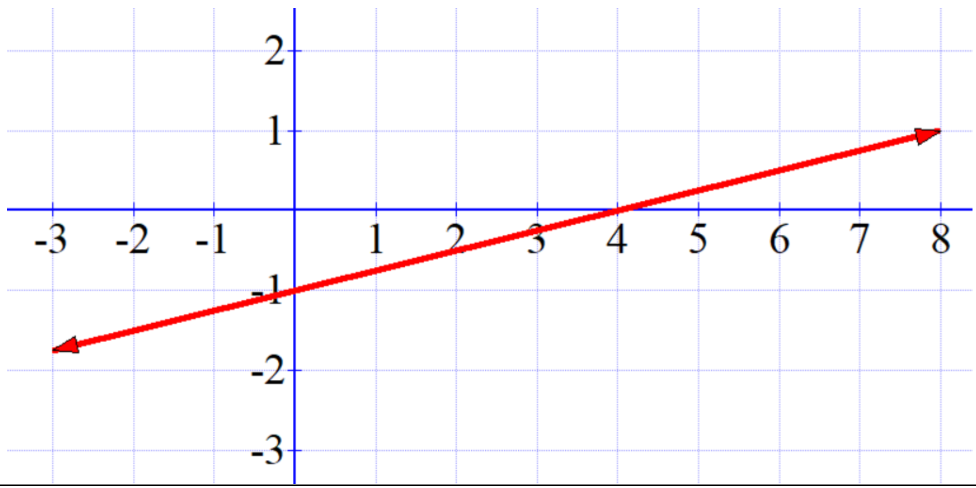
Piecewise Function Practice

Use the following information to answer the first question.

Consider the graph below of $f(x) = \frac{1}{4}x - 1$



1. Written as a piecewise function, y = $|f(x)|$ is

A) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \geq 4 \\ -\left(\frac{1}{4}x - 1\right), if\ x < 4 \end{cases}$

B) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \leq 4 \\ -\left(\frac{1}{4}x - 1\right), if\ x > 4 \end{cases}$

C) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \geq 0 \\ -\left(\frac{1}{4}x - 1\right), if\ x < 0 \end{cases}$

D) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \leq 0 \\ -\left(\frac{1}{4}x - 1\right), if\ x > 0 \end{cases}$

Use the following information to answer the next question.

Consider the function, y = 4x - 6, when analyzing the statements below.

Statement 1	The x-intercept of y = 4x – 6 is $\frac{3}{2}$.
Statement 2	The x-intercept of y = 4x – 6 is $-\frac{3}{2}$.
Statement 3	The point (1,2) is on the graph of $y = \|f(x)\|$
Statement 4	The point (0,-6) is on the graph of $y = \|f(x)\|$

2. The two correct statements are

A) 1 and 4   B) 1 and 3   C) 2 and 4   D) 2 and 3

Use the following information to answer the next question.

The piecewise function of $y = |3x - 3|$ is

$$y = \begin{cases} 3x - 3, if\ x \geq K \\ -(3x - 3), if\ x < K \end{cases}$$

where K is an integer.

3. The value of K is ____.

Use the following information to answer the next question.

A math student was told that the piecewise function for a certain absolute value equation is:

$$y = \begin{cases} 2x + 5, if\ x \geq -2.5 \\ -(2x + 5), if\ x < -2.5 \end{cases}$$

4. The equation for this absolute value function is

A) $y = |5 + x|$ B) $y = |5 - 2x|$ C) $y = |2x + 5|$ D) $y = |2x - 5|$

5. Written as a piecewise function, $y = |x^2 - 5x - 6|$ is

$$A)\ y = \begin{cases} x^2 - 5x - 6, if\ x \leq -2\ and\ x \geq 3 \\ -(x^2 - 5x - 6), if -1 < x < 3 \end{cases}$$

$$B)\ y = \begin{cases} x^2 - 5x - 6, if -1 < x < 3 \\ -(x^2 - 5x - 6), if\ x \leq -2\ and\ \geq 3 \end{cases}$$

$$C)\ y = \begin{cases} x^2 - 5x - 6, if\ x \leq -1\ and\ x \geq 6 \\ -(x^2 - 5x - 6), if -1 < x < 6 \end{cases}$$

$$D)\ y = \begin{cases} x^2 - 5x - 6, if -1 < x < 6 \\ -(x^2 - 5x - 6), if\ x \leq -1\ and\ x \geq 6 \end{cases}$$

