Solving Exponential Equations With No Common Base Practice

Use the following information to answer the first question.

A math student was asked to solve $2^{3x} = 24$. Consider the 4 possible equations when the variable is isolated.

I	$x = \frac{3log24}{log2}$
II	$x = \frac{log24}{3log2}$
III	$x = \frac{3log2}{log24}$
IV	$x = \frac{log2}{3log24}$

1. The correct isolation is

A) I B) II C) III D) IV

Use the following information to answer the next question.

The population of a certain bacteria doubles every 25 minutes. Starting from a single bacterium, the number of bacteria E, present after *m* minutes can be modelled by the formula:

$$E = 2^{\frac{m}{25}}$$

2. To the nearest minute, the time it will take for there to be at **least** 7400 bacteria is ______ min.

Use the following information to answer the next question.

A math student was asked to solve the exponential equation, $6^{x+1} = 50$.
Analyze the student's work below.

Step 1	$\log 6^{x+1} = \log 50$
Step 2	$(x + 1) \log 6 = \log 50$
Step 3	$x + \log 6 = \log 50$
Step 4	$x = \log 50 - \log 6$

3. Unfortunately the work is incorrect. The first error was made in step

A) 1 B) 2 C) 3 D) 4

4. The value of x , rounded to two decimals, in the equation $12 \bullet 3^{(x-4)} = 72$ can be written in the form, x = 5.MN, where M represents the number in the tenths position and N represents the number in the hundredths position. The value of N is

A) 2 B) 3 C) 5 D) 8

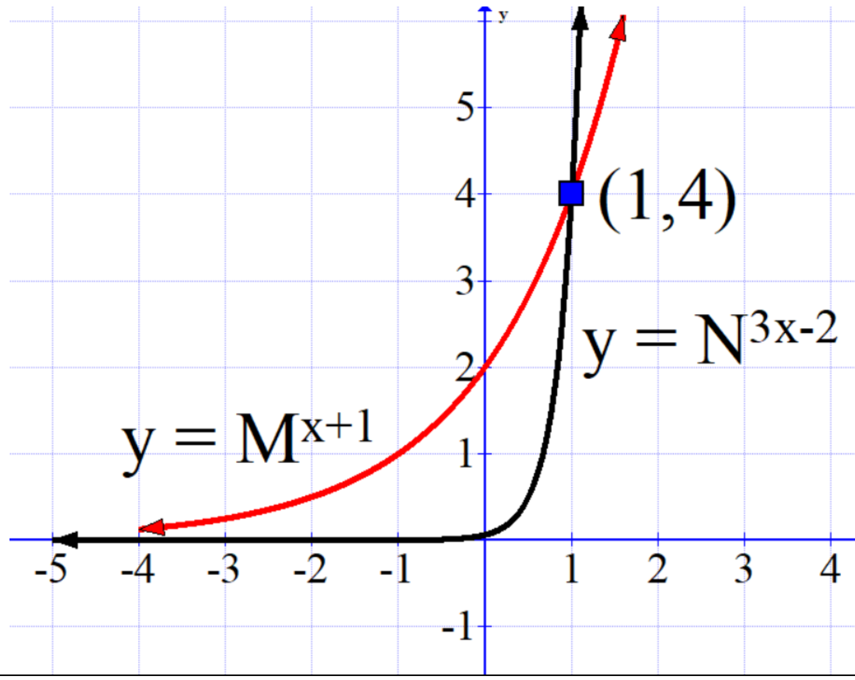
Suppose you were given the equation, $2^{(x+1)} = 3^{(x-2)}$, to solve graphically. You know that you can input $y_1 = 2^{(x+1)}$ and $y_2 = 3^{(x-2)}$ into your graphing calculator. You would also use a window setting that shows both graphs.

5. Which statement below accurately describes how to determine the solution?

A) Determine the largest x-intercept of the two graphs.
B) Determine the largest y-intercept of the two graphs.
C) Determine the point of intersection and state the x-coordinate of this point.
D) Determine the point of intersection and state the y-coordinate of this point.

Use the following information to answer the next question.

A student was asked to solve $M^{x+1} = N^{3x-2}$ graphically. The graph is shown below.



6. The value of M is ____.

Use the following information to answer the next question.

From 2002 the population of a particular city grew at an average rate of 4.3%/a. For the next several years, its population, P, can be modelled by the exponential function

$$P = x(1 + r)^n$$

where x is the population in 2002, r is the average annual growth rate and n is the number of years since 2002. During this period, the population grew from 22 360 to 38 650.

