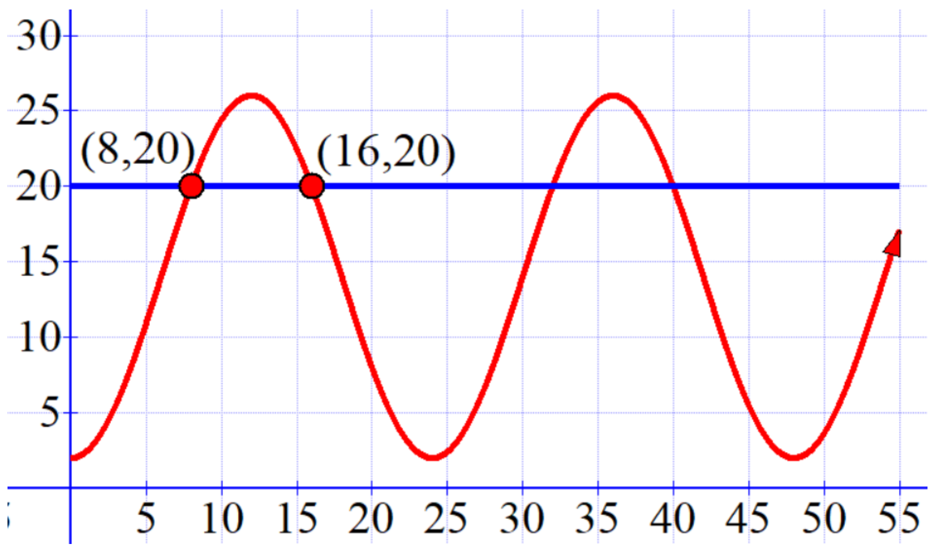
<u>Analyzing Sinusoidal Functions With a Context Practice</u>

Use the following information to answer the first question.

The height of a Ferris Wheel as a function of time can be modelled by the equation $h(t) = 12\,sin\left(\frac{\pi}{12}(t-6)\right) + 14$, where h is the height in metres and t is the time in seconds. Consider the graph of this scenario below.



1. Which statement below accurately describes the above graph?

   A) Within the first revolution of the wheel, the height is below 20 metres for 8 seconds.
   B) Within the first revolution of the wheel, the height is below 20 metres for 16 seconds.
   C) Within the first revolution of the wheel, the height is above 20 metres for 8 seconds.
   D) Within the first revolution of the wheel, the height is above 20 metres for 20 seconds.

2. A function that models the amount of air in the lungs of an average seated adult is $V(t) = -0.37\,cos\frac{\pi}{2}\,t + 0.45,$ where V is the volume in litres and t is the time in seconds. The length of time, to the nearest whole second, that it takes to breathe in and then breathe out is _____ seconds.

Use the following information to answer the next question.

The height of a pendulum, *h*, inches above a table top *t* seconds after the pendulum is released can be modelled by the sinusoidal function,

$$h(t) = 1.5 \sin(3.14t + 1.57) + 4.5.$$

3. The height of the pendulum at the moment of release, to the nearest tenth of an inch, is

   A) 1.5 B) 4.5 C) 3.0 D) 6.0

Use the following information to answer the next question.

Sally and a couple of her friends are playing skip rope. The height of the rope is a function of time and can be modelled by $h(t) = 1.25\, sin\left(\frac{\pi}{2}t\right) + 1.5$, where *h* is the height in metres and *t* is the time in seconds.

4. The minimum height of the rope, in metres, is

   A) 1.25 B) 1.5 C) 0.25 D) 0.20

Use the following information to answer the next question.

The Niagara Skywheel is a 170 ft tall Ferris Wheel in the middle of Clifton Hill, Niagara Falls, Ontario, Canada. It opened on June 17, 2006 at a cost of $10 million. Its 42 fully enclosed passenger cars can each carry 9 people and are heated in the winter and air conditioned in the warmer months. The ride is about 12-15 minutes long. The height of the passenger cars above the ground can be modelled by the sinusoidal function, $y = a \cdot \sin(bx + c) + d$, where x is the time in seconds, after a group of people get into the car.

5. In this context, the y-intercept represents

   A) The height of the car at the highest point above the ground.
   B) The height of the car the moment it starts to move after a group gets in.
   C) The length of time to make 1 complete revolution.
   D) The length of time for the car to reach a maximum height.

Use the following information to answer the next question.

