Solving Rational Equations That Simplify To A Quadratic Equation Practice

Consider the 4 rational equations below.

Rational Equation I	Rational Equation II
$\frac{3}{x}-\frac{1}{4}=\frac{5x}{12}$	$\frac{15}{x-4}=\frac{4}{2x}+10$
Rational Equation III	**Rational Equation IV**
$\frac{x}{x+5}+\frac{50}{x^2-25}=4$	$\frac{1}{x}+\frac{1}{x-4}=\frac{1}{15}$

Now consider possible expressions used to multiply each term in order to clear the fractions.

E	J
(x – 5)	$4x^2$
F	**K**
15(x)(x - 4)	15(x – 4)
G	**L**
12x	2x(x – 4)
H	**M**
x(x – 4)	(x – 5)(x + 5)

1. Use the letters E, F, G, H, J, K, L, or M to fill in the blank.
   A) Equation I matches with ___.
   B) Equation II matches with ___.
   C) Equation III matches with ___.
   D) Equation IV matches with ___.

Use the following information to answer the next question.

While correctly solving the rational equation $\frac{2x+5}{7}+\frac{8x}{x+1}=4x$ algebraically, a student wrote an equivalent quadratic equation of the form $ax^2 + bx + c = 0$

2. The equivalent quadratic equation could have been

A) $30x^2 + 35x - 7 = 0$
B) $26x^2 - 35x - 5 = 0$
C) $18x^2 - 10 + 4 = 0$
D) $12x^2 + 7x + 22 = 0$

Use the following information to answer the following question.

When solving the rational equation $\frac{c}{12}+\frac{c+3}{3c}=\frac{1}{c}$, the following statements are made.

Statement 1	The extraneous root is c = -4 and the solution is c = 0.
Statement 2	The extraneous root is c = -4 and the solution is c = 4.
Statement 3	The extraneous root is c = 0 and the solution is c = 4.
Statement 4	The extraneous root is c = 0 and the solution is c =-4.

3. The correct statement is

A) 1 B) 2 C) 3 D) 4

4. The solution to the rational equation, $\frac{1}{2x}-\frac{5}{6}=-\frac{1}{3}$, is x = ____.

Use the following information to answer the next question.

Analyze a students' work as she solves

$$\frac{x}{2x-2}-\frac{2}{3x+3}=\frac{5x^2-2x+9}{12x^2-12}$$

Step 1	$\frac{x}{2(x-1)}-\frac{2}{3(x+1)}=\frac{5x^2-2x+9}{12(x+1)(x-1)}$
Step 2	$12(x+1)(x-1)\left[\frac{x}{2(x-1)}-\frac{2}{3(x+1)}=\frac{5x^2-2x+9}{12(x+1)(x-1)}\right]$
Step 3	$[6(x)(x+1)] - [4(2)(x-1)] = [5x^2 - 2x + 9]$
Step 4	$[6x^2 + 6x] - [8x - 8] = [5x^2 - 2x + 9]$
Step 5	$6x^2 - 2x + 8 = 5x^2 - 2x + 9$
Step 6	$x^2 = 1$
Step 7	$x = 1$ or $x = -1$

5. An error was made in step number

A) 4 B) 5 C) 6 D) 7

Use the following information to answer the next question.

Ken and Rob ran a 5 K race where Ken's average speed was 0.03 m/s faster than Rob. Ken finished 24 seconds ahead of Rob. The following equation models this scenario:

$$\frac{5000}{x}-\frac{5000}{x+0.03}=24$$

6. Ken's average speed to the nearest hundredth of a metre per second is

A) 2.49 B) 2.52 C) 3.17 D) 3.20

Use the following information to answer the next question.

A father and son are raking a large portion of grass on their acreage. Together, it takes them 1.2 hours. In comparing their times in the past, when the son works by himself, it takes 1 hour longer than when the father does the raking on his own. An equation to model this situation is:

$$\left(\frac{1}{x}\right)1.2 + \left(\frac{1}{x+1}\right)1.2 = 1, \; x > 0$$

The extraneous root can be written in the form x = -0.K, where K is an integer.