7. According to this model, the population reached 38 650 in the year

A) 2013 B) 2015 C) 2017 D) 2019

8. Solve $5 \bullet 2^{(1-3x)} = 92$, to the nearest thousandth. Show all work.

Solving Exponential Equations With No Common Base Practice Solutions

Use the following information to answer the first question.

A math student was asked to solve $2^{3x} = 24$. Consider the 4 possible equations when the variable is isolated.

I	$x = \frac{3log24}{log2}$
II	$x = \frac{log24}{3log2}$
III	$x = \frac{3log2}{log24}$
IV	$x = \frac{log2}{3log24}$

1. The correct isolation is

A) I B) II C) III D) IV

Solution

Take the log of both sides of the equation.

$\log 2^{3x} = \log 24$

Use the power law to move the exponent to the front of the log.

$(3x) \log 2 = \log 24$

Divide both sides by 3 log 2 to isolate the variable.

$$x = \frac{log24}{3log2}$$

The correct answer Is B.

Use the following information to answer the next question.

The population of a certain bacteria doubles every 25 minutes. Starting from a single bacterium, the number of bacteria E, present after *m* minutes can be modelled by the formula: $$E = 2^{\frac{m}{25}}$$

2. To the nearest minute, the time it will take for there to be at **least** 7400 bacteria is _322_ min.

Solution

Substitute E = 7400

$$7400 = 2^{\frac{m}{25}}$$

Take the log of both sides.

$$\log 7400 = log2^{\frac{m}{25}}$$

Use the power law to move the exponent to the front of the log.

$$log7400 = \left(\frac{m}{25}\right) log2$$

Divide both sides by log 2 to isolate the term with the variable.

$$\frac{log7400}{log2} = \frac{m}{25}$$

$$12.853 = \frac{m}{25}$$

321.332… = m

It will take 322 minutes.

Use the following information to answer the next question.

A math student was asked to solve the exponential equation, $6^{x+1} = 50$.
Analyze the student's work below.

Step 1	$\log 6^{x+1} = \log 50$
Step 2	$(x + 1) \log 6 = \log 50$
Step 3	$x + \log 6 = \log 50$
Step 4	$x = \log 50 - \log 6$

3. Unfortunately the work is incorrect. The first error was made in step

A) 1 B) 2 C) 3 D) 4

Solution

In step 1, the log is taken on each side of the equal sign. This is correct.

In step 2, the exponent is moved to the front of the log due to the application of the Power Law of logarithms. This is correct.

In step 3, the removal of the brackets is incorrect. It should be $x\log 6 + \log 6 = \log 50$.

The correct answer is C.

4. The value of x , rounded to two decimals, in the equation $12 \bullet 3^{(x-4)} = 72$ can be written in the form, x = 5.MN, where M represents the number in the tenths position and N represents the number in the hundredths position. The value of N is

A) 2 B) 3 C) 5 D) 8

Solution

Since there is a number in front of the power, divide both sides by this number to isolate the power. Divide both sides by 12.

$$\frac{12 \cdot 3^{(x-4)}}{12} = \frac{72}{12}$$

$3^{(x-4)} = 6$

Take the log of both sides.

log $3^{(x-4)}$ = log 6

Move the exponent to the front of the law by applying the Power Law.

(x – 4) log 3 = log 6

Remove the brackets by multiplying.

x log 3 – 4 log 3 = log 6

Add 4 log 3 to both sides.

x log 3 = log 6 + 4 log 3

Divide both sides by log 3.

$$x = \frac{\log 6 + 4\log 3}{\log 3}$$

x = 5.6309…

Thus, M = 6 and N = 3.

The correct answer is B.

Suppose you were given the equation, $2^{(x+1)} = 3^{(x-2)}$, to solve graphically. You know that you can input $y_1 = 2^{(x+1)}$ and $y_2 = 3^{(x-2)}$ into your graphing calculator. You would also use a window setting that shows both graphs.