The regular rise and fall of the ocean's water level can limit the times at which ships can enter and exit a harbour. On a given day, the depth of the water in a particular harbour, d, in metres, can be modelled by the sinusoidal function

$$d = 2.35 \sin(0.51t + 4.11) + 13.13$$

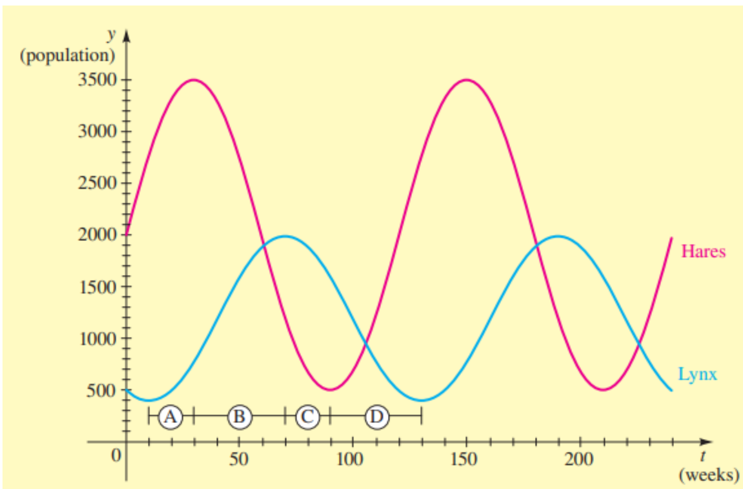
where t is the time in hours after midnight.

6. If a particular ship requires a minimum water depth of 12.85 m to enter the harbour on the given day, the time at which it can enter the harbour for the **second time**, to the nearest hundredth of an hour after midnight, is

A) 10.45 B) 12.76 C) 16.35 D) 18.06

Use the following information to answer the next question.

An isolated island is inhabited by two species mammals; lynx and hares. The lynx are predators that feed on the hares, their prey. The lynx and hare population change cyclically. In part A of the graph below hares are abundant,



so the lynx have plenty to eat and their population increases. By the time portrayed in Part B, so many lynx are feeding on the hares, that their population declines. In Part C, the hare population has declined so much that there is not enough food for the lynx, so the lynx population starts to decrease. In Part D, so many lynx have died that the hares have few enemies and their population increases again.

7. A possible sinusoidal equation that could model the lynx population is

   A) P(t) = a $\cdot$ sin$\left(\frac{\pi}{30}\right)(t-c)+d$
   B) P(t) = a $\cdot$ sin$\left(\frac{\pi}{60}\right)(t-c)+d$
   C) P(t) = a $\cdot$ sin$\left(\frac{\pi}{90}\right)(t-c)+d$
   D) P(t) = a $\cdot$ sin$\left(\frac{\pi}{120}\right)(t-c)+d$

Use the following information to answer the next question.

A gymnast is doing timed bounces on a trampoline. The trampoline mat is 1 metre above the ground. When he bounces up, his feet reach a height of 3 metres *above the mat* and when he bounces down, his feet depress the mat by 0.5 metres. Once in rhythm, his coach uses a stopwatch to record times. A sinusoidal function to model the height of the jump **above the ground** in metres, as a function of time, in seconds is h(t) = 1.75 sin(πt) + 2.25

8. A) Determine the maximum height above the ground. Show work by using the appropriate parameters in the equation.

   B) Once a maximum height is reached, how long is it before the maximum height is reached again? Use the appropriate equation parameter.

   C) How high above the mat was the gymnast when the coach began timing? Explain.

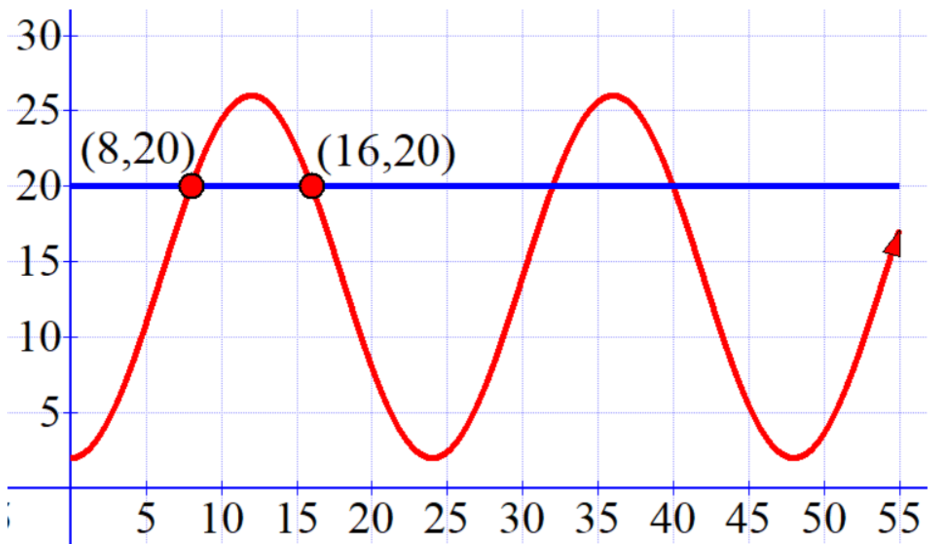
   D) How high above the ground, to the nearest 10^{th}, was the gymnast after 1.8 seconds?

