Range of a Rational Function with a Point of Discontinuity Practice

1. The range of the rational equation $y = \frac{(x+6)(x-1)}{(x-1)}$, $x \neq 1$, is

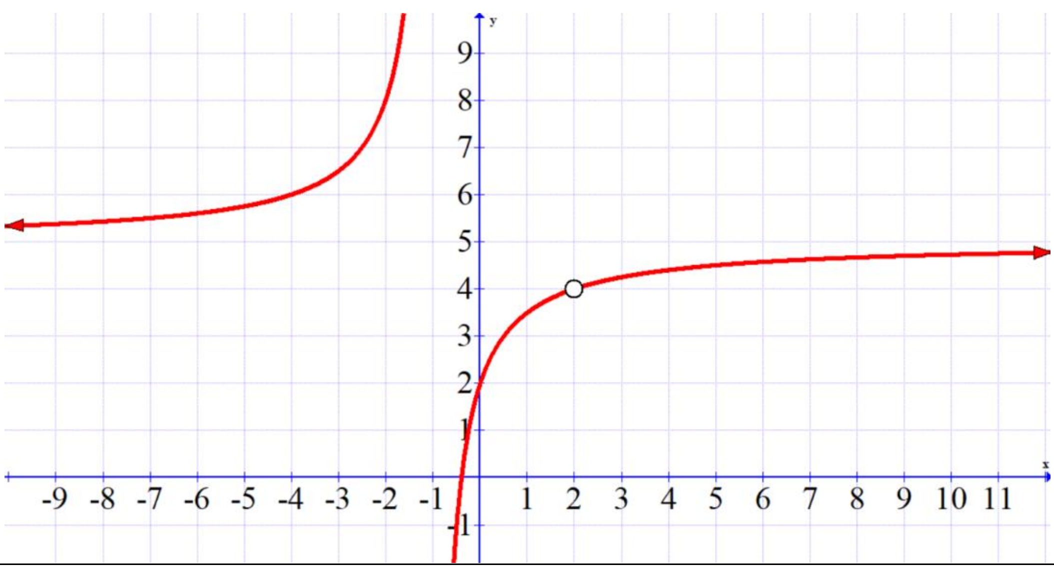
   A) {y| y ≠ 0, 7, y∈ R}
   B) {y| y ≠ 7, y∈ R}
   C) {y| y ≠ 0, 1, y∈ R}
   D) {y| y ≠ 1, y∈ R}

2. The range of the rational function $y = \frac{(3x+4)(x-2)}{x^2-2x}$, $x \neq 0,2$, is

   A) {y| y ≠ 3, 5, y∈ R}
   B) {y| y ≠ 3, y∈ R}
   C) {y| y ≠ 0, 5, y∈ R}
   D) {y| y ≠ 5, y∈ R}

Use the following graph to answer the next question.

The range of the function below is {y| y ≠ 5, K, y ∈ R}, where K is an integer.



3. The value of K is _____.

Use the following information to answer the next question.

Given the following rational function, $f(x) = \frac{x^2+9x-10}{x+10}$, the following statements are made.

Statement 1	The simplification is f(x) = x + 1, x ≠ -10.
Statement 2	The simplification is f(x) = x – 1, x ≠ -10.
Statement 3	The range is {y| y ≠ -11, y ∈ R}
Statement 4	The range is {y| y ≠ -9, y ∈ R}

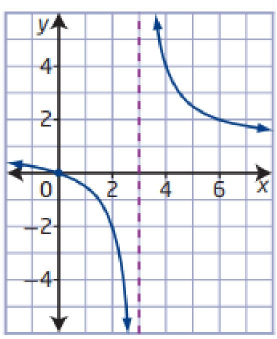
4. The two true statements are

A) 2 and 3 B) 1 and 4 C) 1 and 3 D) 2 and 4

5. Given the rational equation $y = \frac{(3x+7)(x-2)}{(4x-3)(x-2)}$, x ≠ $\frac{3}{4}, 2$, then y cannot be equal to $\frac{3}{4}, or\ \frac{13}{K}$. The value of K is _____.

Use the following information to answer the next question.

I.



