Solving a Trig Equation Using Identity Substitution Practice

Use the following information to answer the first question.

Consider the following 4 trigonometric equations and a possible substitution option into the equation to begin the solving process.

Equation	Possible Substitution
A. $\csc^2 x - 1 = \frac{1}{3}$	1. $1 - 2\sin^2 x$
B. $-3\sin x = \sin 2x$	2. $1 - \cos^2 x$
C. $\cos 2x + \sin x = 1$	3. $2\sin x\cos x$
D. $3\sin^2 x + 4 = -4\cos x$	4. $1 + \cot^2 x$

1. Using the numbers 1-4, the correct substitution options for equations A, B, C, and D, respectively are ___, ___, ___, and ___.

Use the following information to answer the next question.

Analyze the student's work below as he attempts to solve a trigonometric equation using identity substitution. Solve, $9 - 9\cos^2 x = 4\sin x$, in the domain $0^0 \leq x < 360^0$. Round any decimal answers to the nearest tenth.

Step 1	$9(1 - \cos^2 x) = 4\sin x$
Step 2	$9(\sin^2 x) = 4\sin x$
Step 3	$9\sin^2 x - 4\sin x = 0$
Step 4	$\sin x(9\sin x - 4) = 0$
Step 5	$\sin x = 0$ or $\sin x = \frac{4}{9}$
Step 6	$x = 26.4^0$ and 180^0

2. Unfortunately, the student made an error in step

A) 2 B) 4 C) 5 D) 6

3. If the task is to solve the equation, cos 2x = 2 $\cos^2 x$ – $\sin^2 x$, by using an identity substitution, the most likely substitution for the double angle cosine term would be

   A) 2 $\cos^2 x$ – 1 B) 1 – 2 $\sin^2 x$ C) $\cos^2 x$ – $\sin^2 x$ D) None of these

4. Given the trigonometric equation, 2$\sin^2$ x = 2 + cos x, the initial identity substitution **and** one of the solutions in the domain $[0,2\pi)$ are

   A) 2sin x cos x **and** $x = \frac{\pi}{2}$
   B) 2sin x cos x **and** $x = \frac{\pi}{3}$
   C) 1 – $\cos^2 x$ **and** $x = \frac{\pi}{2}$
   D) 1 – $\cos^2 x$ **and** $x = \frac{\pi}{3}$

5. When solving the equation, (sin 2x)(tan x) = 1 algebraically over the domain $[0,2\pi)$, the solution in quadrant 1 can be written in the form x = $\frac{\pi}{K}$, where K is an integer. The value of K is ____.

6. Solve the trig equation, cos 2x – sin x + 2 = 0, algebraically over the domain [0, 360^0). Show all work.

Use the following information to answer the next question.

Jaden was asked by her teacher to solve $2\sec^2 x + \tan^2 x - 3 = 0$, over the domain $[-\pi, 0]$. After an appropriate identity substitution, a simplification was either: ① $\tan^2 x = \frac{1}{2}$, or ② $\tan^2 x = \frac{1}{3}$

7. The correct simplification and the correct solution within the given domain are

   A) ① **and** x = $-\frac{\pi}{6}\ and\ -\frac{5\pi}{6}$
   B) ② **and** x = $-\frac{\pi}{6}\ and\ -\frac{5\pi}{6}$
   C) ① **and** x = $-\frac{\pi}{3}\ and\ -\frac{2\pi}{3}$
   D) ② **and** x = $-\frac{\pi}{3}\ and\ -\frac{2\pi}{3}$

8. Joelene said there are 3 solutions to $8\sin\theta = \sin 2\theta, in\ the\ domain\ [0,2\pi]$ Jack said there are 4 solutions. Who is correct? Explain using an algebraic solution requiring a substitution.

Solving a Trig Equation Using Identity Substitution Practice Solutions

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C. $\cos 2x + \sin x = 1$	3. $2\sin x\cos x$
D. $3\sin^2 x + 4 = -4\cos x$	4. $1 + \cot^2 x$

1. Using the numbers 1-4, the correct substitution options for equations A, B, C, and D, respectively are _4_, _3_, _1_, and _2_.

Solution

Equation A requires a Pythagorean Identity for $\csc^2 x$. Substitution #4 will work for this equation.

Equation B requires a Double Angle sine identity for sin 2x. Substitution #3 will work for this equation.