7. The value of K is

A) 2 B) 4 C) 6 D) 8

8. Solve $\frac{x^2}{x-1} = \frac{3x-2}{x-1}$. Show all work. Explain what many might consider to be a common error.

Solving Rational Equations That Simplify To A Quadratic Equation Practice Solutions

Consider the 4 rational equations below.

Rational Equation I	Rational Equation II
$\frac{3}{x} - \frac{1}{4} = \frac{5x}{12}$	$\frac{15}{x-4} = \frac{4}{2x} + 10$
Rational Equation III	**Rational Equation IV**
$\frac{x}{x+5} + \frac{50}{x^2-25} = 4$	$\frac{1}{x} + \frac{1}{x-4} = \frac{1}{15}$

Now consider possible expressions used to multiply each term in order to clear the fractions.

E (x – 5)	**J** $4x^2$
F 15(x)(x - 4)	**K** 15(x – 4)
G 12x	**L** 2x(x – 4)
H x(x – 4)	**M** (x – 5)(x + 5)

1. Use the letters E, F, G, H, J, K, L, or M to fill in the blank.
   A) Equation I matches with _G_.
   B) Equation II matches with _L_.
   C) Equation III matches with _M_.
   D) Equation IV matches with _F_.

Solution

For equation 1, the smallest expression that the denominators (x, 4, and 12) divide evenly into is 12x. **The correct match is G.**

For equation 2, the smallest expression that the denominators ((2x) and (x – 4)) divide evenly into is (2x)(x – 4). **The correct match is L.**

For equation 3, first factor the denominator, $x^2 - 25$. Factored as difference of squares, it is equivalent to (x + 5) (x – 5). The smallest expression that the denominators ((x + 5) and (x + 5)(x – 5)) divide evenly into is (x + 5) (x – 5). **The correct match is M.**

For equation 4, the smallest expression that the denominators ((x) (x – 4) and (15)) divide evenly into is (x) (x – 4) (15). **The correct match is F.**

Use the following information to answer the next question.

While correctly solving the rational equation $\frac{2x+5}{7} + \frac{8x}{x+1} = 4x$ algebraically, a student wrote an equivalent quadratic equation of the form $ax^2 + bx + c = 0$

2. The equivalent quadratic equation could have been

   A) $30x^2 + 35x - 7 = 0$
   B) $26x^2 - 35x - 5 = 0$
   C) $18x^2 - 10 + 4 = 0$
   D) $12x^2 + 7x + 22 = 0$

Solution

To clear the fractions, multiply each of the 3 terms by (7)(x + 1).

$$(7)(x+1)\left[\frac{2x+5}{7} + \frac{8x}{x+1} = 4x\right]$$

$[(2x + 5)(x + 1)] + [(8x)(7)] = [(4x)(7)(x + 1)]$

$[2x^2 + 7x + 5] + [56x] = [28x(x + 1)]$

$[2x^2 + 7x + 5] + [56x] = [28x^2 + 28x]$

$2x^2 + 63x + 5 = 28x^2 + 28x$

$0 = 26x^2 - 35x - 5$

The correct answer is B.

Use the following information to answer the following question.

When solving the rational equation $\frac{c}{12} + \frac{c+3}{3c} = \frac{1}{c}$, the following statements are made.

Statement 1	The extraneous root is c = -4 and the solution is c = 0.
Statement 2	The extraneous root is c = -4 and the solution is c = 4.
Statement 3	The extraneous root is c = 0 and the solution is c = 4.
Statement 4	The extraneous root is c = 0 and the solution is c =-4.

3. The correct statement is

A) 1 B) 2 C) 3 D) 4

Solution

Multiply each of the 3 terms by the smallest expression that the denominators ((12), (3c), and (c)) divide evenly into; which is 12c.

$$12c\left[\frac{c}{12} + \frac{c+3}{3c} = \frac{1}{c}\right]$$

$c^2 + 4(c + 3) = 12$

$c^2 + 4c + 12 = 12$

$c^2 + 4c = 0$

$c(c + 4) = 0$

$c = 0$ or $c = -4$

c = 0 is an extraneous root because it is not part of the domain of the original equation. If c = 0 is allowed, parts of the equation would be undefined and this is not possible.