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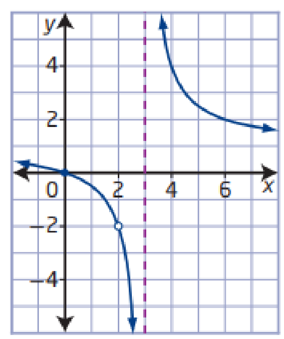

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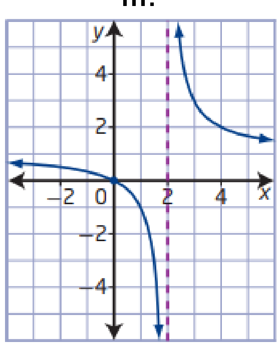

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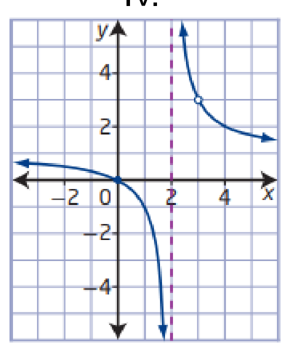


6. The graph having a range of {y| y ≠ 1, -2, y ∈ R} is graph number _____.

Use the following information to answer the next question.

Given the following rational function, $f(x) = \frac{x^2+6x+5}{2x^2+10x}$, x ≠ 0, -5, consider the following statements.

Statement 1	There is a vertical asymptote at x = 5.
Statement 2	There is a horizontal asymptote at y = ½ .
Statement 3	The point of discontinuity is $\left(-5, -\frac{2}{5}\right)$.
Statement 4	The range is {y| y ≠ $\frac{1}{2}, y \in R$}

7. The correct statement is

A) 1 B) 2 C) 3 D) 4

8. The range of the rational equation, $y = \frac{4x-4}{x^2(x-1)}, x \neq 0,1$, is

A) {y| y ≠ 0, 4, y ∈ R}
B) {y| y > 0, y ≠ 4, y ∈ R}
C) {y| y > 0, y ∈ R}
D) {y| y ≠ 4, y ∈ R}

Range of a Rational Function with a Point of Discontinuity Practice Solutions

1. The range of the rational equation $y = \frac{(x+6)(x-1)}{(x-1)}$, $x \neq 1$, is

   A) {y| y ≠ 0, 7, y ∈ R}
   B) {y| y ≠ 7, y ∈ R}
   C) {y| y ≠ 0, 1, y ∈ R}
   D) {y| y ≠ 1, y ∈ R}

Solution

When the common binomial (x – 1) is divided out of the numerator and the denominator to simplify the equation, the result is y = x + 6. This is a linear function that would normally have a domain and range as the set of real numbers. But x ≠ 1, because if it did, the denominator would be undefined.

To find the point of discontinuity, substitute x = 1 into the simplified equation, y = x + 6.

y = (1) + 6

y = 7

The point of discontinuity is (1,7). The only value that is not allowed for y is 7.

The correct answer is B.

2. The range of the rational function $y = \frac{(3x+4)(x-2)}{x^2-2x}$, $x \neq 0,2$, is

   A) {y| y ≠ 3, 5, y∈ R}
   B) {y| y ≠ 3, y∈ R}
   C) {y| y ≠ 0, 5, y∈ R}
   D) {y| y ≠ 5, y∈ R}

Solution

Factor the denominator and simplify.

$$y = \frac{(3x+4)(x-2)}{x(x-2)} = \frac{3x+4}{x}$$

If the values of x were extended to positive or negative infinity, the value of y would approach 3, but never reach it. Thus, there is a horizontal asymptote at y = 3.