   E) How long after the timing started, to the nearest hundredth, did the gymnast first touch the mat?

Analyzing Sinusoidal Functions With a Context Practice Solutions

Use the following information to answer the first question.

The height of a Ferris Wheel as a function of time can be modelled by the equation $h(t) = 12\,sin\left(\frac{\pi}{12}(t-6)\right) + 14$, where h is the height in metres and t is the time in seconds. Consider the graph of this scenario below.

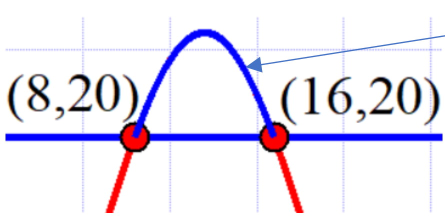


1. Which statement below accurately describes the above graph?

    A) Within the first revolution of the wheel, the height is below 20 metres for 8 seconds.
    B) Within the first revolution of the wheel, the height is below 20 metres for 20 seconds.
    C) Within the first revolution of the wheel, the height is above 20 metres for 8 seconds.
    D) Within the first revolution of the wheel, the height is above 20 metres for 16 seconds.

Solution

A horizontal line is drawn at y = 20. Any value of *y* between x = 8 and x = 16, will be greater than 20, as shown by the blue part of the parabola.



y is greater than 20 here. The difference between 16 and 8 is 8. For 8 seconds, the height is greater than 20. **The correct answer is C.**

2. A function that models the amount of air in the lungs of an average seated adult is $V(t) = -0.37\ cos\frac{\pi}{2}\ t + 0.45$, where V is the volume in litres and t is the time in seconds. The length of time, to the nearest whole second, that it takes to breathe in and then breathe out is __4__ seconds.

Solution

The length of time to breathe in and breathe out would be considered the period. After breathing in and breathing out, the cycle will repeat itself.

The value for 'b' in the equation is $\frac{\pi}{2}$. We know that $Period = \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{2}} = \left(\frac{2\pi}{1}\right)\left(\frac{2}{\pi}\right) = 4$

The length of time, to the nearest whole second, that it takes to breathe in and then breathe out is 4 seconds.

[Note: Math 30-2 is limited to sine functions, but cosine functions behave in a very similar manner]

Use the following information to answer the next question.

The height of a pendulum, *h*, inches above a table top *t* seconds after the pendulum is released can be modelled by the sinusoidal function, h(t) = 1.5 sin (3.14t + 1.57) + 4.5.

3. The height of the pendulum at the moment of release, to the nearest tenth of an inch, is

A) 1.5 B) 4.5 C) 3.0 D) 6.0

Solution

It is important to understand that at the moment of release, no time has elapsed and thus time = 0. Substitute t = 0 into the equation.

h(0) = 1.5 sin (3.14(0) + 1.57) + 4.5.

h(0) = 1.5 sin (1.57) + 4.5.

h(0) = 1.5(0.999…) + 4.5.

h(0) = 1.5 + 4.5.

h(0) = 6

The correct answer is D.

Use the following information to answer the next question.

Sally and a couple of her friends are playing skip rope. The height of the rope is a function of time and can be modelled by $h(t) = 1.25\, sin\left(\frac{\pi}{2}t\right) + 1.5$, where *h* is the height in metres and *t* is the time in seconds.

4. The minimum height of the rope, in metres, is

A) 1.25 B) 1.5 C) 0.25 D) 0.20

Solution

The value of 'd' is 1.5 and the value of 'a' is 1.25.

The minimum value is d – a.

1.5 – 1.25 = 0.25

The correct answer is C.

Use the following information to answer the next question.

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   D) The length of time for the car to reach a maximum height.

Solution

At t = 0, the ride has not yet begun. The y-intercept is (0, y). Thus, y represents the height of the car after the group enters and waiting for the ride to begin. In other words, this is the height of the car the moment it starts to move.

