Domain After the Division of Two Functions Practice

1. If f(x) = $\sqrt{x-4}$ and g(x) = $\sqrt{x-2}$, then the domain of h(x) = $\frac{f(x)}{g(x)}$ is

   A) {x| x ≥ 2, x ϵ R}
   B) {x| x ≥ 4, x ϵ R}
   C) {x| x ≥ 2, x ≠ 4, x ϵ R}
   D) {x| x ≥ 4, x ≠ 6, x ϵ R}

Use the following information to answer the next question.

Given the functions: f(x) = x + 1, g(x) = 2^x, h(x) = $\frac{f(x)}{g(x)}$, and j(x) = $\frac{g(x)}{f(x)}$, consider the following statements.

Statement 1	The domains of h(x) and g(x) are the same.
Statement 2	The value x = -1 is part of the domain of j(x).
Statement 3	The domain of h(x) is {x| x ϵ R}
Statement 4	The domain of j(x) is {x| x $\neq 0$ x ϵ R}

2. The correct statement is

   A) 1 B) 2 C) 3 D) 4

Use the following information to answer the next question.

Given the logarithmic function f(x) = $\log_2 x$, and the linear function g(x) = x – 3, the domain of h(x) = $\left(\frac{f}{g}\right)x$, can be written in the form, {x| x > K, x ≠ M, x ϵ R}, where K and M are integers.

3. The value of K is ____ and the value of M is ____.

4. The domain of h(x) = $\frac{\sqrt{25-x^2}}{x-4}$ is

A) {x| x $\neq 4, x \in R$}
B) {x| $-5 \leq x \leq 5, x \in R$}
C) {x| $-5 \leq x \leq 5, x \neq 4, x \in R$}
D) {x| $-25 \leq x \leq 25, x \neq 4, x \in R$}

Use the following information to answer the next question.

The domain of h(x) = $\frac{f(x)}{g(x)}$ is {x| x $\neq -7, 7$, x ϵ R}

Consider the possible functions for f(x) and g(x).

For f(x)	**For g(x)**
① f(x) = 3x + 21	③ g(x) = x - 49
② $f(x) = \sqrt{3x+21}$	④ g(x) = x^2 - 49

5. Which statement below is true?

A) The function f(x) is ① and the function g(x) is ③.
B) The function f(x) is ① and the function g(x) is ④.
C) The function f(x) is ② and the function g(x) is ③.
D) The function f(x) is ② and the function g(x) is ④.

6. Given f(x) = $\sqrt{x} - 1$, and g(x) = 2x, the domain of h(x) = $\frac{f(x)}{g(x)}$ can be written in the form {x| x > K, x ∈ R}, where K is an integer. The value of K is ____.

Use the following information to answer the next question.

Consider the function $j(x) = \frac{\frac{1}{x+2}}{\frac{1}{x}}$

7. A) Simplify this complex rational expression.

   B) Identify the domain before and after the simplification to show they are different.

   C) What is the final domain of j(x)? Explain.

Use the following to answer the next question.

Anaylze the domains of the following functions.

Function I	y = 3^x - 1
Function II	y = $\sqrt{x+1}$
Function III	y = $\log_2 x$
Function IV	y = x^2

A new function $h(x) = \frac{f(x)}{g(x)}$ is formed where the domain of h(x) is {x| x > 0, x∈ R}.

8. If g(x) is function IV, which function, f(x), is the numerator of h(x)? Explain.

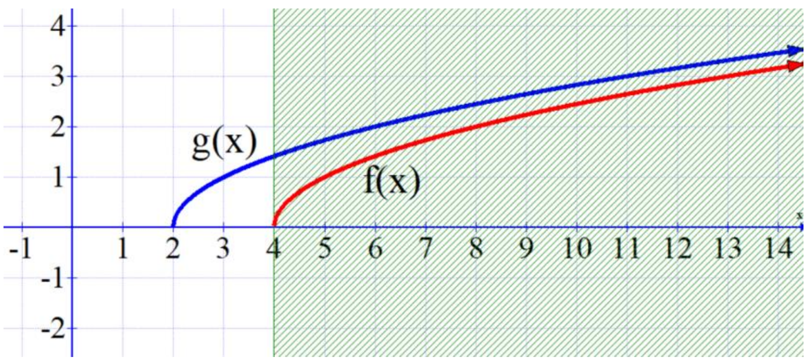
Domain After the Division of Two Functions Practice Solutions