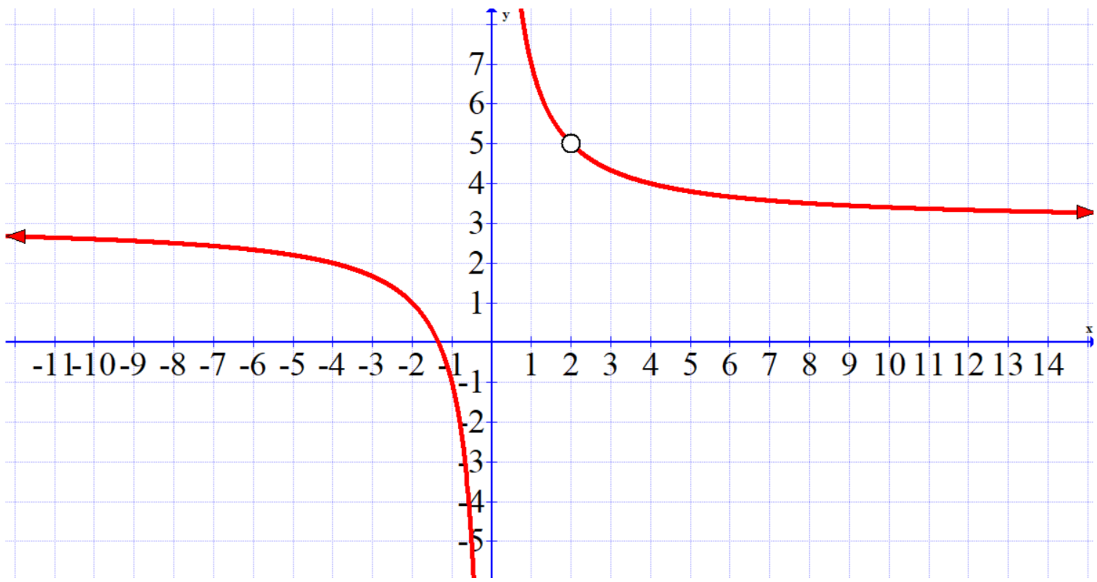
By looking at the denominator prior to simplifying, there is a non-permissible value of x = 2. Substituting x = 2 into the simplified equation,

$$y = \frac{3x + 4}{x}$$

$$y = \frac{3(2) + 4}{(2)} = \frac{10}{2} = 5$$

There is a point of discontinuity at (2,5).

If we are able, it is a good idea to check the shape of the graph.

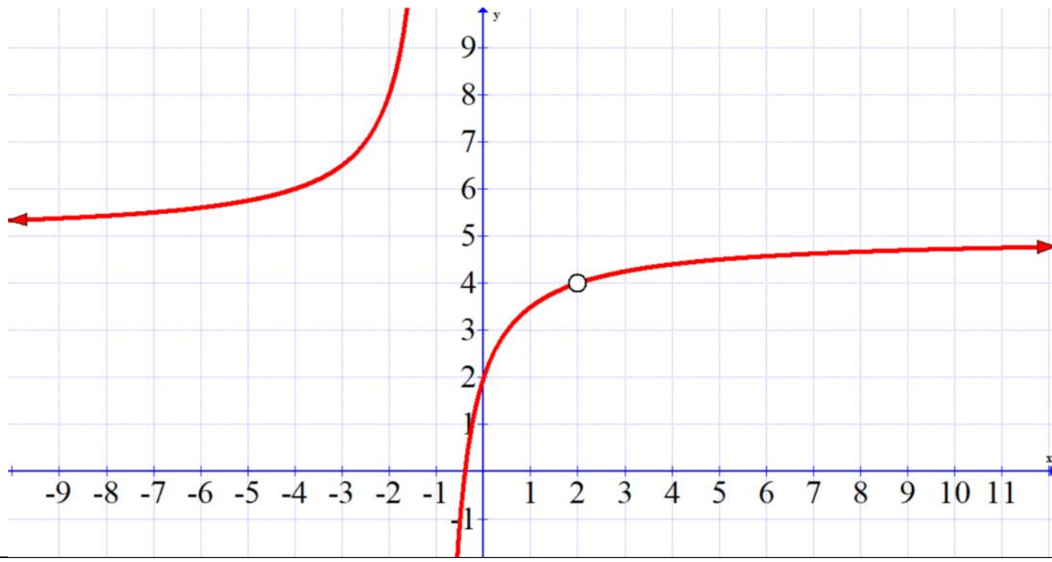


We can see the horizontal asymptote at y = 3, and the point of discontinuity at (2,5). The only 2 values not allowed for *y* are 3 and 5.

The correct answer is A.

Use the following graph to answer the next question.