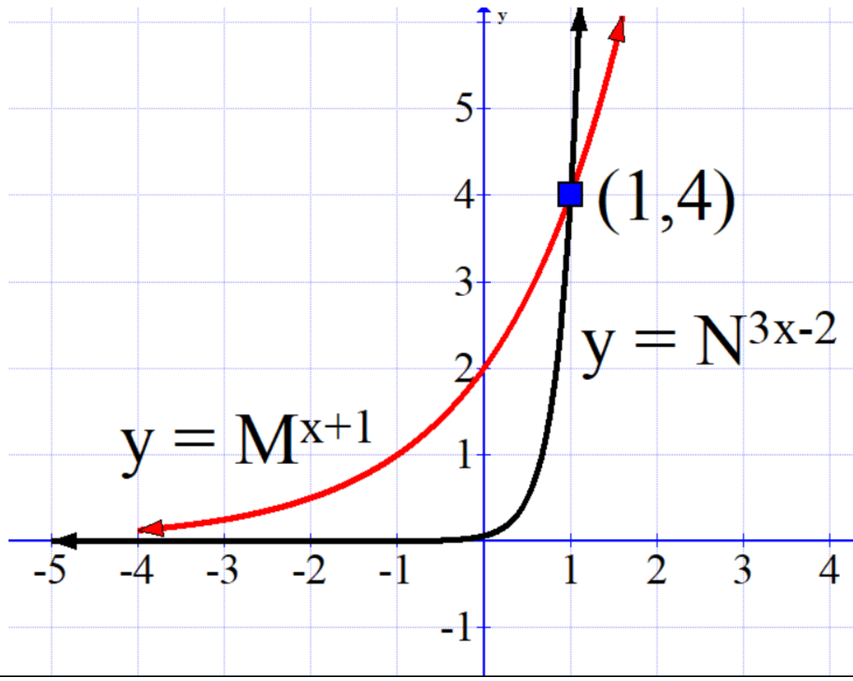
5. Which statement below accurately describes how to determine the solution?

   A) Determine the largest x-intercept of the two graphs.
   B) Determine the largest y-intercept of the two graphs.
   C) Determine the point of intersection and state the x-coordinate of this point.
   D) Determine the point of intersection and state the y-coordinate of this point.

The correct answer is C.

Use the following information to answer the next question.

A student was asked to solve $M^{x+1} = N^{3x-2}$ graphically. The graph is shown below.



6. The value of M is __2__.

Solution

The graphs intersect at the point (1,4). This point must satisfy both equations.

Substitute this point for x and y into the equation having the variable M.

$4 = M^{((1)+1)}$

$4 = M^2$

Take the square root of both sides and maintain the positive value.

$2 = M$.

The value of M is 2.

Use the following information to answer the next question.

From 2002 the population of a particular city grew at an average rate of 4.3%/a. For the next several years, its population, P, can be modelled by the exponential function $P = x(1 + r)^n$ where x is the population in 2002, r is the average annual growth rate and n is the number of years since 2002. During this period, the population grew from 22 360 to 38 650.

7. According to this model, the population reached 38 650 in the year

A) 2013 B) 2015 C) 2017 D) 2019

Solution

Substitute appropriate values into the equation.

Since the average rate of growth is 4.3%/a, the decimal equivalent that goes into the formula is 0.043.

$P = x(1 + r)^n$

$38\,650 = 22\,360(1 + 0.043)^n$

Isolate the power by dividing both sides by 22 360.

$$\frac{38\,650}{22\,360} = \frac{22\,360(1 + 0.043)^n}{22\,360}$$

$$\frac{38\,650}{22\,360} = (1 + 0.043)^n$$

Take the log of both sides and move the exponent to the front of the log.

$$log\left(\frac{38\,650}{22\,360}\right) = n\log(1 + 0.043)$$

$$\frac{log\left(\frac{38\,650}{22\,360}\right)}{\log(1.043)} = n$$

n = 12.998…

Rounded to the nearest integer, n = 13.

We are told in the question that n is the number of years since 2002. When 13 is added to 2002, the year is 2015.

The correct answer is B.

8. Solve 5 • $2^{(1-3x)}$ = 92, to the nearest thousandth. Show all work.

Solution

Since there is a number in front of the power, divide both sides by that number (5), to isolate the power.

$$\frac{5 \cdot 2^{(1-3x)}}{5} = \frac{92}{5}$$

$2^{(1-3x)} = 18.4$

Take the log of both sides and move the exponent to the front of the log.

$(1 – 3x) \log 2 = \log 18.4$

Multiply to remove the brackets.

$\log 2 – 3x \log 2 = \log 18.4$

Subtract log 2 from both sides.

$-3x \log 2 = \log 18.4 – \log 2$

Divide both sides by -3log 2.

$\frac{-3x \log 2}{-3 \log 2} = \frac{\log 18.4 - \log 2}{-3 \log 2}$

$x = \frac{\log 18.4 - \log 2}{-3 \log 2}$

x = -1.0672…

The value of x to the nearest thousandth is -1.067.