1. If f(x) = $\sqrt{x-4}$ and g(x) = $\sqrt{x-2}$, then the domain of h(x) = $\frac{f(x)}{g(x)}$ is

   A) {x| x ≥ 2, x ϵ R}
   B) {x| x ≥ 4, x ϵ R}
   C) {x| x ≥ 2, x ≠ 4, x ϵ R}
   D) {x| x ≥ 4, x ≠ 6, x ϵ R}

Solution

The domain common to both functions, or the overlap, is x greater than or equal to 4, as shown below.



In terms of taking the domain of the denominator into account, on its own, x ≥ 2. But as we see from the graph, for this quotient, it can’t be any number between x = 2 and x = 4. Therefore, the domain of this quotient is the overlap shown in the graph.

The correct answer is B.

Use the following information to answer the next question.

Given the functions: f(x) = x + 1, g(x) = 2^x, h(x) = $\frac{f(x)}{g(x)}$, and j(x) = $\frac{g(x)}{f(x)}$, consider the following statements.

Statement 1	The domains of h(x) and g(x) are the same.
Statement 2	The value x = -1 is part of the domain of j(x).
Statement 3	The domain of h(x) is {x| x ϵ R}
Statement 4	The domain of j(x) is {x| x $\neq$ 0 x ϵ R}

2. The correct statement is

A) 1 B) 2 C) 3 D) 4

Solution

Statement 1

The function f(x) is linear and has a domain of x ∈ R. The function g(x) is exponential and has a domain of x ∈ R.

For the quotient function h(x), when g(x) is in the denominator, the domain is x ∈ R, since there is no value for x that would make y = 0.

For the quotient function j(x), when f(x) is in the denominator, there is a value for x that would make y = 0. When x = -1, y = 0. Thus, the domain of j(x) is x ≠ -1.

Their domains are different and thus statement 1 is **false**.

Statement 2

If x = -1 is part of the domain for j(x), then the denominator would be equal to zero. This is not allowed because the equation would be undefined.

Statement 2 is **false**.

Statement 3

Since the domains of f(x) and g(x) are both real numbers, and there is no restricted value for the denominator, the domain of h(x) is {x| x ϵ R}.

Statement 3 is **true**.

Statement 4

The domain of j(x) is {x| x $\neq -1$ x ϵ R}

Statement 4 is **false.**

The correct answer is C.

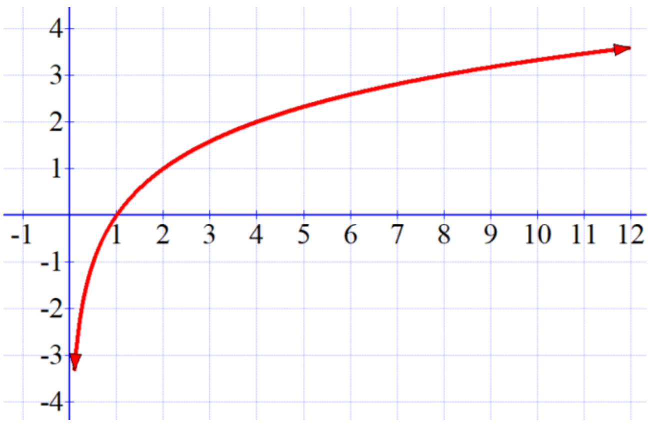
Use the following information to answer the next question.

Given the logarithmic function f(x) = $\log_2 x$, and the linear function g(x) = x – 3, the domain of h(x) = $\left(\frac{f}{g}\right)x$, can be written in the form, {x| x > K, x ≠ M, x ϵ R}, where K and M are integers.

