Combinatorics Problems With Cases Practice

Use the following information to answer the first question.

Little Jimmy invites 5 boys and 4 girls to his 7th birthday party. One of the cooperative games that his parents are planning requires groups of 6 people (6 will take part and the other 4 will watch).

1. If Jimmy's parents decide that they would like **at least** 1 girl in this cooperative game, how many cases need to be added in order to determine the number of ways the group can be made?

   A) 1 B) 2 C) 3 D) 4

Use the following information to answer the next question.

As an extra event at a high school basketball tournament, the organizers decided to run a mini 3 on 3 tournament on the day before the main tournament begins. Each team is allowed to submit a team of 3 players, with the condition that **at most** there can be 2 players over 6 feet 2 inches. One particular team has 4 players over 6 feet 2 inches, and 7 players under this height.

2. The number of ways this particular team can field their 3 on 3 team is

   A) $({}_4C_1)\,({}_7C_2) + ({}_4C_2)\,({}_7C_1)$
   B) $({}_4C_1)\,({}_7C_2)\ \ ({}_4C_2)\,({}_7C_1)$
   C) $({}_4C_0)\,({}_7C_3) + ({}_4C_1)\,({}_7C_2) + ({}_4C_2)\,({}_7C_1)$
   D) $({}_4C_0)\,({}_7C_3)\ \ ({}_4C_1)\,({}_7C_2)\ \ ({}_4C_2)\,({}_7C_1)$

3. A school chess club has 6 boys and 2 girls. They will be sending a team of 4, consisting of at least 1 girl, to compete against a rival school. The number of ways the team can be selected is represented by $({}_2C_k)\,({}_6C_v) + ({}_2C_2)\,({}_6C_2)$. The value of v is ____.

Use the following information to answer the next question.

To answer a particular question requires the addition of 4 cases.

Case 1	Case 2
$({}_5C_2)\,({}_{16}C_5) + ({}_5C_3)\,({}_{16}C_4)$	$({}_5C_3)\,({}_{16}C_4) + ({}_5C_4)\,({}_{16}C_3)$
Case 3	**Case 4**
$({}_5C_2)\,({}_{16}C_5) + ({}_5C_3)\,({}_{16}C_4) + ({}_5C_4)\,({}_{16}C_3) + ({}_5C_5)\,({}_{16}C_2)$	$({}_5C_2)\,({}_{16}C_5) + ({}_5C_3)\,({}_{16}C_4) + ({}_5C_4)\,({}_{16}C_3)$

Consider the 4 scenarios below.

Scenario 1	Scenario 2
A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at most** 2 adults.	A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at least** 2 adults.
Scenario 3	**Scenario 4**
A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at least** 2 youth.	A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at most** 2 youth.

4. The correct ***scenario*** that matches with Case 3 is

A) 1 B) 2 C) 3 D) 4

Use the following information to answer the next question.

A company has 12 millwrights and 9 pipefitters. They secured a contract to send a group of 7 workers to a plant to work on a project. This project requires **at most** 2 pipefitters.

5. The number of different ways this group of 7 workers can be formed is

   A) 14 560 B) 22 008 C) 37 620 D) 41 286

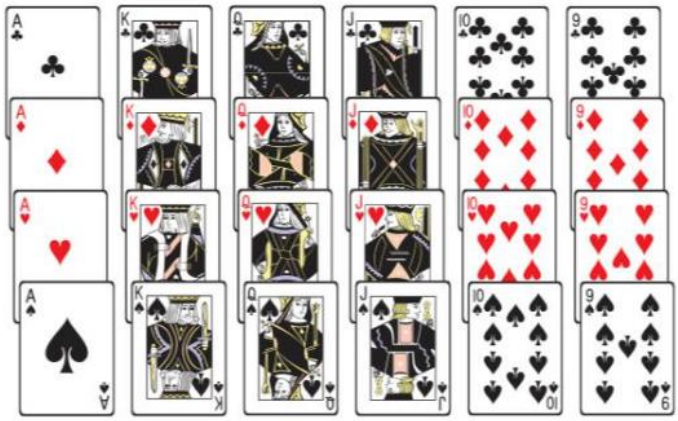
Use the following information to answer the next question.

