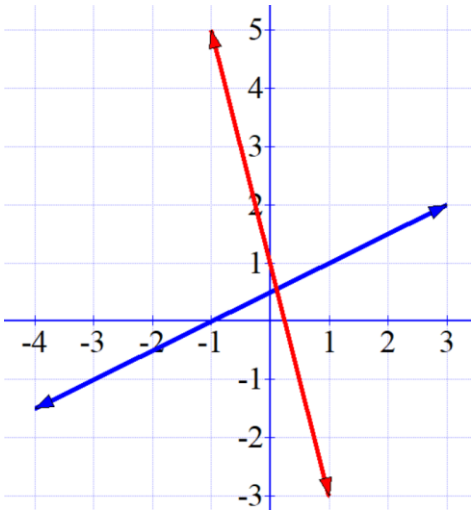
A



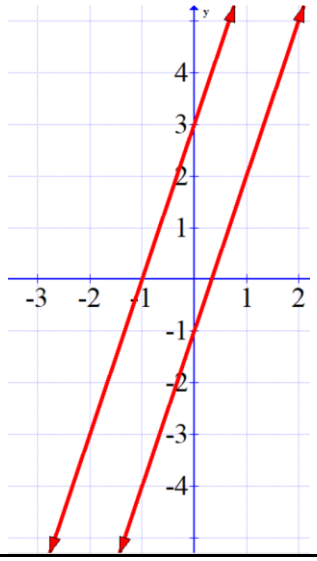
B



C



D



2. The graph that has one solution is

A) A B) B C) C D) D

Solution

Graphs A and D are parallel and will not intersect. Therefore there is no solution. Graph B is one line on top of the other, or coincident lines. There is an infinite number of solutions.

Graph C has two lines intersecting at one point. This means that there is one solution.

The correct answer is C.

3. If the linear system, ① y = -5x + K and ② 3y = -15x + 6, have an infinite number of solutions, the value for K is __2__.

Solution

Isolate y in equation ② by dividing every term by 3.

y = -5x + 2

Compare to equation ①, y = -5x + K.

In order to have an infinite number of solutions, the slopes and y-intercepts both must be the same. In each equation, the slope is -5. The y-intercept in equation ② is 2. Therefore the y-intercept in equation ① must also be 2. Thus, K = 2.

The value for K is 2.

Use the following information to answer the next question.

Consider the following linear system, where m_1 and m_2 are integer values in front of the variable x, and b_1 and b_2 are integer values.

① $y = m_1x + b_1$
② $y = m_2x + b_2$

4. If we are told that there is no solution to this linear system, then we know that

A) $m_1 = m_2$ **and** $b_1 = b_2$
B) $m_1 = m_2$ **and** $b_1 \neq b_2$
C) $m_1 \neq m_2$ **and** $b_1 = b_2$
D) $m_1 \neq m_2$ **and** $b_1 \neq b_2$

Solution

When *y* is isolated, as it is in both equations, the value in front of *x* (in this question m_1 and m_2) is the slope. The value at the end of the equation is the y-intercept. (in this question b_1 and b_2).

To have no solution, the slopes must be the same (to give parallel lines) and the y-intercepts must be different.

The correct answer is B.

Use the following information to answer the next question.

Four math student were given a task. When presented with the equation, -6x + y = 3, the students had to create another linear equation that will form a linear system with no solution. The student submissions are shown below.

Dean	y = -6x - 1
Randy	$\frac{1}{2}x + \frac{1}{3}y = 1$
Elaine	2y – 5 = 12x
Chelsea	-18x = -3y + 9

5. The student having the correct equation is

A) Dean B) Randy C) Elaine D) Chelsea

Solution

The initial equation given in the question is -6x + y = 3. Add 6x to both sides to put the equation in an equivalent form, which will make it easier to compare.

y = 6x + 3

Rewrite each of the other equations in the form y = mx + b, in order to compare slopes and y-intercepts.

Dean

y = -6x - 1

His equation is already in this form.

Randy

Clear fractions by multiplying each of the 3 terms by 6. We choose 6 because it is the smallest number that the denominators of (2) and (3) will divide evenly into.

$\frac{1}{2}x + \frac{1}{3}y = 1$

$6\left[\frac{1}{2}x + \frac{1}{3}y = 1\right]$

3x + 2y = 6

Subtract 3x from both sides.

2y = -3x + 6

Divide every term by 2.

$y = -\left(\frac{3}{2}x\right) + 3$

Elaine

2y – 5 = 12x

Add 5 to both sides.

2y = 12x + 5

Divide every term by 2.

y = 6x + 5/2

Chelsea

-18x = -3y + 9

Add 3y to both sides.

3y – 18x = 9

Add 18x to both sides.

