Probability Involving Permutations and Combinations Practice

Use the following information to answer the first question.

Sarah, Hannah, and Becky are competing with 5 other girls to be an 800 m runner on the school track team. All girls have an equal chance of winning the trial race.

1. An expression for determining the probability that Sarah, Hannah, and Becky will place 1st, 2nd, and 3rd, in any order, is

   A) $\frac{{}_3P_3}{{}_8P_3}$ B) $\frac{{}_3P_1}{{}_8P_3}$ C) $\frac{{}_3P_3}{{}_5P_3}$ D) $\frac{{}_3P_1}{{}_5P_3}$

2. Ronald buys 3 tickets out of a total of 40 raffle tickets and there are two winning numbers. The probability that Ronald wins exactly 1 prize is

   A) $\frac{1}{40}$ B) $\frac{76}{1560}$ C) $\frac{19}{195}$ D) $\frac{56}{780}$

Use the following information to answer the next question.

A child randomly selects 2 toys from a box of 8, 2 of which are defective.

3. The probability that none are defective, rounded to the nearest hundredth, is _____.

Use the following information to answer the next question.

A group of 4 males and 8 females volunteered for a school improvement committee, which meets with the administration 4 times a year. A specific project requires a sub-group of 7 students.

4. What is the probability, as a decimal rounded to the nearest hundredth, that a 7 member committee will consist of at least 2 males?

   A) 0.85 B) 0.77 C) 0.62 D) 0.55

Use the following information to answer the next question.

You and a friend are trying to decide which of 4 possible movies that you want to watch this evening. Your friend says, “How about I blindfold you and if you put each movie into its correct case, then you get to choose the movie”? You agree.

5. The probability that you will get to choose the movie is

   A) $\frac{1}{4}$ B) $\frac{1}{16}$ C) $\frac{1}{24}$ D) $\frac{1}{40}$

6. Tony has letter tiles that spell INBOX. He has selected 3 of these tiles at random. The probability that he selects 1 vowel and 2 consonants can be written as a reduced fraction in the form, $\frac{K}{M}$, where K and M are integers. The values of K and M, respectively, are ___ and ___.

7. A bank card personal identification number consists of any 4 digits. Repeat digits are allowed and the code can start with zero. The probability that the code will end with the digit 8 is

   A) 0.1 B) 0.25 C) 0.3 D) 0.45

8. City Council consists of 7 women and 6 men. Four representatives are chosen at random to form a sub-committee on low income housing feasibility. If Mayor Janet Smith must be on the committee, what is the probability that exactly 2 men will be on this committee? Show all work. Explain.

Probabilities That Involve Permutations and Combinations Practice Solutions

Use the following information to answer the first question.

Sarah, Hannah, and Becky are competing with 5 other girls to be an 800 m runner on the school track team. All girls have an equal chance of winning the trial race.

1. An expression for determining the probability that Sarah, Hannah, and Becky will place 1st, 2nd, and 3rd, in any order, is

A) $\frac{{}_3P_3}{{}_8P_3}$ B) $\frac{{}_3P_1}{{}_8P_3}$ C) $\frac{{}_3P_3}{{}_5P_3}$ D) $\frac{{}_3P_1}{{}_5P_3}$

Solution

The sample space, or the total amount of outcomes as shown in the denominator, is determined by taking the total number of girls (8) and ordering them 3 at a time. The permutation notation to indicate this is ${}_8P_3$.

The favorable outcomes as shown in the numerator, are all the ways we can order these 3 girls in finishing 1st, 2nd, or 3rd. From a Fundamental Counting Principle perspective, it is like having 3 stages and determining the number of options for each stage.

___ ___ ___

1st 2nd 3rd

3 X 2 X 1 = 6

1st 2nd 3rd

There are 6 ways for the 3 girls to finish in the top 3 in any order. Using permutation notation, it is equivalent to ${}_3P_3$.

The correct answer is A.

2. Ronald buys 3 tickets out of a total of 40 raffle tickets and there are two winning numbers. The probability that Ronald wins exactly 1 prize is

A) $\frac{1}{40}$ B) $\frac{76}{1560}$ C) $\frac{19}{195}$ D) $\frac{56}{780}$

Solution

Method 1

He could win the 1st and lose the 2nd OR he could lose the 1st and win the 2nd

$$\left(\frac{2}{40}\right)\left(\frac{38}{39}\right) + \left(\frac{38}{40}\right)\left(\frac{2}{39}\right)$$

$$\left(\frac{76}{1560}\right) + \left(\frac{76}{1560}\right)$$

$$= \left(\frac{152}{1560}\right)$$

$$= \left(\frac{19}{195}\right)$$

Method 2

$$\frac{(number\ of\ ways\ to\ win\ once)(number\ of\ ways\ to\ lose\ once)}{total\ number\ of\ ways\ to\ choose\ 2\ tickets\ given\ a\ total\ of\ 40}$$

$$\frac{({}_2C_1)({}_{38}C_1)}{{}_{40}C_2}$$

$$= \frac{(2)(38)}{780}$$

$$= \left(\frac{19}{195}\right)$$

The correct answer is C.

Use the following information to answer the next question.

A child randomly selects 2 toys from a box of 8, 2 of which are defective.