6. Given the absolute value function $y = |2x^2 - 7x - 4|$, one part of the corresponding piecewise function is y = -($2x^2$ – 7x – 4), if $-\frac{1}{2} < x < 4$. The other part for this piecewise function is

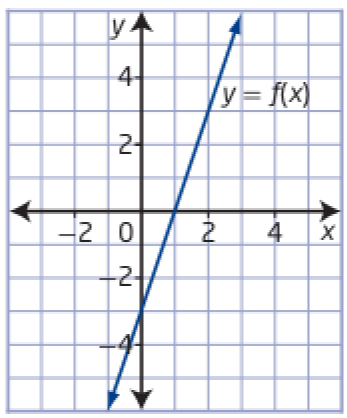
   A) y = -($2x^2$ – 7x – 4), if $x \leq -\frac{1}{2}$ $and\ x\ \geq 4$.
   B) y = $2x^2$ – 7x – 4, if $x \leq -\frac{1}{2}$ $and\ x\ \geq 4$.
   C) y = -($2x^2$ – 7x – 4), if -4 < x < ½ .
   D) y = $2x^2$ – 7x – 4, if -4 < x < ½.

7. The point (-1,-4) is on the graph of x = f(x). The corresponding point on the graph of $y = |f(x)|$ is

   A) (-1, 4) B) (1, -4) C) (1, 4) D) (-4, -1)

Use the following information to answer the next question.

Consider the linear function below. Both intercepts are integers.



8. A) Determine the equation of y = f(x). Explain.

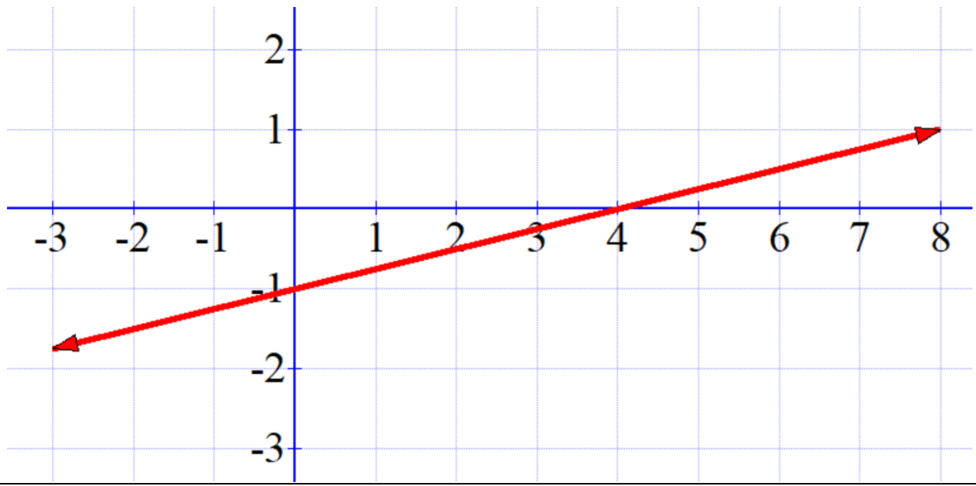
   B) Sketch the graph of $y = |f(x)|$. Label both intercepts. State any invariant points.

   C) Write as a piecewise function. Explain.

Piecewise Function Practice Solutions

Use the following information to answer the first question.

