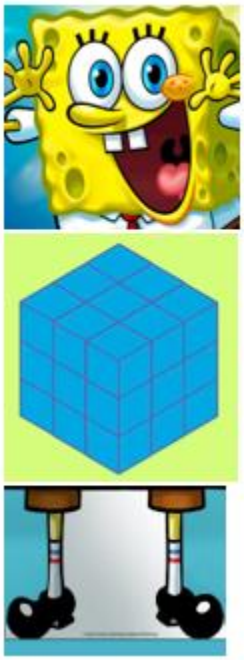
Cube Roots Practice

Use the following information to answer the first question.

Remember our friend, Sponge Bill Cube Pants.



When we first met him, the edge of his cube pants was 3 units (as shown in the picture above). He then lost some weight and the seamstress had to make new pants that had an edge of 2 units. Since that time, he has put some of the weight back on and the volume inside his pants is now 20 cubic units.

1. Which of the following statements is true?

   A) $\sqrt[3]{20}$ is a perfect cube.
   B) The new length of the edge of his cube pants is greater than 3 units.
   C) The new length of the edge of his cube pants is less than 2 units.
   D) The new length of the edge of his cube pants is between 2.5 and 3 units.

2. The value of $\sqrt[3]{-27} + \sqrt[3]{343}$ is ____.

Use the following information to answer the next question.

Consider the prime factorization of the following numbers.

First Number	$2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7$
Second Number	$3 \cdot 3 \cdot 3 \cdot 13 \cdot 13 \cdot 13$
Third Number	$3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 13 \cdot 13 \cdot 13$
Fourth Number	$2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7$

3. The number that is **not** a perfect cube is the

   A) First B) Second C) Third D) Fourth

4. If the volume of a cube is 729 cm^3, then the surface area of the cube is

   A) 9 cm^2 B) 81 cm^2 C) 162 cm^2 D) 486 cm^2

5. Between 1001 and 2500, there are three perfect cubes. The smallest of these can be written in the form $\sqrt[3]{KMMK}$, where K and M are integers. The value of K is ___ and the value of M is ___.

6. The volume of a cube is 91.125 cm^3. If the length of the edge of this cube is increased by 1.5 cm, the volume of this new cube will exceed the volume of the original cube (to the nearest whole number) by

   A) 125 cm^3 B) 144 cm^3 C) 175 cm^3 D) 216 cm^3

7. If the surface area of a cube is 1350 cm^2, then the volume of the cube, in cm^3, is _____.

Use the following information to answer the next question.

Consider the numbers and the statements below.

Number 1	Number 2	Number 3	Number 4	Number 5
729	-125	224	-121	5832

Use the numbers 1, 2, 3, 4, or 5 to fill in the blank below with the best answer. Each number can only be used once.

Statement 1	This number is a perfect cube, but not a perfect square.	___
Statement 2	This number is neither a perfect cube or a perfect square.	___
Statement 3	This number has an undefined square root.	___
Statement 4	This number is both a perfect cube and a perfect square.	___
Statement 5	This number has a negative value for its cube root.	___

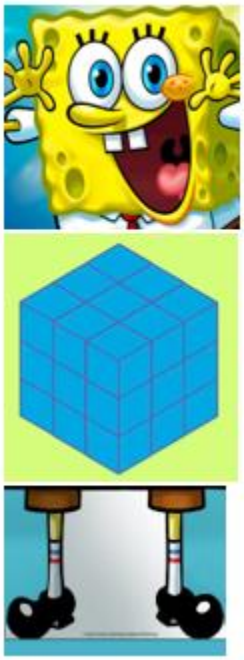
8. The numbers, in correct order, that answer statements one through five, are _____.

9. Without technology, determine the cube root of 9261 using two different methods. Explain.

Cube Roots Practice Solutions

Use the following information to answer the first question.

Remember our friend, Sponge Bill Cube Pants.



When we first met him, the edge of his cube pants was 3 units (as shown in the picture above). He then lost some weight and the seamstress had to make new pants that had an edge of 2 units. Since that time, he has put some of the weight back on and the volume inside his pants is now 20 cubic units.

1. Which of the following statements is true?

   $A)$ $\sqrt[3]{20}$ is a perfect cube.
   B) The new length of the edge of his cube pants is greater than 3 units.
   C) The new length of the edge of his cube pants is less than 2 units.
   D) The new length of the edge of his cube pants is between 2.5 and 3 units.

Solution

The prime factorization of 20 is 2 X 2 X 5. Since there are not identical sets of 3 factors, this is not an example of a perfect cube.

Using technology,

$\sqrt[3]{20} = 2.714\ldots$

The cube root of 20 is not a perfect cube. Statement A is false.