Verify that c = -4 is a solution.

$$\frac{(-4)}{12} + \frac{(-4)+3}{3(-4)} = \frac{1}{(-4)}$$

$$\frac{(-4)}{12} + \frac{(-1)}{-12} = \frac{1}{(-4)}$$

$$\frac{(-4)}{12} + \frac{(1)}{12} = \frac{-3}{(12)}$$

$$-\frac{3}{12} = -\frac{3}{12}$$

The correct answer is D.

4. The solution to the rational equation, $\frac{1}{2x} - \frac{5}{6} = -\frac{1}{3}$, is x = _1_.

Solution

$$6x\left[\frac{1}{2x} - \frac{5}{6} = -\frac{1}{3}\right]$$

3 – 5x = -2x

3 = 3x

x = 1

Verify.

$$\frac{1}{2(1)} - \frac{5}{6} = -\frac{1}{3}$$

$$\frac{1}{2} - \frac{5}{6} = -\frac{1}{3}$$

$$\frac{3}{6} - \frac{5}{6} = -\frac{2}{6}$$

$$-\frac{2}{6} = -\frac{2}{6}$$

The value for x is 1.

Use the following information to answer the next question.

Analyze a students' work as she solves

$$\frac{x}{2x-2} - \frac{2}{3x+3} = \frac{5x^2 - 2x + 9}{12x^2 - 12}$$

Step 1	$\frac{x}{2(x-1)} - \frac{2}{3(x+1)} = \frac{5x^2 - 2x + 9}{12(x+1)(x-1)}$
Step 2	$12(x+1)(x-1)\left[\frac{x}{2(x-1)} - \frac{2}{3(x+1)} = \frac{5x^2 - 2x + 9}{12(x+1)(x-1)}\right]$
Step 3	$[6(x)(x+1)] - [4(2)(x-1)] = [5x^2 - 2x + 9]$
Step 4	$[6x^2 + 6x] - [8x - 8] = [5x^2 - 2x + 9]$
Step 5	$6x^2 - 2x + 8 = 5x^2 - 2x + 9$
Step 6	$x^2 = 1$
Step 7	$x = 1$ or $x = -1$

5. An error was made in step number

A) 4 B) 5 C) 6 D) 7

Solution

Everything is good up until step 7. Although the roots appear to be correct, when we look at the factored form of the equation in step 1, $x \neq 1$ or -1 as these are non-permissible values. These values are not part of the domain and thus are extraneous.

Step 7 should state that there is no solution.

The correct answer is D.

Use the following information to answer the next question.

Ken and Rob ran a 5 K race where Ken's average speed was 0.03 m/s faster than Rob. Ken finished 24 seconds ahead of Rob. The following equation models this scenario:

$$\frac{5000}{x} - \frac{5000}{x + 0.03} = 24$$

6. Ken's average speed to the nearest hundredth of a metre per second is

A) 2.49 B) 2.52 C) 3.17 D) 3.20

Solution

$$(x)(x + 0.03)\left[\frac{5000}{x} - \frac{5000}{x + 0.03} = 24\right]$$

[(5000)(x + 0.03)] – [(5000)(x)] = [(24)(x)(x + 0.03)]

[5000x + 150] – [5000x] = [(24x) (x + 0.03)]

[5000x + 150] – [5000x] = [($24x^2$ + 0.72x)]

150 = $24x^2$ + 0.72x

0 = $24x^2$ + 0.72x – 150

Use the quadratic formula.

a = 24, b = 0.72, c = -150

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.72) \pm \sqrt{(0.72)^2 - 4(24)(-150)}}{2(24)}$$

$$x = \frac{-(0.72) \pm \sqrt{14400.5184}}{48}$$

$$x = \frac{-(0.72) \pm 120.00216}{48}$$

$$x = \frac{-0.72 + 120.00216}{48} = 2.4850 \ldots$$

$$x = \frac{-0.72 - 120.00216}{48} = -2.515 \ldots$$

The negative root does not make sense in this context.