The range of the function below is {y| y ≠ 5, K, y ∈ R}, where K is an integer.



3. The value of K is ___4___.

Solution

The point of discontinuity is shown on the graph at (2,4). When x = 2 on the original function, the denominator of that function would be undefined. Thus, when x = 2, 4 cannot be a part of the range.

The value of K is 4.

Use the following information to answer the next question.

Given the following rational function, $f(x) = \frac{x^2+9x-10}{x+10}$, the following statements are made.

Statement 1	The simplification is f(x) = x + 1, x ≠ -10.
Statement 2	The simplification is f(x) = x – 1, x ≠ -10.
Statement 3	The range is {y| y ≠ -11, y ∈ R}
Statement 4	The range is {y| y ≠ -9, y ∈ R}

4. The two true statements are

A) 2 and 3 B) 1 and 4 C) 1 and 3 D) 2 and 4

Solution

The factoring and simplification is

$$f(x) = \frac{x^2+9x-10}{x+10} = \frac{(x-1)(x+10)}{(x+10)} = x - 1$$

To find the range, we need the point of discontinuity. There is a vertical asymptote at

x = -10 and therefore -10 is a non-permissible value. Substitute x = -10 into the simplified equation.

f (-10) = (-10) – 1

f (-10) = -11

The point of discontinuity is (-10, -11).

Since the simplification of this function is linear, there is no horizontal asymptote. The only value *y* cannot be equal to is -11.

The true statements are #2 and #3.

The correct answer is A.

5. Given the rational equation $y = \frac{(3x+7)(x-2)}{(4x-3)(x-2)}$, x ≠ $\frac{3}{4}, 2$, then *y* cannot be equal to $\frac{3}{4}, or\ \frac{13}{K}$. The value of K is _5_.

Solution

After simplifying the equation when the common binomial is divided out of the numerator and the denominator, we have $y = \frac{(3x+7)}{(4x-3)}$.

When the degree in the numerator and the degree in the denominator is the same (degree of 1 in this example), the horizontal asymptote is equal to the division of the coefficients on the degree terms. In this example, it is $\frac{3}{4}$. The horizontal asymptote is y = $\frac{3}{4}$. This is the reason that y cannot be equal to $\frac{3}{4}$.

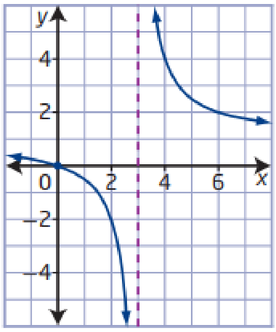
Since x ≠ 2, substitute x = 2 into the simplified equation $y = \frac{(3x+7)}{(4x-3)}$, to find the y-coordinate.

$$y = \frac{(3(2)+7)}{(4(2)-3)} = \frac{13}{5}$$

The value of K is 5.

Use the following information to answer the next question.

I.



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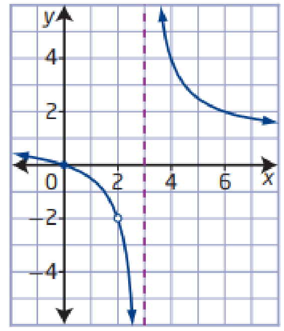

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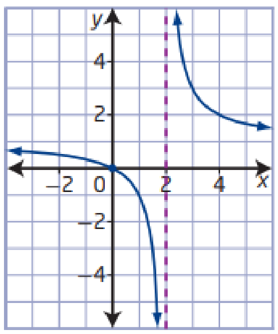

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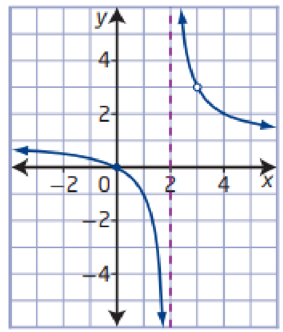


6. The graph having a range of $\{y|\ y \neq 1, -2,\ y \in R\}$ is graph number __II__.

Solution

The horizontal asymptote for each graph is y = 1. This is why one of the values that y cannot be is 1.

Graphs I and III have no points of discontinuity so we can dismiss these as possible answers.

Reading off the y-axis, the y-coordinate of the point of discontinuity for graph IV is 3.