Equation C requires a Double Angle cosine identity for cos 2x. Substitution #1 will work for this equation.

Equation D requires a Pythagorean Identity for $\sin^2 x$. Substitution #2 will work for this equation.

The correct substitution options for equations A, B, C, and D, respectively are 4, 3, 1, and 2.

Use the following information to answer the next question.

Analyze the student's work below as he attempts to solve a trigonometric equation using identity substitution. Solve, $9 - 9\cos^2 x = 4 \sin x$, in the domain $0^0 \leq x < 360^0$. Round any decimal answers to the nearest tenth.

Step 1	$9(1 - \cos^2 x) = 4 \sin x$
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Step 6	$x = 26.4^0$ and 180^0

2. Unfortunately, the student made an error in step

A) 2 B) 4 C) 5 D) 6

Solution

For step 1, a common factor of 9 is factored out of the first two terms. This is correct.

For step 2, a Pythagorean substitution of ($\sin^2 x$) is made for ($1 - \cos^2 x$). This is correct.

For step 3, the equation is set equal to zero. This is correct.

For step 4, a common factor of sin x is divided out of the two terms. This is correct.

For step 5, the specific ratios are isolated. This is correct.

For step 6, in the given domain, sine is positive in quadrants 1 and 2. The quadrant 2 solution is not given. It should be 153.6^0.

The error occurred in step 6.

The correct answer is D.

3. If the task is to solve the equation, $\cos 2x = 2 \cos^2 x - \sin^2 x$, by using an identity substitution, the most likely substitution for the double angle cosine term would be

A) $2 \cos^2 x - 1$ B) $1 - 2 \sin^2 x$ C) $\cos^2 x - \sin^2 x$ D) None of these

Solution

This question involves a Double Angle cosine term. From the formula sheet, there are 3 possible options for substitution. Since the equation already has a $\cos^2$ and a $\sin^2$, the most likely option is C. By doing so, the ($-\sin^2 x$) terms on each side of the equal sign can be removed, and presenting an equation only in terms of $\cos^2 x$. Either of the other substitutions would result in 2 ratios instead of 1.

The correct answer is C.

4. Given the trigonometric equation, $2\sin^2 x = 2 + \cos x$, the initial identity substitution **and** one of the solutions in the domain $[0,2\pi)$ are

   A) $2\sin x \cos x$ **and** $x = \frac{\pi}{2}$
   B) $2\sin x \cos x$ **and** $x = \frac{\pi}{3}$
   C) $1 - \cos^2 x$ **and** $x = \frac{\pi}{2}$
   D) $1 - \cos^2 x$ **and** $x = \frac{\pi}{3}$

Solution

Options A and B cannot be correct because ($2\sin x \cos x$) is a substitution for a Double Angle sine term($\sin 2x$), which we do not have in this question.

Since this question has a $\sin^2 x$ term and a $\cos x$ term, substituting a Pythagorean Identity of ($1 - \cos^2 x$), will result in an equation in terms of only 1 trig function.

Solve the equation to determine the correct solution.