3. The value of K is _0_ and the value of M is _3_.

Solution

The domain of the logarithmic function is x > 0, as shown in the graph below.



The linear function has a domain of x ∈ R. The domain common to the two functions is

x > 0. Next, we have to be concerned about values of the variable that would make the denominator equal to zero.

With the linear function in the denominator, we know that x – 3 ≠ 0. Thus, x ≠ 3.

The value of x must be greater than zero, but it cannot be equal to 3. The domain is written as {x| x > **0**, x ≠ **3**, x ϵ R}.

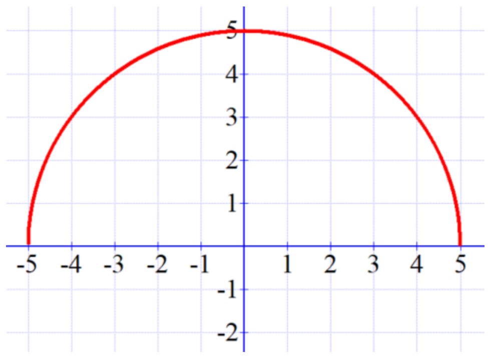
The value of K is 0 and the value of M is 3.

4. The domain of h(x) = $\frac{\sqrt{25-x^2}}{x-4}$ is

A) {x| x $\neq 4, x \in R\}$
B) {x| $-5 \leq x \leq 5, x \in R\}$
C) {x| $-5 \leq x \leq 5, x \neq 4, x \in R\}$
D) {x| $-25 \leq x \leq 25, x \neq 4, x \in R\}$

Solution

The graph of $y = \sqrt{25 - x^2}$ is shown below.



The domain is {x| $-5 \leq x \leq 5, x \in R\}$.

Thus, x can be any real number between -5 and 5, except for x = 4. If x = 4, the denominator would be equal to zero, but that is not possible because then the denominator would be undefined.

The domain is {x| $-5 \leq x \leq 5, x \neq 4, x \in R\}$.

The correct answer is C.

Use the following information to answer the next question.

The domain of h(x) = $\frac{f(x)}{g(x)}$ is {x| x $\neq -7, 7$, x ϵ R}

Consider the possible functions for f(x) and g(x).

For f(x)	**For g(x)**
① f(x) = 3x + 21	③ g(x) = x - 49
② $f(x) = \sqrt{3x + 21}$	④ g(x) = x^2 - 49

5. Which statement below is true?

   A) The function f(x) is ① and the function g(x) is ③.
   B) The function f(x) is ① and the function g(x) is ④.
   C) The function f(x) is ② and the function g(x) is ③.
   D) The function f(x) is ② and the function g(x) is ④.

Solution

From the given choices, we know that g(x) must be either ③ or ④. The factored form of $x^2 - 49$ is (x + 7) (x – 7). This would lead us to the non-permissible values, -7 and 7, for the domain of h(x). Thus, g(x) is ④.

We would now be looking for a domain of the numerator that is x ∈ R. Otherwise, there would be a restriction on the domain other than just x ≠ -7, 7. Thus, the numerator is represented by ①.

The correct answer is B.

6. Given f(x) = $\sqrt{x} - 1$, and g(x) = 2x, the domain of h(x) = $\frac{f(x)}{g(x)}$ can be written in the form {x| x > K, x ∈ R}, where K is an integer. The value of K is _<u>0</u>_.

Solution

The domain of f(x) is x ≥ 0 since x cannot be a negative value. The domain of the linear function g(x) is x ∈ R. The common elements of these domains, or overlap, is x ≥ 0.

However, since g(x) is in the denominator, x ≠ 0.

Thus, the domain of h(x) is {x| x > 0, x ∈ R}.

The value of K is 0.

Use the following information to answer the next question.

Consider the function $j(x) = \frac{\frac{1}{x+2}}{\frac{1}{x}}$

7. A) Simplify this complex rational expression.

Solution

$$j(x) = \frac{\frac{1}{x+2}}{\frac{1}{x}} = \left(\frac{1}{x+2}\right)\left(\frac{x}{1}\right) = \frac{x}{x+2}$$

B) Identify the domain before and after the simplification to show they are different.