The correct answer is B.

Use the following information to answer the next question.

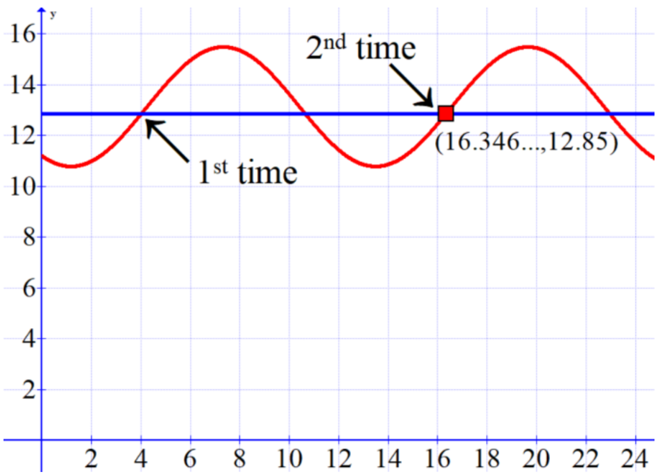
The regular rise and fall of the ocean's water level can limit the times at which ships can enter and exit a harbour. On a given day, the depth of the water in a particular harbour, d, in metres, can be modelled by the sinusoidal function $d = 2.35 \sin(0.51t + 4.11) + 13.13$ where t is the time in hours after midnight.

6. If a particular ship requires a minimum water depth of 12.85 m to enter the harbour on the given day, the time at which it can enter the harbour for the **second time**, to the nearest hundredth of an hour after midnight, is

   A) 10.45 B) 12.76 C) 16.35 D) 18.06

Solution

When given the dependent variable, water depth, and needing to determine the independent variable, time, we look to the graphing calculator.



Input y_1 = 2.35 sin(0.51x + 4.11) + 13.13

Input y_2 = 12.85

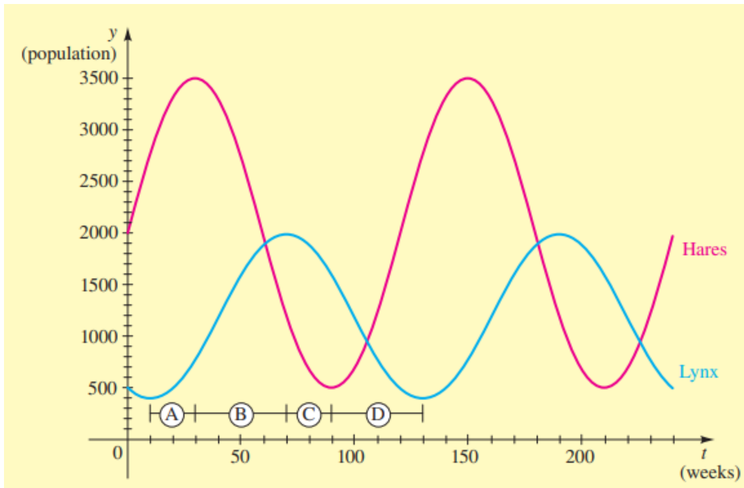
The first time that the ship can enter the harbour is about 4 hours after midnight. In theory, the ship is safe until the sine wave goes below 12.85 m, which is sometime between 10 and 11 hours after midnight.

As the sine wave begins to rise again (meaning that the water depth is rising), the ship can safely enter the harbour for the **second time** at 16.35 hours after midnight.

The correct answer is C.

Use the following information to answer the next question.

An isolated island is inhabited by two species mammals; lynx and hares. The lynx are predators that feed on the hares, their prey. The lynx and hare population change cyclically. In part A of the graph below hares are abundant,



so the lynx have plenty to eat and their population increases. By the time portrayed in Part B, so many lynx are feeding on the hares, that their population declines. In Part C, the hare population has declined so much that there is not enough food for the lynx, so the lynx population starts to decrease. In Part D, so many lynx have died that the hares have few enemies and their population increases again.

7. A possible sinusoidal equation that could model the lynx population is

A) P(t) = a $\cdot \sin\left(\frac{\pi}{30}\right)(t-c)+d$

B) P(t) = a $\cdot \sin\left(\frac{\pi}{60}\right)(t-c)+d$

C) P(t) = a $\cdot \sin\left(\frac{\pi}{90}\right)(t-c)+d$

D) P(t) = a $\cdot \sin\left(\frac{\pi}{120}\right)(t-c)+d$

Solution

The only parameter that changes from the given options is the value for 'b'.