Jennifer owns a local ice cream shop. She employs 3 adults and 5 students. Every month during the spring and summer, she goes to the supplier to buy ice cream and it takes 4 people to collect all the tubs of ice cream.

6. If the crew of 4 must include Jennifer and **at least** 2 students, then the number of possible crews is ____.

Use the following information to answer the next question.

Euchre is a card game that uses only the following 24 cards shown below.



7. The number of 5 card hands that have **at most** 1 Jack is

   A) 34 884 B) 21 044 C) 18 678 D) 10 216

[Note: The two stages are Jacks and No Jacks]

8. A volleyball squad of 12 players is to be chosen from 19 available players. In how many ways can this be done if Alisha or Carmen cannot both be selected? Explain and show all work.

Combinatorics Problems With Cases Practice Solutions

Use the following information to answer the first question.

Little Jimmy invites 5 boys and 4 girls to his 7th birthday party. One of the cooperative games that his parents are planning requires groups of 6 people (6 will take part and the other 4 will watch).

1. If Jimmy's parents decide that they would like **at least** 1 girl in this cooperative game, how many cases need to be added in order to determine the number of ways the group can be made?

   A) 1 B) 2 C) 3 D) 4

Solution

In this context, **at least** 1 means 1 or 2 or 3 or 4. Although the groups will have 6 total people, the highest number is 4 because that it the maximum number of girls possible.

Case 1	Case 2	Case 3	Case 4
1 girl and 5 boys	2 girls and 4 boys	3 girls and 3 boys	4 G and 2 B

The correct answer is D.

Use the following information to answer the next question.

As an extra event at a high school basketball tournament, the organizers decided to run a mini 3 on 3 tournament on the day before the main tournament begins. Each team is allowed to submit a team of 3 players, with the condition that **at most** there can be 2 players over 6 feet 2 inches. One particular team has 4 players over 6 feet 2 inches, and 7 players under this height.

2. The number of ways this particular team can field their 3 on 3 team is

   A) $({}_4C_1)({}_7C_2) + ({}_4C_2)({}_7C_1)$
   B) $({}_4C_1)({}_7C_2)\ ({}_4C_2)({}_7C_1)$
   C) $({}_4C_0)({}_7C_3) + ({}_4C_1)({}_7C_2) + ({}_4C_2)({}_7C_1)$
   D) $({}_4C_0)({}_7C_3)\ ({}_4C_1)({}_7C_2)\ ({}_4C_2)({}_7C_1)$

Solution

In this context, **at most** 2 means 0 or 1 or 2. We will consider 3 cases.

Let Over = over 6 feet 2 inches

Let Under = under 6 feet 2 inches

Case 1		Case 2		Case 3
0 over and 3 under		1 over and 2 under		2 over and 1 under
$({}_4C_0)\ ({}_7C_3)$	+	$({}_4C_1)\ ({}_7C_2)$	+	$({}_4C_2)\ ({}_7C_1)$

The correct answer is C.

3. A school chess club has 6 boys and 2 girls. They will be sending a team of 4, consisting of at least 1 girl, to compete against a rival school. The number of ways the team can be selected is represented by $({}_2C_k)\ ({}_6C_v) + ({}_2C_2)\ ({}_6C_2)$. The value of v is _3_.

Solution

Case 1		Case 2
1 girl and 3 boys		2 girls and 2 boys
$({}_2C_1)\ ({}_6C_3)$	+	$({}_2C_2)\ ({}_6C_2)$

The value of v is 3.

Use the following information to answer the next question.

To answer a particular question requires the addition of 4 cases.