3. The probability that none are defective, rounded to the nearest hundredth, is __0.54___.

Solution

If 2 out of 8 are defective, then 6 out of 8 are **not** defective.

$$Probability = \frac{number\ of\ ways\ to\ choose\ 2\ toys\ from\ 6\ non\ defective\ toys}{number\ of\ ways\ to\ choose\ 2\ toys\ from\ a\ total\ of\ 8\ toys}$$

$\frac{{}_6C_2}{{}_8C_2}$

$= \frac{15}{28}$

= 0.5357…

The probability, to the nearest hundredth, is 0.54.

Use the following information to answer the next question.

A group of 4 males and 8 females volunteered for a school improvement committee, which meets with the administration 4 times a year. A specific project requires a sub-group of 7 students.

4. What is the probability, as a decimal rounded to the nearest hundredth, that a 7 member committee will consist of at least 2 males?

A) 0.85 B) 0.77 C) 0.62 D) 0.55

Solution

When seeing the phrase "at least", we should begin to realize that we are likely to be dealing with cases.

What does "at least 2 males" mean in this context? With a total of 4 males in a 7 member committee, it means 2 males or 3 males or 4 males.

Let M = Males

Let F = Females

Case 1	+	Case2	+	Case 3
2M and 5F	+	3M and 4F	+	4M and 3F
$({}_4C_2)({}_8C_5)$	+	$({}_4C_3)({}_8C_4)$	+	$({}_4C_4)({}_8C_3)$

The sample space, or the total number of outcomes for each case is ${}_{12}C_7$, because there are 12 total people and we are forming a committee of 7 people.

Now the probabilities are:

Case 1 + Case2 + Case 3

$\frac{({}_4C_2)({}_8C_5)}{{}_{12}C_7}$ + $\frac{({}_4C_3)({}_8C_4)}{{}_{12}C_7}$ + $\frac{({}_4C_4)({}_8C_3)}{{}_{12}C_7}$

$\frac{(6)(56)}{792}$ + $\frac{(4)(70)}{792}$ + $\frac{(1)(56)}{792}$

$\frac{336}{792}$ + $\frac{280}{792}$ + $\frac{56}{792}$

$= \frac{672}{792}$

$= 0.8484\ldots$

The correct answer is A.

Use the following information to answer the next question.

You and a friend are trying to decide which of 4 possible movies that you want to watch this evening. Your friend says, “How about I blindfold you and if you put each movie into its correct case, then you get to choose the movie”? You agree.

5. The probability that you will get to choose the movie is

$A)\ \frac{1}{4}$ B) $\frac{1}{16}$ C) $\frac{1}{24}$ D) $\frac{1}{40}$

Solution

$$Probability = \frac{favorable\ outcomes}{total\ outcomes}$$

There is only 1 possible way to put all the movies into their correct case. Thus, there is one favorable outcome.

Order is important so we are concerned with a permutation. Given 4 total objects and seeking to order them 4 at a time, the notation needed is ${}_4P_4$.

Using the calculator, ${}_4P_4 = 24$. The total number of outcomes is 24.

The probability of you successfully completing the task is $\frac{1}{24}$. This is also the probability of you getting to choose the movie.

The correct answer is C.

6. Tony has letter tiles that spell INBOX. He has selected 3 of these tiles at random. The probability that he selects 1 vowel and 2 consonants can be written as a reduced fraction in the form, $\frac{K}{M}$, where K and M are integers. The values of K and M, respectively, are _3_ and _5_.

Solution

There are 2 vowels (I and O) and 3 consonants (N, B, and X)

$$probability = \frac{(\#\ of\ ways\ to\ choose\ 1\ vowel\ from\ 2)(\#\ of\ ways\ to\ choose\ 2\ consonants\ from\ 3)}{total\ ways\ to\ choose\ 3\ tiles\ from\ a\ total\ of\ 5\ tiles}$$

$$P = \frac{({}_2C_1)\ ({}_3C_2)}{{}_5C_3}$$

$$P = \frac{(2)(3)}{10}$$

$$P = \frac{6}{10} = \frac{3}{5}$$

Thus, K = 3 and M = 5.

The values of K and M, respectively are, 3 and 5.

7. A bank card personal identification number consists of any 4 digits. Repeat digits are allowed and the code can start with zero. The probability that the code will end with the digit 8 is

A) 0.1 B) 0.25 C) 0.3 D) 0.45

Solution

The sample space is 10 X 10 X 10 X 10, or 10 000.

With the last digit having to be 8, there is only one way for 8 to occupy that spot. The first 3 numbers can be occupied by any one of 10 choices.

The favorable outcomes are 10 X 10 X 10 X 1.

$$Probability = \frac{1000}{10000} = \frac{1}{10}$$

The correct answer is A.

8. City Council consists of 7 women and 6 men. Four representatives are chosen at random to form a sub-committee on low income housing feasibility. If Mayor Janet Smith must be on the committee, what is the probability that exactly 2 men will be on this committee? Show all work. Explain.

Since the Mayor is a women and she must be on the committee, we can lock her into one spot – meaning that there is one less women to choose from and in reality, we need only 3 more people to fill the 4 member committee.

The sample space is now ${}_{12}C_3$.

The favorable outcomes will be the number of ways 2 men can be chosen from a total of 6 and the number of ways 1 woman can be chosen from a total of 6 women.

$$\frac{({}_6C_2)\,({}_6C_1)}{{}_{12}C_3}$$

$$= \frac{(15)(6)}{220}$$

$$= \frac{90}{220} = \frac{9}{22}$$

The probability that there is exactly 2 men on this committee is $\frac{9}{22}$.