$2\sin^2 x = 2 + \cos x$

$2(1 - \cos^2 x) = 2 + \cos x$

$2 - 2\cos^2 x - 2 - \cos x = 0$

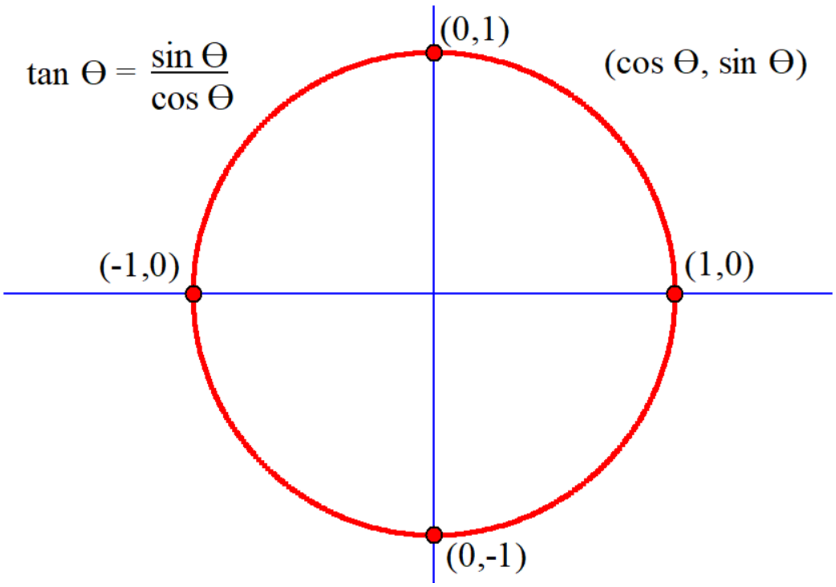
$-2\cos^2 x - \cos x = 0$

$0 = 2\cos^2 x + \cos x$

$0 = \cos x(2\cos x + 1)$

$\cos x = 0$ or $\cos x = -\frac{1}{2}$

For $\cos x = 0$, use the unit circle.



When x = $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, the cosine ratio is 0. These are the solutions for this part of the equation.

For cos x = $-\frac{1}{2}$, find the reference angle. $Cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$. Since cosine is negative in quadrants 2 and 3, the solutions are x = $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

Since all 4 solutions are x = $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ and x = $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$, the only correct solution from the given options is $\frac{\pi}{2}$.

The correct answer is C.

5. When solving the equation, (sin 2x)(tan x) = 1 algebraically over the domain [0,2π), the solution in quadrant 1 can be written in the form x = $\frac{\pi}{K}$, where K is an integer. The value of K is _4_.

Solution

Substitute (2sin x cos x) for (sin 2x).

(2sin x cos x)(tan x) = 1

Now substitute $\left(\frac{sinx}{cosx}\right)$ $for\ tanx$.

(2sin x cos x)($\frac{sinx}{cosx}$) = 1

Simplify.

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

Take the square root of both sides.

$$sinx = \pm\frac{1}{\sqrt{2}} which\ is\ equal\ to\ \pm\frac{\sqrt{2}}{2}$$

The reference angle is $\frac{\pi}{4}$, and there are 4 solutions, $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, and\ \frac{7\pi}{4}$.

The solution in quadrant 1 is $\frac{\pi}{4}$. Thus, K = 4.

The value of K is 4.

6. Solve the trig equation, cos 2x – sin x + 2 = 0, algebraically over the domain $[0, 360^0)$. Show all work.

Solution

There is a Double Angle cosine term. We will substitute $(1 - 2\sin^2 x)$ because we will then have an equation in terms of only sine.

$(1 - 2\sin^2 x) - \sin x + 2 = 0$

$-2\sin^2 x - \sin x + 3 = 0$

An equivalent form is

$2\sin^2 x + \sin x - 3 = 0$

Factor.

$(2\sin x + 3)(\sin x - 1) = 0$

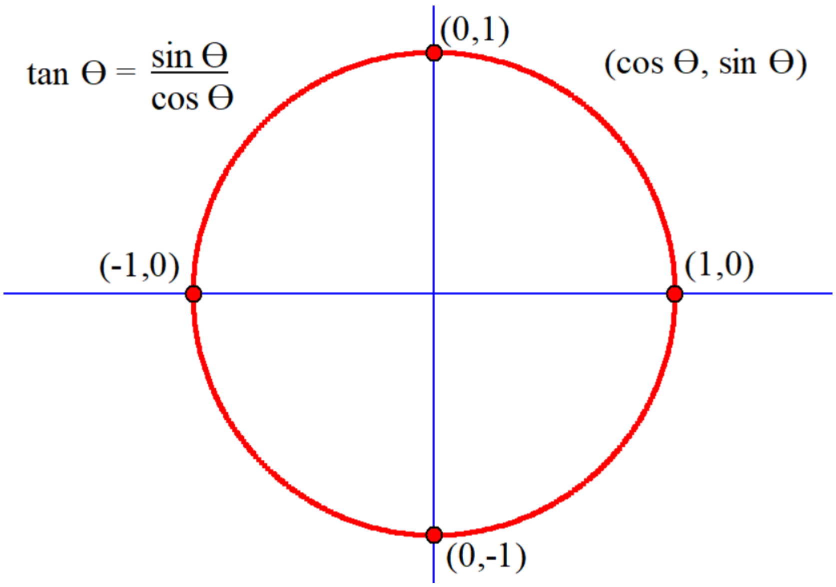
$\sin x = -\frac{3}{2}$ and $\sin x = 1$

Since the sine ratio cannot be greater than the absolute value of 1, there is no solution for $\sin x = -\frac{3}{2}$.

