Combining Multiple Logarithmic Laws Practice

1. The simplification of $5log_3x + \frac{log_3x^4}{4} - log_3(xy)$ is

   A) $log_3\left(\frac{x^5}{y}\right)$ B) $log_3\left(\frac{x^{20}}{y}\right)$ C) $log_3\left(\frac{x^5}{y^3}\right)$ D) $log_3\left(\frac{x^{20}}{y^3}\right)$

2. The solution to the logarithmic equation $log_4(x+1) = log_4(5x-1) - 1$ is ______.

3. The simplification of -3log x – log xy is

   A) $-log\left(\frac{x^4}{y}\right)$ B) $\frac{1}{logx^4logy}$ C) $log\left(\frac{1}{x^4y}\right)$ D) $\frac{-logx^4}{logy}$

4. When $mlog_ck^2 - log_ck^m + 4 = q$ is simplified, the result is

   A) $c^{q-4} = k^m$ B) $c^{q-4} = km$ C) $c^{q-4} = \frac{k}{m}$ D) $c^{q-4} = m^k$

5. Given $log_4(x^2-25) - log_4(x+5) + log_4(x-1) = log_4(x^2-kx+m)$, the value for m is

   A) 1 B) 4 C) 5 D) 6

Use the following to answer the next question.

A math student was asked to simplify $log_kk^4 + log_km^{-2} + log_k(k^{16}m^{40})$. Analyze the work below.

Step 1	$log_k(k^4)(m^{-2})(k^{16})(m^{40})$
Step 2	$log_k(k^{20})(m^{38})$
Step 3	$20 + log_k(m^{38})$
Step 4	$20 + 38log_km$
Step 5	$10 + 19log_km$

6. Unfortunately, the work is flawed. An error occurred in step

A) 2　　B) 3　　C) 4　　D) 5

7. As a single logarithm in simplest form, and stating the restrictions on the variable, $log_{13}\frac{x}{\sqrt{x}} + log_{13}\sqrt{x^5} - \frac{4}{3}log_{13}x$ is

A) $\frac{2}{3}log_{13}x$ **and** the restriction is x ≥ 0.

B) $\frac{2}{3}log_{13}x$ **and** the restriction is x > 0.

C) $\frac{5}{3}log_{13}x$ **and** the restriction is x ≥ 0.

D) $\frac{5}{3}log_{13}x$ **and** the restriction is x > 0.

Use the following to answer the next question.

Consider the logarithmic equation:

$$log_7(x-3) + log_7(2x-13) = 2$$

8. A) State the restrictions on the variable.

B) Solve the equation and show all work.

C) State the extraneous root. Explain.

Combining Multiple Logarithmic Laws Practice Solutions

1. The simplification of $5log_3x + \frac{log_3x^4}{4} - log_3(xy)$ is

$A)\ log_3\left(\frac{x^5}{y}\right)$ B) $log_3\left(\frac{x^{20}}{y}\right)$ C) $log_3\left(\frac{x^5}{y^3}\right)$ D) $log_3\left(\frac{x^{20}}{y^3}\right)$

Solution

Rewrite the second term in an equivalent form.

$$5log_3x + \left(\frac{1}{4}\right)log_3x^4 - log_3(xy)$$

Use the Power Law to move the coefficient in front of the log to the exponential position.

$$log_3x^5 + log_3x^{4\left(\frac{1}{4}\right)} - log_3(xy)$$

$$log_3x^5 + log_3x - log_3(xy)$$

Use the Product Law to combine the first two terms.

$$log_3(x^5)(x) - log_3(xy)$$

Use the Quotient Law.

$$log_3\left(\frac{x^6}{xy}\right) = log_3\left(\frac{x^5}{y}\right)$$

The correct answer is A.

2. The solution to the logarithmic equation $log_4(x+1) = log_4(5x-1) - 1$ is __5__.

Solution

Gather the logarithmic terms to one side of the equal sign. This is done so that the terms can be combined into a single logarithmic term.

$$log_4(x+1) - log_4(5x-1) = -1$$

Use the Quotient Law.

$$log_4\left(\frac{x+1}{5x-1}\right) = -1$$

Convert to expontential form.

$$4^{-1} = \left(\frac{x+1}{5x-1}\right)$$

$$\frac{1}{4} = \left(\frac{x+1}{5x-1}\right)$$

Cross multiply.

5x – 1 = 4(x + 1)

5x – 1 = 4x + 4

x = 5

Verify the solution by substituting x = 5 into the original equation.

$$log_4((5)+1) = log_4(5(5)-1) - 1$$

$$log_4(6) = log_4(24) - 1$$

$$1.292\ldots = 1.292\ldots$$

3. The simplification of -3log x – log xy is

$A)\ -log\left(\frac{x^4}{y}\right)$ B) $\frac{1}{logx^4logy}$ C) $log\left(\frac{1}{x^4y}\right)$ D) $\frac{-logx^4}{logy}$

Solution

Use the Power Law to move the coefficient of the first term to the exponential position.

log x^{-3} – log xy

Use the Quotient Law.

$$log\left(\frac{x^{-3}}{xy}\right) = log\frac{1}{(x^3)(x)(y)} = log\left(\frac{1}{x^4y}\right)$$

The correct answer is C.

4. When $mlog_c k^2 - log_c k^m + 4 = q$ is simplified, the result is

A) $c^{q-4} = k^m$ B) $c^{q-4} = km$ C) $c^{q-4} = \frac{k}{m}$ D) $c^{q-4} = m^k$

Solution

Since the two logarithmic terms have the same base, and there is a subtraction sign, it is time for RED ALERT. But before that, use the Power Law to move the coefficient from in front of the first term to the exponential position.