Use the graph to determine the period for the sinusoidal graph representing the lynx population.

The lowest part of the graph to the far left of the graph near the origin is x = 10 weeks.

One complete cycle is completed at x = 130 weeks. The difference between these two values is the period, which is 120 weeks.

$$b = \frac{2\pi}{period} = \frac{2\pi}{120} = \frac{\pi}{60}$$

The correct answer is B.

Use the following information to answer the next question.

A gymnast is doing timed bounces on a trampoline. The trampoline mat is 1 metre above the ground. When he bounces up, his feet reach a height of 3 metres *above the mat* and when he bounces down, his feet depress the mat by 0.5 metres. Once in rhythm, his coach uses a stopwatch to record times. A sinusoidal function to model the height of the jump **above the ground** in metres, as a function of time, in seconds is h(t) = 1.75 sin(πt) + 2.25

8. A) Determine the maximum height above the ground. Show work by using the appropriate parameters in the equation.

Solution

The value of ‘d’ is 2.25 and the value of ‘a’ is 1.75.

The maximum value is (a + d), which is 4.

The maximum height above the ground is 4 m.

B) Once a maximum height is reached, how long is it before the maximum height is reached again? Use the appropriate equation parameter.

Solution

Going from maximum height to the next maximum height is equivalent to the period.

The value of ‘b’ is π.

$Period = \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2.$

Once the maximum height has been reached, it will occur again in 2 seconds.

C) How high above the mat was the gymnast when the coach began timing? Explain.

Solution

This is the point where t = 0. Substitute this value into the equation.

$$h(0) = 1.75 \sin(\pi(0)) + 2.25$$

$$h(0) = 1.75 \sin(0) + 2.25$$

$$h(0) = 1.75(0) + 2.25$$

$$h(0) = 2.25$$

The equation is a function of the height above the ground. Since the mat is 1 m above the ground, subtract 1 m from 2.25 to find the height above the mat.

The gymnast is 1.25 m above the mat when the coach began timing.

D) How high above the ground, to the nearest 10th, was the gymnast after 1.8 seconds?

Solution

Substitute 1.8 for t into the equation.

$$h(1.8) = 1.75 \sin(\pi(1.8)) + 2.25$$

$$h(1.8) = 1.75(-0.587\ldots) + 2.25$$

$$h(1.8) = -1.028\ldots + 2.25$$

$$h(1.8) = 1.221\ldots$$

After 1.8 seconds, the gymnast is 1.22 m above the ground.

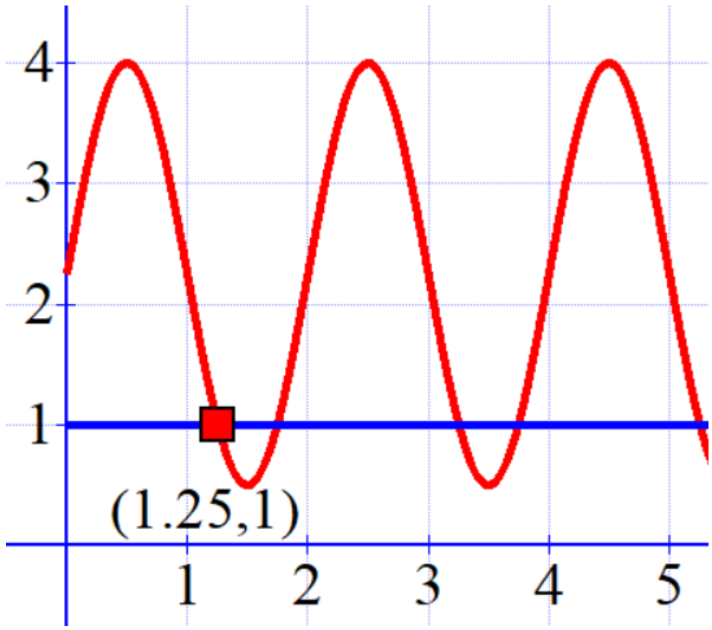
E) How long after the timing started, to the nearest hundredth, did the gymnast first touch the mat?

Solution

Touching the mat would be equivalent to being 1 metre from the ground. Use technology to determine the answer because we are given the dependent variable, height and we asked to find the independent variable time.

Input $y_1 = 1.75 \sin(\pi t) + 2.25$

Input $y_2 = 1$



The x-coordinate of the intersection point is the answer.

The gymnast first touched the mat 1.25 seconds after timing began.