The solution is x = 2.49. This value represents Rob's average speed.

Since we are asked to find Ken's average speed, we must add 0.03 to 2.49.

Ken's average speed is 2.52 m/s.

The correct answer is B.

Use the following information to answer the next question.

A father and son are raking a large portion of grass on their acreage. Together, it takes them 1.2 hours. In comparing their times in the past, when the son works by himself, it takes 1 hour longer than when the father does the raking on his own. An equation to model this situation is: $\left(\frac{1}{x}\right)1.2 + \left(\frac{1}{x+1}\right)1.2 = 1$, x > 0 The extraneous root can be written in the form x = -0.K, where K is an integer.

7. The value of K is

A) 2 B) 4 C) 6 D) 8

Solution

An equivalent to $\left(\frac{1}{x}\right)1.2 + \left(\frac{1}{x+1}\right)1.2 = 1$, is

$$\left(\frac{1.2}{x}\right) + \left(\frac{1.2}{x+1}\right) = 1$$

Clear the fractions.

$$(x)(x+1)\left[\frac{1.2}{x} + \frac{1.2}{x+1} = 1\right]$$

[1.2(x + 1)] + [1.2(x)] = (1)(x)(x + 1)

1.2x + 1.2 + 1.2x = x^2 + x

2.4x + 1.2 = x^2 + x

0 = x^2 – 1.4x – 1.2

We could graph or use the quadratic formula, but we will factor. First, multiply each term by 10 to remove the decimals.

0 = $10x^2$ – 14x – 12.

Divide out a common 2.

$0 = 2(5x^2 - 7x - 6)$

Decompose the middle term. Re-write (-7x) such that the coefficients multiply to (ac) or -30 and add to (b) or -7. We will re-write is as -10x and 3x.

$0 = 2(5x^2 - 10x + 3x - 6)$

$0 = 2((5x^2 - 10x) + (3x - 6))$

$0 = 2(5x(x - 2) + 3(x - 2))$

$0 = 2(5x + 3)(x - 2)$

$x = 2$

$x = -\frac{3}{5}\ or - \mathbf{0.6}$

Verify x = 2 is correct.

$$\left(\frac{1}{(2)}\right)1.2 + \left(\frac{1}{(2)+1}\right)1.2 = 1$$

$$\left(\frac{1.2}{(2)}\right) + \left(\frac{1.2}{(2)+1}\right) = 1$$

$$\left(\frac{1.2}{2}\right) + \left(\frac{1.2}{3}\right) = 1$$

$$0.6 + 0.4 = 1$$

$$1 = 1$$

The extraneous root is -0.6.

The value of K is 6.

The correct answer is C.

8. Solve $\frac{x^2}{x-1} = \frac{3x-2}{x-1}$. Show all work. Explain what many might consider to be a common error.

Solution

Clear the fractions. The smallest expression that the denominators (i.e. (x – 1)) will divide evenly into is (x – 1).

$$(x-1)\left[\frac{x^2}{x-1} = \frac{3x-2}{x-1}\right]$$

$x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$

(x – 2) (x – 1) = 0

x = 2 or x = 1

At this point, a common error might be that it is assumed there are two solutions without checking their validity.

For the original equation, the domain is x ≠ 1. Thus, the solution x = 1 is not valid. If x = 1 were allowed, we would be dividing by zero and that creates a situation of an expression being undefined.

Now we need to check to see if x = 2 is a valid solution.

Verify x = 2.

$$\frac{(2)^2}{(2)-1} = \frac{3(2)-2}{(2)-1}$$

$$\frac{4}{1} = \frac{6-2}{1}$$

$$\frac{4}{1} = \frac{4}{1}$$

x = 1 is an extraneous root.

The solution is x = 2.