Consider the graph below of $f(x) = \frac{1}{4}x - 1$



1. Written as a piecewise function, y = $|f(x)|$ is

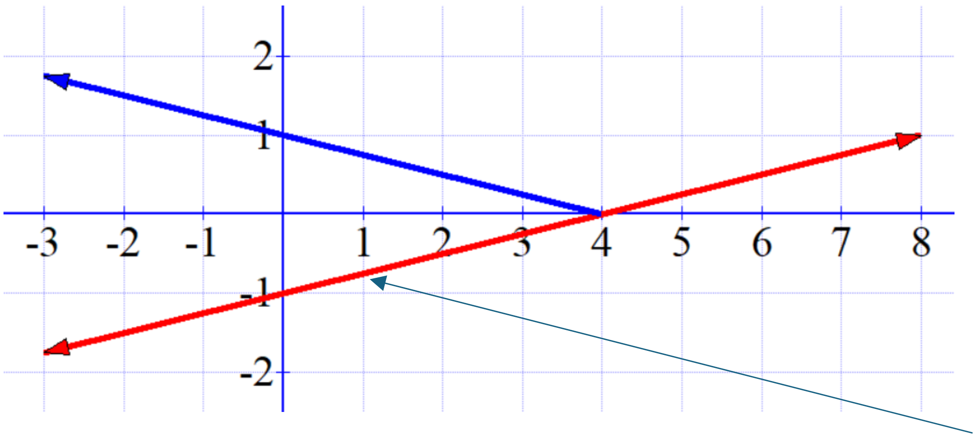
A) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \geq 4 \\ -\left(\frac{1}{4}x - 1\right), if\ x < 4 \end{cases}$

B) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \leq 4 \\ -\left(\frac{1}{4}x - 1\right), if\ x > 4 \end{cases}$

C) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \geq 0 \\ -\left(\frac{1}{4}x - 1\right), if\ x < 0 \end{cases}$

D) $y = \begin{cases} \frac{1}{4}x - 1, if\ x \leq 0 \\ -\left(\frac{1}{4}x - 1\right), if\ x > 0 \end{cases}$

Solution



All of the points on the linear function $f(x) = \frac{1}{4}x - 1$, where x is less than 4, have been reflected in the x-axis, since the y-coordinates for all these points are negative. For example, the point (0, -1) moves to (0, 1) and the point (2, -0.5) moves to (2, 0.5). Remember that the absolute value of a function means taking the absolute value of the y-coordinate.

For all points on the linear function where x is greater than or equal to 4, since these corresponding y-coordinates are positive, their absolute values are also positive. Thus, this portion of the graph is on both the original linear function and the absolute value function.

Therefore, in terms of a piecewise function,

y = $\frac{1}{4}x - 1$, if x $\geq$ 4

y = $-(\frac{1}{4}x - 1)$, if x $<$ 4

The correct answer is A.

Use the following information to answer the next question.

Consider the function, y = 4x - 6, when analyzing the statements below.

Statement 1	The x-intercept of y = 4x – 6 is $\frac{3}{2}$.
Statement 2	The x-intercept of y = 4x – 6 is $-\frac{3}{2}$.
Statement 3	The point (1,2) is on the graph of $y = \lvert f(x) \rvert$
Statement 4	The point (0,-6) is on the graph of $y = \lvert f(x) \rvert$

2. The two correct statements are

A) 1 and 4 B) 1 and 3 C) 2 and 4 D) 2 and 3

Solution

Statement 1

To determine the x-intercept, set y = 0 and solve for x.

y = 4x – 6

(0)= 4x - 6
6 = 4x

$$x = \frac{6}{4} = \frac{3}{2}$$

Statement 1 is true.

Statement 2

Since we know that statement 1 is true, statement 2 cannot be true.

Statement 2 is not true.

Statement 3

If the point (1,2) is on the graph of $y = \lvert f(x) \rvert$, then it should satisfy the equation.
$y = \lvert 4x - 6 \rvert$
$(2) = \lvert 4(1) - 6 \rvert$

$2 = |-2|$
2 = 2

Statement 3 is true.

Statement 4

Statement 4 must be false because this absolute value function cannot have a negative y-coordinate.

The two true statements are 1 and 3.

The correct answer is B.

Use the following information to answer the next question.