Solution

The domain prior to simplification is:

{x| x ≠ -2, 0, x ∈ R}.

The domain after simplification is:

{x| x ≠ -2, x ∈ R}.

C) What is the final domain of j(x)? Explain.

The final domain of j(x) is $\{x|\ x \neq -2, 0, x \in R\}$.

Just like non-permissible values need to be determined before simplification, the domain needs to be determined prior to simplification.

Use the following to answer the next question.

Anaylze the domains of the following functions.

Function I	$y = 3^x - 1$
Function II	$y = \sqrt{x+1}$
Function III	$y = \log_2 x$
Function IV	$y = x^2$

A new function $h(x) = \frac{f(x)}{g(x)}$ is formed where the domain of h(x) is $\{x|\ x > 0, x \in R\}$.

8. If g(x) is function IV, which function, f(x), is the numerator of h(x)? Explain.

Solution

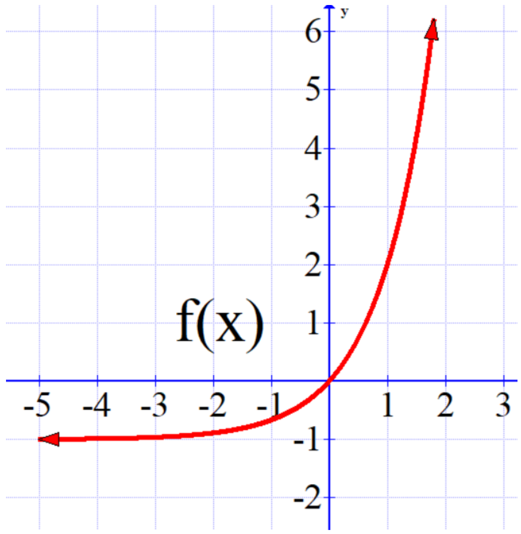
$$h(x) = \frac{f(x)}{x^2}$$

Function I

Suppose f(x) is represented by Function I.

$$h(x) = \frac{3^x - 1}{x^2}$$

The function f(x) is exponential, and its domain is $x \in R$.



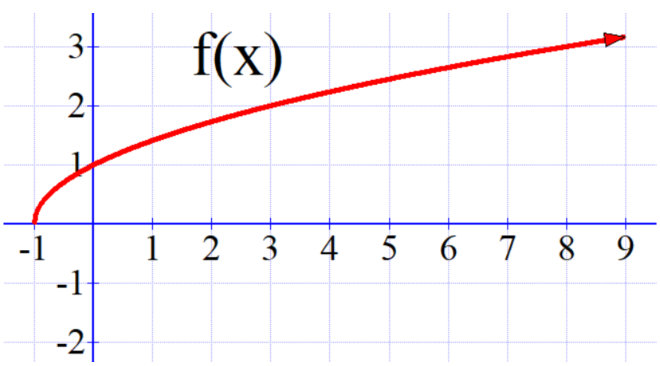
Since g(x) has the same domain on its own, the only restriction would be any value of x that would make g(x) = 0. Thus $x \neq 0$, which would be the domain. Since we are looking for a domain of $x > 0$, f(x) cannot be Function I.

<u>Function II</u>

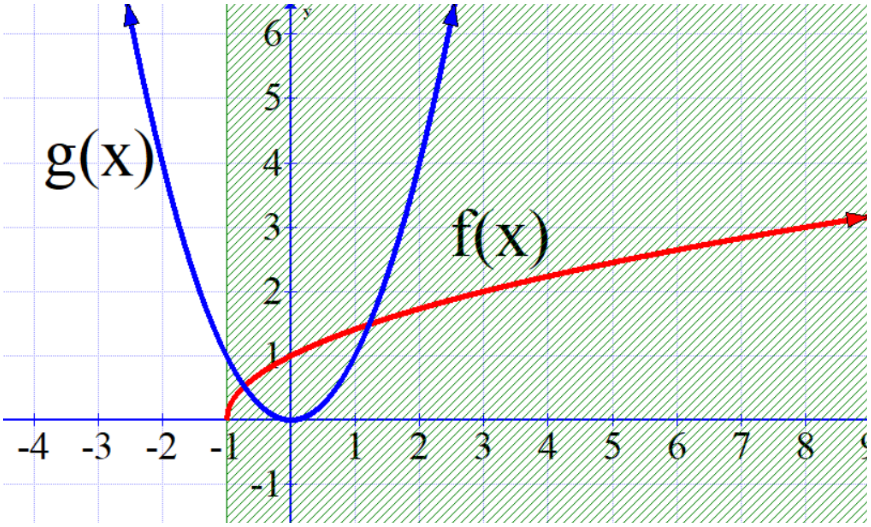
Suppose f(x) is represented by Function II.