Reading off the y-axis, the y-coordinate of the point of discontinuity for graph II is -2.

The correct graph number is II.

Use the following information to answer the next question.

Given the following rational function, $f(x) = \frac{x^2+6x+5}{2x^2+10x}$, x ≠ 0, -5, consider the following statements.

Statement 1	There is a vertical asymptote at x = 5.
Statement 2	There is a horizontal asymptote at y = ½ .
Statement 3	The point of discontinuity is $\left(-5, -\frac{2}{5}\right)$.
Statement 4	The range is {y| y $\neq \frac{1}{2}, y \in R\}$

7. The correct statement is

A) 1 B) 2 C) 3 D) 4

Solution

Begin by factoring and simplifying.

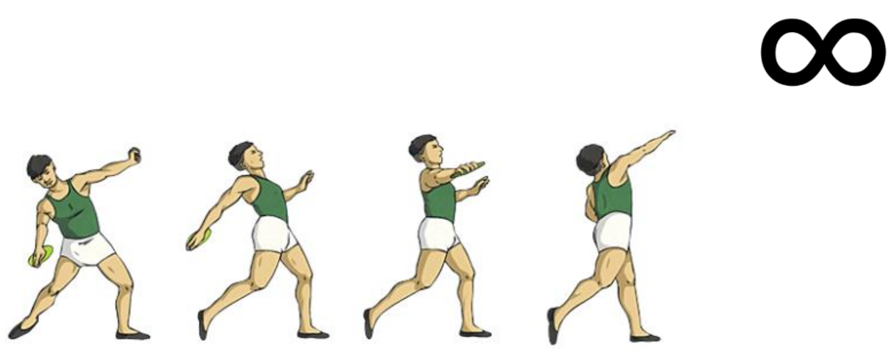
$f(x) = \frac{(x+1)(x+5)}{2x(x+5)} = \frac{x+1}{2x}$, x ≠ 0, -5

There is a vertical asymptote at x = -5. Thus, statement 1 is **false.**

There are a few different ways to determine the horizontal asymptote. I will mention 2 ways here.

Method 1

Using the simplified equation, $f(x) = \frac{x+1}{2x}, x \neq 0{,}5,$ if we "threw" the x-values to positive and negative infinity,



all that would be left is y = ½. This is the horizontal asymptote.

Method 2

Using knowledge of the degrees of the polynomials in the numerator and the denominator. In this example, since the degrees are the same (both first degree), the asymptote is found by dividing the numerical coefficients of the highest degree terms. The coefficients are 1 and 2, and when divided, we get ½.

Statement 2 is **true.**

To find the point of discontinuity, substitute x = -5 into the simplified equation.

$$f(-5) = \frac{(-5)+1}{2(-5)} = \frac{-4}{-10} = \frac{2}{5}$$

The point of discontinuity is $\left(-5, \frac{2}{5}\right)$. Statement 3 is **false.**

The range of the graph will be y not equal to the horizontal asymptote and the y-coordinate of the point of discontinuity. The range is $\{y \mid y \neq \frac{1}{2}, \frac{2}{5}, y \in R\}$. Statement 4 is **false.**

The correct answer is B.