The piecewise function of $y = |3x - 3|$ is

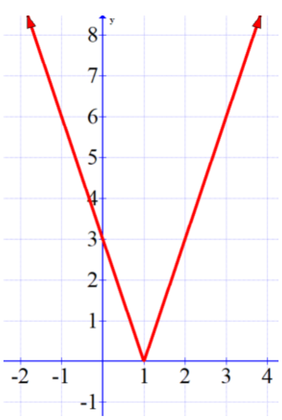
$$y = \begin{cases} 3x - 3, if\ x \geq K \\ -(3x - 3), if\ x < K \end{cases}$$

where K is an integer.

3. The value of K is _1_.

Solution

The value of K represents the x-intercept. It is at this point that the original linear function will chart a new course. Any part of the original graph that is below the x-intercept, will have negative y values. These negative values are then reflected in the x-axis to become positive.



The graph is the absolute value of y = 3x – 3. We can tell from the graph that the x-intercept is 1. But we can also solve it algebraically, by setting y = 0 and solve for x.

(0) = 3x – 3

3 = 3x

x = 1 **The value of K is 1.**

Use the following information to answer the next question.

A math student was told that the piecewise function for a certain absolute value equation is: $$y = \begin{cases} 2x + 5, if\ x \geq -2.5 \\ -(2x + 5), if\ x < -2.5 \end{cases}$$

4. The equation for this absolute value function is

$A)\ y = |5 + x|$ B) $y = |5 - 2x|$ C) $y = |2x + 5|$ D) $y = |2x - 5|$

Solution

We require a function that has an x-intercept of -2.5.

Remove the absolute value signs, set y = 0, and solve for x.

A) y = 5 + x
(0) = 5 + x
-5 = x

B) y = 5 – 2x

(0) = 5 – 2x

-5 = -2x

2.5 = x

C) y = 2x + 5

(0) = 2x + 5

-5 = 2x

-2.5 = x

D) y = 2x – 5

(0) = 2x – 5

5 = 2x

2.5 = x

The correct answer is C.

5. Written as a piecewise function, $y = |x^2 - 5x - 6|$ is

$$A)\ y = \begin{cases} x^2 - 5x - 6\,, if\ x \leq -2\ and\ x \geq 3 \\ -(x^2 - 5x - 6)\,, if\ -1 < x < 3 \end{cases}$$

$$B)\ y = \begin{cases} x^2 - 5x - 6\,, if\ -1 < x < 3 \\ -(x^2 - 5x - 6)\,, if\ x \leq -2\ and\ \geq 3 \end{cases}$$

$$C)\ y = \begin{cases} x^2 - 5x - 6\,, if\ x \leq -1\ and\ x \geq 6 \\ -(x^2 - 5x - 6)\,, if\ -1 < x < 6 \end{cases}$$

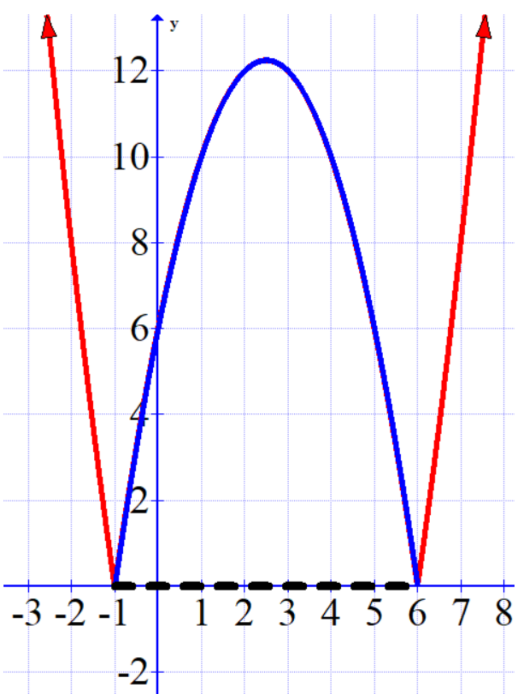
$$D)\ y = \begin{cases} x^2 - 5x - 6\,, if\ -1 < x < 6 \\ -(x^2 - 5x - 6)\,, if\ x \leq -1\ and\ x \geq 6 \end{cases}$$

Solution

Factor $y = x^2 - 5x - 6$, to determine the x-intercepts.