$$h(x) = \frac{\sqrt{x+1}}{x^2}$$

The function f(x) in this case is radical and the domain is $x \geq -1$.



Consider the common elements for the domains of f(x) and g(x) in this case.



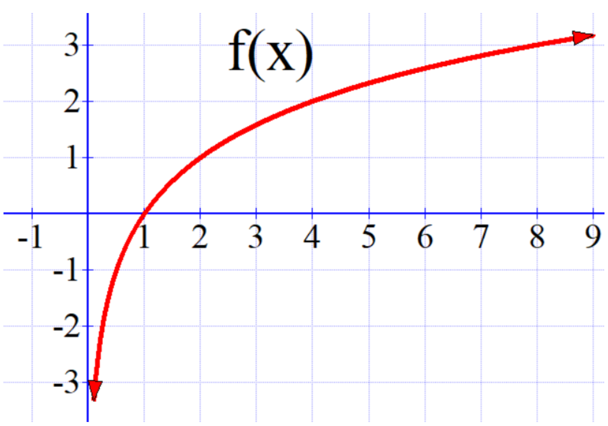
The domain in this case would be {x| x ≥ -1, x ≠ 0, x ∈ R}. With g(x) in the denominator, g (0) = 0, which is not allowed because division by zero is undefined. Since this is not the required domain given in the question, f(x) cannot be represented by Function II.

<u>Function III</u>

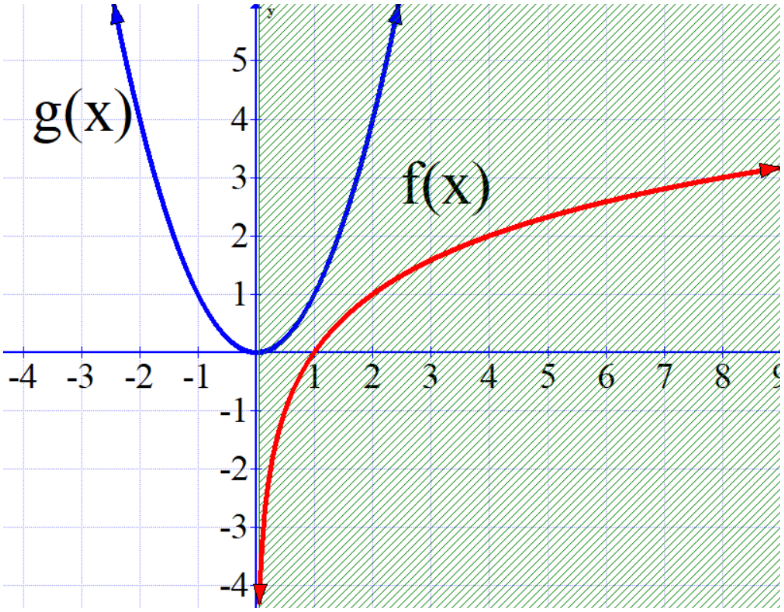
Suppose f(x) is represented by Function III.

$$h(x) = \frac{log_2 x}{x^2}$$

The function f(x) in this case is logarithmic and the domain is x > 0.



Consider the common elements of the domains of f(x) and g(x) in this case.



The common elements, or overlap, of the domains of these two functions is:

{x| x > 0, x ∈ R}.

The numerator of h(x) is Function **III**.