8. The range of the rational equation, $y = \frac{4x-4}{x^2(x-1)}, x \neq 0,1$, is

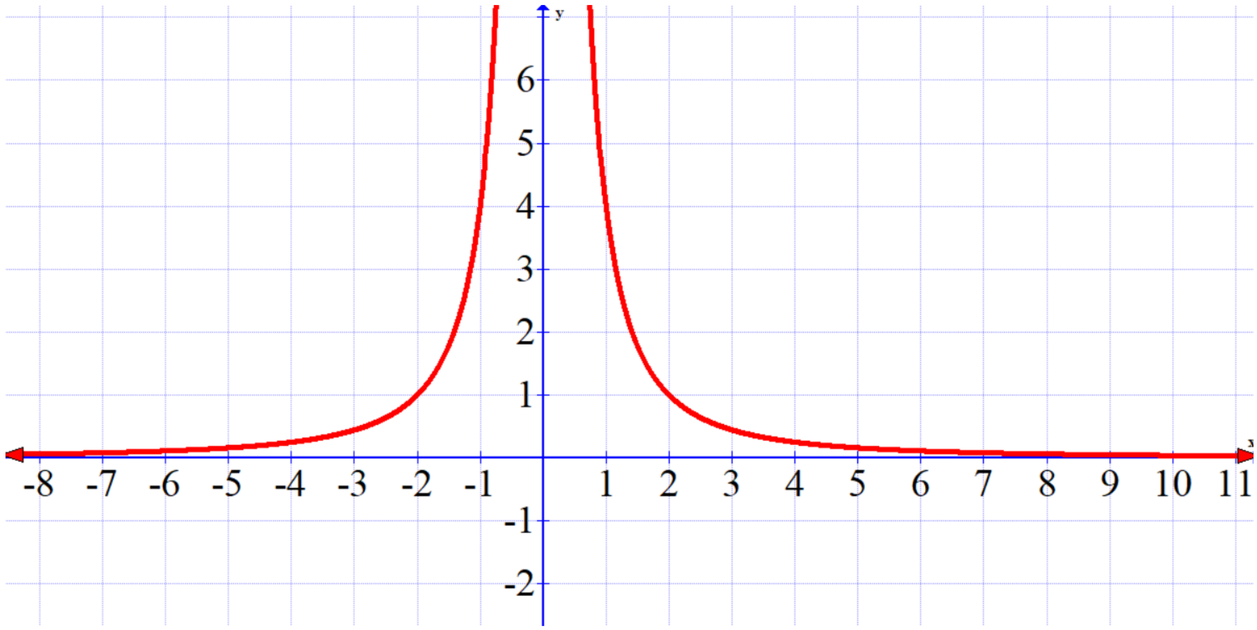
   A) $\{y \mid y \neq 0, 4, y \in R\}$
   B) $\{y \mid y > 0, y \neq 4, y \in R\}$
   C) $\{y \mid y > 0, y \in R\}$
   D) $\{y \mid y \neq 4, y \in R\}$

Solution

Factor and simplify.

$$y = \frac{4(x-1)}{x^2(x-1)} = \frac{4}{x^2}$$

In most cases, when determining the range, having a visual graph of the function is very useful.



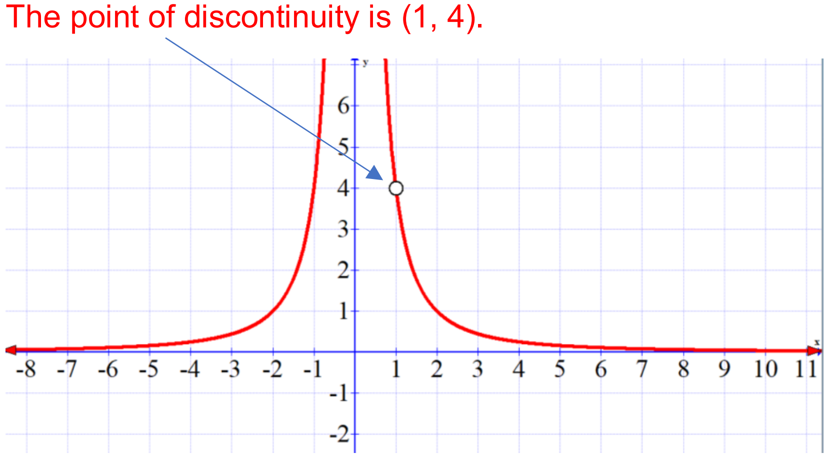
Since x ≠ 0, we see that there is a vertical asymptote at x = 0.

There is a horizontal asymptote at y = 0. There are no values for y less than or equal to zero. This makes sense when observing the equation, $y = \frac{4}{x^2}$. Whether x is positive or negative, squaring it will always result in a positive answer. And then dividing 4 by any positive number, will also yield a positive number. This tells us the y > 0.

Now we need the y-coordinate of the point of discontinuity. Since x ≠ 1, substitute x = 1 into the simplified equation.

$$y = \frac{4}{(1)^2} = \frac{4}{1} = 4.$$

The point of discontinuity is (1, 4).



The correct answer is B.