$y = (x - 6)(x + 1)$

The x-intercepts are 6 and -1.



Any value of x less than -1 or greater than 6 will produce positive y-values both on the original quadratic function and the absolute value function. Therefore, within this domain, the function is $y = x^2 - 5x - 6$.

Any value of x between -1 and 6 would have produced negative y-values on the original quadratic function. In order to reflect these y-values in the x-axis, a negative sign must be placed on the quadratic function;

$y = -(x^2 - 5x - 6)$

The correct answer is C.

6. Given the absolute value function $y = |2x^2 - 7x - 4|$, one part of the corresponding piecewise function is y = -(2x² – 7x – 4), if $-\frac{1}{2} < x < 4$. The other part for this piecewise function is

   A) y = -(2x² – 7x – 4), if $x \leq -\frac{1}{2}$ *and* $x \geq 4$.
   B) y = 2x² – 7x – 4, if $x \leq -\frac{1}{2}$ *and* $x \geq 4$.
   C) y = -(2x² – 7x – 4), if -4 < x < ½ .
   D) y = 2x² – 7x – 4, if -4 < x < ½.

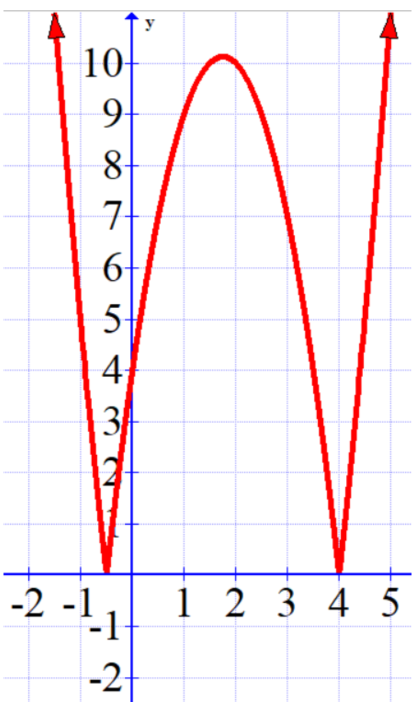
Solution

Factor y = 2x² – 7x – 4 in order to determine the x-intercepts.

y = (2x + 1) (x – 4)

[NOTE: If unsure how to factor this trinomial, you can check out the following link]

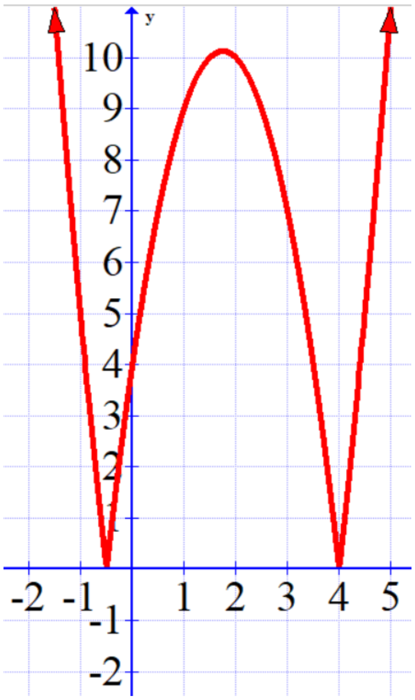
The x-intercepts are 4 and $-\frac{1}{2}$.



The graph of $y = |2x^2 - 7x - 4|$, is shown to the left. For the original quadratic function, y = 2x² – 7x – 4, within the domain of -½ < x < 4, the corresponding y-coordinates are negative. When taking the absolute value of these negative y-coordinates, they are all reflected in the x-axis. Signifying this as a partial piecewise function, it would be

y = -(2x² – 7x – 4), if $-\frac{1}{2} < x < 4$.

The negative sign in front of the function illustrates this reflection.



For any value of x greater than or equal to 4, and any value of x less than or equal to -½ , all points on the original quadratic function will also be points on the absolute value function. The reason for this is because of these corresponding y-values on the original quadratic function are positive.