3y = 18x + 9

Divide every term by 3.

y = 6x + 3

A system with no solution has equations with the same slopes and different y-intercepts. When comparing the original equation, y = 6x + 3, to the given options, only Elaine's answer of y = 6x + 5/2, satisfies this requirement.

The correct answer is C.

Use the following information to answer the next question.

Given the linear system, ① 2x – y = -5 and ② 4x + y = -7, Julian said that since the slopes are different, there is exactly 1 solution. Julian also said that the solution is (1,7) because he verified it in the equation ① 2x – y = -5.

$$2(1) - (7) = -5$$
$$-5 = -5$$

6. Do you agree or disagree with Jilian? Explain.

Solution

When there is exactly one solution for a linear system, that solution or point must satisfy both equations. Julian has shown that the point (1,7) satisfies equation ①.

Now we have to substitute this point into equation ②.

$4x + y = -7$

$4(1) + (7) = -7$

$4 + 7 = -7$

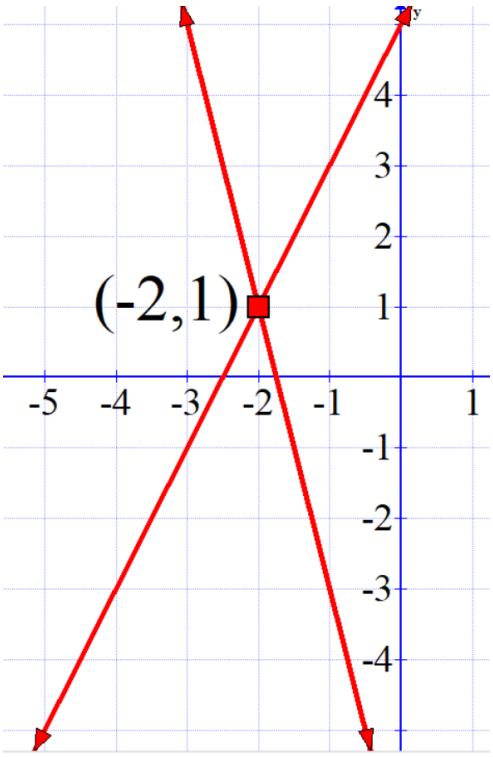
$11 \neq -7$

Therefore, (1,7) is not a solution to this linear system.

To find the correct solution, graph the linear system.

The intersection point is (-2,1).

① $2x - y = -5$	② $4x + y = -7$
$2(-2) - (1) = -5$	$4(-2) + (1) = -7$
$-4 - 1 = -5$	$-8 + 1 = -7$
$-5 = -5$	$-7 = -7$



I do not agree with Julian. It is necessary that the solution satisfy both equations, not just one. The correct solution is (-2,1).

7. The linear system, ① y = -7x + 1 and ② y + kx = 3, has no solution. The value for K is _7_.

Solution

Isolate y in both equations to allow for easy comparison of slopes and y-intercepts.

① y = -7x + 1

② y =-kx + 3

For a system to have no solution, the slopes are equal and the y-intercepts are different. In this case, the y-intercepts are different (i.e. 1 and 3). For the slopes to be equal, K must be equal to 7.

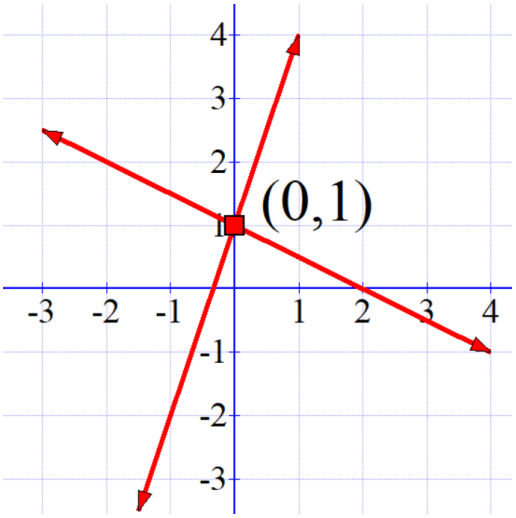
The value for K is 7.

8. Usually if a linear system has exactly 1 solution, the slopes and the y-intercepts are different. Is it possible for the slopes to be different, the y-intercepts to be the same and for there to be exactly 1 solution? Explain.

Solution

Yes this is possible if the solution is on the y-axis. For example, given the system,

① y = 3x + 1 and ② y = (-½)x + 1, there is one intersection point at (0,1).



We know that this is the solution as it satisfies both equations.

y = 3x + 1	y = (-½)x + 1
(1) = 3(0) + 1	(1) = (-½)(0) + 1
1 = 1	1 = 1

The slopes of each equation are different. Equation ① has a slope of 1, and equation ② has a slope of -½. The y-intercepts are the same, i.e. both are 1. There is exactly 1 solution.