The new length of the edge is 2.714… The new length is not greater than 3 units. Statement B is false.

Statement C is false.

Statement D is true.

The correct answer is D.

2. The value of $\sqrt[3]{-27} + \sqrt[3]{343}$ is _4_.

Solution

The cube root of -27 is -3, because (-3) (-3) (-3) = -27

The cube root of 343 is 7, because (7) (7) (7) = 343.

-3 + 7 = 4

$\sqrt[3]{-27} + \sqrt[3]{343}$ is _4_.

Use the following information to answer the next question.

Consider the prime factorization of the following numbers.

First Number	$2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7$
Second Number	$3 \cdot 3 \cdot 3 \cdot 13 \cdot 13 \cdot 13$
Third Number	$3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 13 \cdot 13 \cdot 13$
Fourth Number	$2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7$

3. The number that is **not** a perfect cube is the

A) First B) Second C) Third D) Fourth

Solution

Once prime factorization is determined, if there are 3 sets of identical factors, it indicates there is a perfect cube.

For the third number, there are three 3's, three 13's, but only two 7's. The third number is not a perfect cube.

The correct answer is C.

4. If the volume of a cube is 729 cm^3, then the surface area of the cube is

   A) 9 cm^2 B) 81 cm^2 C) 162 cm^2 D) 486 cm^2

Solution

To find the surface area, the length of each edge is needed. The cube root of 729 will be the length of each edge; thus, $\sqrt[3]{729} = 9$.

There are 6 identical faces on a cube. The area of each face is 9 X 9, or 81 cm^2.

Multiply the area of one square by 6.

(81) (6) = 486

The correct answer is D.

5. Between 1001 and 2500, there are three perfect cubes. The smallest of these can be written in the form $\sqrt[3]{KMMK}$, where K and M are integers. The value of K is _1_ and the value of M is _3_.

Solution

Since we know that 10^3 = 1000, $\sqrt[3]{1000} = 10$, which is a perfect cube.

11^3 = 1331

12^3 = 1728

13^3 = 2197

14^3 = 2744

Between 1001 and 2500, the perfect cubes are $\sqrt[3]{1331}$, $\sqrt[3]{1728}$, and $\sqrt[3]{2197}$. The smallest of these is $\sqrt[3]{1331}$.

The value of K is 1 and the value of M is 3.

6. The volume of a cube is 91.125 cm^3. If the length of the edge of this cube is increased by 1.5 cm, the volume of this new cube will exceed the volume of the original cube (to the nearest whole number) by

A) 125 cm^3 B) 144 cm^3 C) 175 cm^3 D) 216 cm^3

Solution

Using the calculator, $\sqrt[3]{91.125} = 4.5$.

The length of the edge of this original cube is 4.5.

Increasing the length of the edge by 1.5 cm, means that the edge of the new cube is 6 cm.

The volume of a cube with side lengths of 6 cm is 6^3, or 216 cm^3.

The difference between the volumes of the new cube and the original cube is 216 – 91.125 = 124.875. To the nearest whole number, the difference is 125 cm^3.

The correct answer is A.

7. If the surface area of a cube is 1350 cm^2, then the volume of the cube, in cm^3, is __3375__.

Solution

The surface area of a cube is comprised of 6 identical faces. Using this knowledge we can determine the length of the edge of the cube. Let x = length of the cube. The area of one face is x^2.

$1350 = 6x^2$

Divide both sides of the equal sign by 6.

$225 = x^2$.

Take the square root of both sides.

$$\sqrt{225} = \sqrt{x^2}$$

$15 = x$

The volume is 15^3, or 3375 cm^3.

Use the following information to answer the next question.

Consider the numbers and the statements below.

Number 1	Number 2	Number 3	Number 4	Number 5
729	-125	224	-121	5832

Use the numbers 1, 2, 3, 4, or 5 to fill in the blank below with the best answer. Each number can only be used once.

Statement 1	This number is a perfect cube, but not a perfect square.	_5_
Statement 2	This number is neither a perfect cube or a perfect square.	_3_
Statement 3	This number has an undefined square root.	_4_
Statement 4	This number is both a perfect cube and a perfect square.	_1_
Statement 5	This number has a negative value for its cube root.	_2_

8. The numbers, in correct order, that answer statements one through five, are __53412__.

Solution

$\sqrt[3]{5832} = 18$

$\sqrt{5832} = 76.367\ldots$

The correct answer for statement 1 is # 5.

$\sqrt[3]{224}$ = 6.073…

$\sqrt{224}$ = 14.966…

The correct answer for statement 2 is # 3.

When a negative number is the radicand for a square root, the result is undefined. This is because there is no real number multiplied by itself to get a negative result.

The correct answer for statement 3 is # 4.