The partial piecewise component to describe this situation is y = $2x^2 - 7x - 4$, if

$x \leq -\frac{1}{2}$ *and* $x \geq 4.$

The correct answer is B.

7. The point (-1,-4) is on the graph of x = f(x). The corresponding point on the graph of $y = |f(x)|$ is

   A) (-1, 4) B) (1, -4) C) (1, 4) D) (-4, -1)

Solution

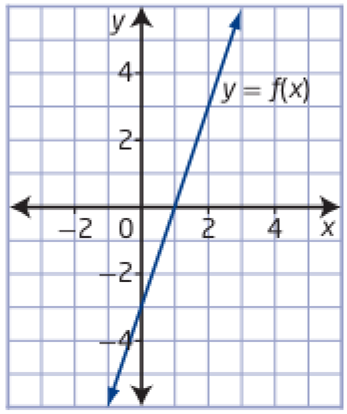
When the absolute value of any function is determined, it is important to remember that for any given value of x, take the absolute value of y. In other words, the value of x stays the same, and any negative y values will become positive.

Given the point (-1,-4), keep x = -1, and change the y value from -4 to +4. The corresponding point is (-1, 4).

The correct answer is A.

Use the following information to answer the next question.

Consider the linear function below. Both intercepts are integers.



8. A) Determine the equation of y = f(x). Explain.

Solution

Given the fact that both intercepts are integers, we can see from the graph that the x-intercept is (1,0) and the y-intercept is (0, -3). These two points can now be used to determine the slope.

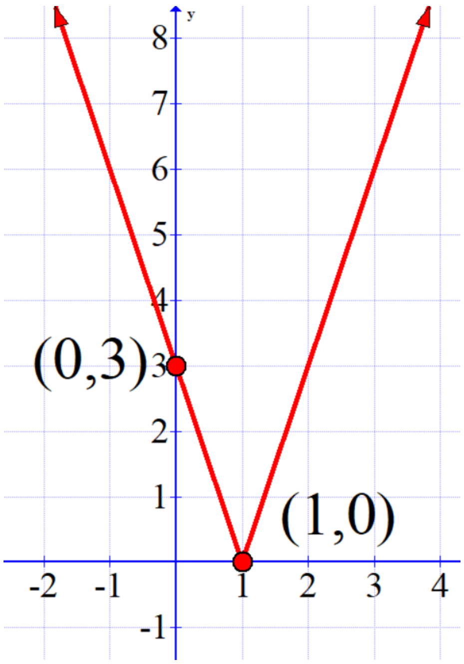
$$slope = \frac{rise}{run}$$

$$slope = \frac{0-(-3)}{1-0} = \frac{3}{1} = 3.$$

The equation of this line in the slope-intercept form is, y = 3x – 3.

B) Sketch the graph of $y = |f(x)|$. Label both intercepts. State any invariant points.

Solution



The y-intercept is (0,3) and the x-intercept is (1,0).

An invariant point remains unchanged when a transformation is applied to it. The x-intercept (1,0) is on both the original linear function, y = 3x – 3, and the absolute value function, y = $|3x - 3|$.

C) Write as a piecewise function. Explain.

At the x-intercept, the original linear function will chart a new course. Any part of the line below the x-axis, will have points containing negative y-values. When taking the absolute value of these negative y-values, they become positive. The line that was below the x-axis has now been reflected in the x-axis, to now move up into quadrant 2.

Thus, the first part of the piecewise notation is y = -(3x – 3), if x < 1. The negative sign in front of the function is used to indicate this reflection.

For any value of x greater than or equal to 1 on the original linear function, the corresponding values for y are positive. Taking the absolute value of these positive y-values does not change the sign, i.e., there are still positive. Thus, the part of the line above the x-axis is the same for both the original linear function, and for its absolute value.

The, the second part of the piecewise notation is y = 3x – 3, if x ≥ 1.

$$y = \begin{cases} 3x - 3, if\ x \geq 1 \\ -(3x - 3), if\ x < 1 \end{cases}$$