Use the unit circle for sin x = 1.

At 90^0 sine is 1.

Thus, $x = 90^0$.



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① $\tan^2 x = \frac{1}{2}$, or

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A. ① **and** $x = -\frac{\pi}{6}$ *and* $-\frac{5\pi}{6}$
B. ② **and** $x = -\frac{\pi}{6}$ *and* $-\frac{5\pi}{6}$
C. ① **and** $x = -\frac{\pi}{3}$ *and* $-\frac{2\pi}{3}$
D. ② **and** $x = -\frac{\pi}{3}$ *and* $-\frac{2\pi}{3}$

Solution

Use a Pythagorean identity substitution. Be aware of the domain.

Substitute $(1 + \tan^2 x)$ for $(\sec^2 x)$.

$2\sec^2 x + \tan^2 x - 3 = 0$

2 (1 + $\tan^2 x$) + $\tan^2 x$ – 3 = 0

2 + 2$\tan^2 x$ + $\tan^2 x$ – 3 = 0

3$\tan^2 x$ – 1 = 0

$\tan^2 x = \frac{1}{3}$

Take the square root of both sides.

$$tanx = \pm\frac{1}{\sqrt{3}}$$

Find the reference angle.

$$tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Within the given domain of [-$\pi, 0$], there are 2 solutions; $-\frac{\pi}{6}$ and $-\frac{5\pi}{6}$.

The correct answer is B.

8. Joelene said there are 3 solutions to 8sinθ = sin2θ, $in\ the\ domain\ [0,2\pi]$ Jack said there are 4 solutions. Who is correct? Explain using an algebraic solution requiring a substitution.

Solution

Substitute for the Double Angle sine identity.

8sinθ = sin2θ

8sinθ = (2sin θ cos θ)

Set the equation equal to zero.

8sinθ - 2sin θ cos θ = 0

Factor.

2sin θ (4 - cos θ) $= 0$

Either 2sin θ = 0, or 4 - cos θ = 0

$\underline{2\sin\theta = 0}$

Divide by 2

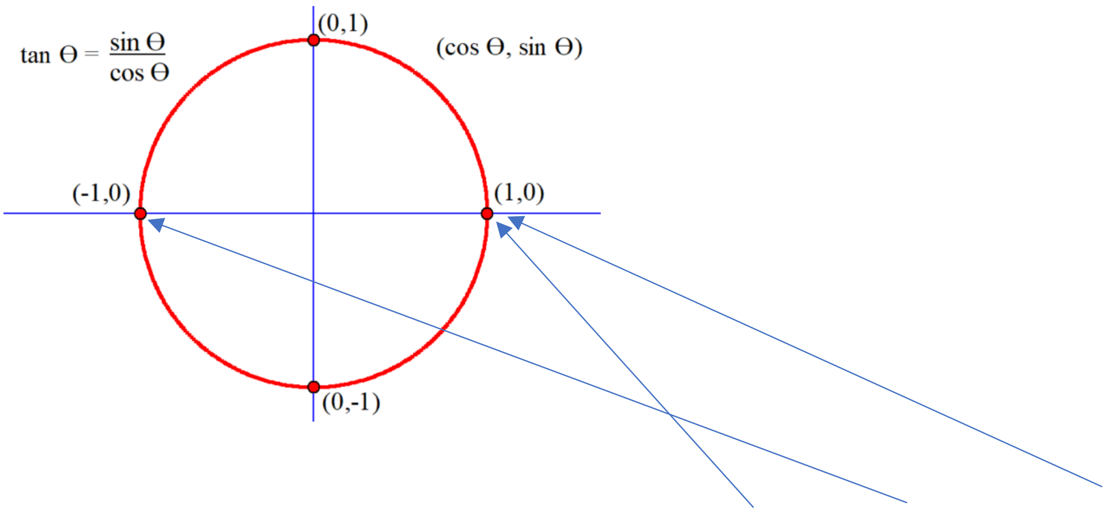
$\sin\theta = 0$

$\underline{4 - \cos\theta = 0}$

Add $\cos\theta$ to both sides

$4 = \cos\theta$

No solution since the cos ratio can't be greater than 1.



Within the given domain of $[0,2\pi]$, sine is zero at 0 radians, π radians, and 2π radians.

There are 3 solutions.

Joelene is correct.