$\sqrt[3]{729} = 9$

$\sqrt{729} = 27$

The correct answer for statement 4 is # 1.

$\sqrt[3]{-125} = -5$

The correct answer for statement 5 is # 2.

9. Without technology, determine the cube root of 9261 using two different methods. Explain.

Solution

We can first try guess and test by using benchmarks.

$20^3 = 8000$

$30^3 = 27\ 000$

Since 9261 is between 8000 and 27 000, our number should be between 20 and 30.

It is closer to 20, so let's test 22.

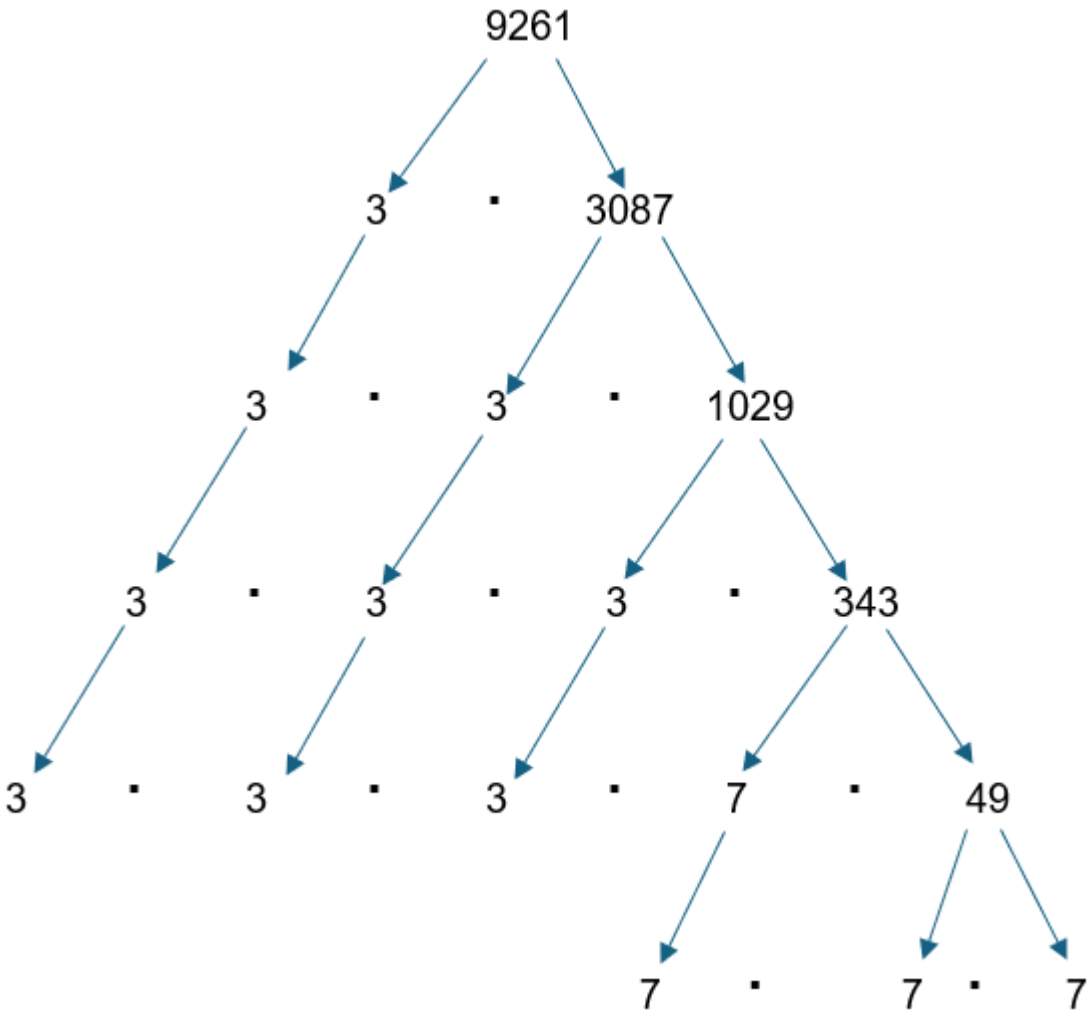
$22^3 = 10\ 648$

Try 21.

$21^3 = 9261$. Thus, $\sqrt[3]{9261} = 21$.

Use a prime factor tree for the second method.

The last digit is odd, so we know that 2 is not a factor. The sum of the digits (9 + 2 + 6 + 1) is 18, and since 3 is a factor of 18, 3 is also a factor of 9261. Thus, we will begin by dividing by 3.



Looking at the prime numbers at the bottom, the prime factorization is (3^3) (7^3). The cube root is found by multiplying 3 and 7. The cube root of 9261 is 21.