$mlog_c k^2 - log_c k^m + 4 = q$

$= \ log_c k^{2m} - log_c k^m + 4 = q$

Now use the Quotient Law to combine the two logarithmic terms into one term.

$log_c \left(\frac{k^{2m}}{k^m}\right) + 4 = q$

$log_c(k^m) + 4 = q$

$log_c(k^m) = q - 4$

Change to exponential form.

$c^{q-4} = k^m$

The correct answer is A.

5. Given $log_4(x^2 - 25) - log_4(x + 5) + log_4(x - 1) = log_4(x^2 - kx + m)$, the value for m is

A) 1 B) 4 C) 5 D) 6

Solution

Use the Quotient Law first, and then the Product Law, to combine the 3 logarithmic terms on the left side of the equal sign into a single logarithmic expression.

$$log_4 \left(\frac{x^2 - 25}{x + 5}\right) + log_4(x - 1) = log_4(x^2 - kx + m)$$

$$log_4 \left(\frac{(x + 5)(x - 5)}{x + 5}\right) + log_4(x - 1) = log_4(x^2 - kx + m)$$

$$log_4(x-5) + log_4(x-1) = log_4(x^2 - kx + m)$$

$$log_4(x-5)(x-1) = log_4(x^2 - kx + m)$$

Since we have one logarithmic expression on each side of an equal sign having the same base, the arguments must be equal.

(x - 5) (x – 1) = x^2 - kx + m

x^2 - 6x + 5 = x^2 - kx + m

m = 5.

The correct answer is C.

Use the following to answer the next question.

A math student was asked to simplify $log_k k^4 + log_k m^{-2} + log_k(k^{16}m^{40})$. Analyze the work below.

Step 1	$log_k(k^4)(m^{-2})(k^{16})(m^{40})$
Step 2	$log_k(k^{20})(m^{38})$
Step 3	$20 + log_k(m^{38})$
Step 4	$20 + 38log_k m$
Step 5	$10 + 19log_k m$

6. Unfortunately, the work is flawed. An error occurred in step

A) 2 B) 3 C) 4 D) 5

Solution

The first 4 steps are correct.

In step 5, we cannot divide two out of each of the terms. To illustrate, suppose that k = 10 and m = 100.

$$20 + 38log_{10}100$$

=

$$20 + 38log_{10}100$$

$$20 + 38(2)$$

= 96

If we divided out a two, the expression would not be equal to to 96.

$$10 + 19log_{10}100$$

$$10 + 19(2)$$

= 48

The correct answer is D.

7. As a single logarithm in simplest form, and stating the restrictions on the variable, $log_{13}\frac{x}{\sqrt{x}} + log_{13}\sqrt{x^5} - \frac{4}{3}log_{13}x$ is

   $A)$ $\frac{2}{3}log_{13}x$ **and** the restriction is x ≥ 0.

   $B)$ $\frac{2}{3}log_{13}x$ **and** the restriction is x > 0.

   $C)$ $\frac{5}{3}log_{13}x$ **and** the restriction is x ≥ 0.

   $D)$ $\frac{5}{3}log_{13}x$ **and** the restriction is x > 0.

Solution

Move the coefficients of any terms to the exponential position using the Power Law.

$$log_{13}\frac{x}{\sqrt{x}} + log_{13}\sqrt{x^5} - log_{13}x^{\frac{4}{3}}$$

Rewrite the radicals with equivalent exponents.

$$log_{13}\frac{x}{x^{\frac{1}{2}}} + log_{13}x^{\frac{5}{2}} - log_{13}x^{\frac{4}{3}}$$

$$log_{13}x^{\frac{1}{2}} + log_{13}x^{\frac{5}{2}} - log_{13}x^{\frac{4}{3}}$$

Use the Product Law to combine the first two terms.

$$log_{13}\left(x^{\frac{1}{2}}\right)\left(x^{\frac{5}{2}}\right) - log_{13}x^{\frac{4}{3}}$$

$$log_{13}(x^3) - log_{13}x^{\frac{4}{3}}$$

Use the Quotient Law.

$$log_{13}\left(\frac{x^3}{x^{\frac{4}{3}}}\right) = log_{13}x^{\frac{5}{3}}$$

$= \frac{5}{3}log_{13}x$

Given the radical sign, and the fact that there is a radical in a denominator, x must be greater than 0.

The correct answer is D.

Use the following to answer the next question.

Consider the logarithmic equation:
$log_7(x-3) + log_7(2x-13) = 2$

8. A) State the restrictions on the variable.

Solution

It is not possible to take the log of a negative number or zero. Look at the expressions in brackets. In the first term, (x – 3) must be greater than 0. Thus x > 3. In the second term, (2x – 13) must be greater than 0. Thus, x > $\frac{13}{2}$ $or\ 6.5$.

To account for both of these restrictions, we must state that x > 6.5.

B) Solve the equation and show all work.

Solution

Combine the two logarithmic terms into a single term using the Product Law.

$$log_7(x-3)(2x-13)=2$$

Convert from logarithmic form to exponential form.

$7^2 = 2x^2 - 19x + 39$

$0 = 2x^2 - 19x - 10$

$0 = (x - 10)(2x + 1)$

x = 10 and $x = -\frac{1}{2}$

Since x cannot be equal to a real number less than 6.5, x = 10.

C) State the extraneous root. Explain.

Solution

The extraneous root is $x = -\frac{1}{2}$. This number would be a solution simply given the quadratic equation. But in this context of logarithmic equations, it is extraneous because it is not possible to take the logarithm of a negative number.