Case 1	Case 2
$({}_5C_2)\,({}_{16}C_5) + ({}_5C_3)\,({}_{16}C_4)$	$({}_5C_3)\,({}_{16}C_4) + ({}_5C_4)\,({}_{16}C_3)$
Case 3	**Case 4**
$({}_5C_2)\,({}_{16}C_5) + ({}_5C_3)\,({}_{16}C_4) + ({}_5C_4)\,({}_{16}C_3) + ({}_5C_5)\,({}_{16}C_2)$	$({}_5C_2)\,({}_{16}C_5) + ({}_5C_3)\,({}_{16}C_4) + ({}_5C_4)\,({}_{16}C_3)$

Consider the 4 scenarios below.

Scenario 1	Scenario 2
A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at most** 2 adults.	A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at least** 2 adults.
Scenario 3	**Scenario 4**
A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at least** 2 youth.	A church youth group has 5 adults and 16 youth. A sub-committee of 7 is formed to look into fundraising possibilities. This committee must have **at most** 2 youth.

4. The correct ***scenario*** that matches with Case 3 is

A) 1 B) 2 C) 3 D) 4

The correct answer is B.

Use the following information to answer the next question.

A company has 12 millwrights and 9 pipefitters. They secured a contract to send a group of 7 workers to a plant to work on a project. This project requires **at most** 2 pipefitters.

5. The number of different ways this group of 7 workers can be formed is

A) 14 560 B) 22 008 C) 37 620 D) 41 286

Solution

In this context, **at most** 2 means 0 or 1 or 2.

Let M = millwright

Let P = pipefitter

Case 1		Case 2		Case 3
0 P and 7 M		1 P and 6 M		2 P and 5 M
$({}_9C_0)\ ({}_{12}C_7)$	+	$({}_9C_1)\ ({}_{12}C_6)$	+	$({}_9C_2)\ ({}_{12}C_5)$
(1) (792)	+	(9) (924)	+	(36) (792)
792	+	8316	+	28 512

= 37 620

The correct answer is C.

Use the following information to answer the next question.

Jennifer owns a local ice cream shop. She employs 3 adults and 5 students. Every month during the spring and summer, she goes to the supplier to buy ice cream and it takes 4 people to collect all the tubs of ice cream.

6. If the crew of 4 must include Jennifer and **at least** 2 students, then the number of possible crews is _40_.

Solution

Since Jennifer must be included, there are 3 remaining people to be selected.

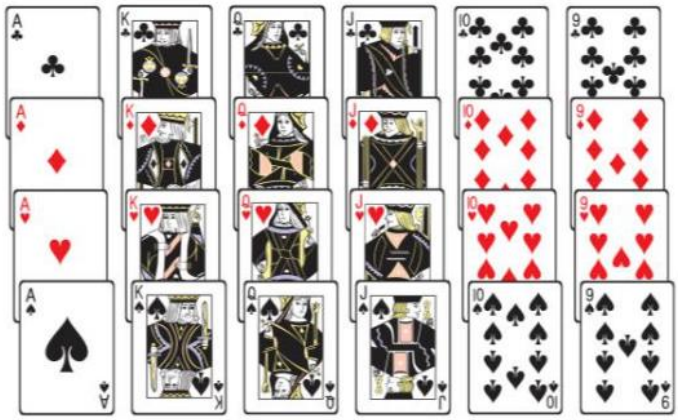
In this context, **at least** 2 means 2 or 3.

Case 1		Case 2
2 students and 1 adult		3 students and 0 adults
$({}_5C_2)\ ({}_3C_1)$	+	$({}_5C_3)\ ({}_3C_0)$
(10) (3)	+	(10) (1)
30	+	10
= 40		

The number of possible crews is 40.

Use the following information to answer the next question.

Euchre is a card game that uses only the following 24 cards shown below.



7. The number of 5 card hands that have **at most** 1 Jack is

A) 34 884 B) 21 044 C) 18 678 D) 10 216

[Note: The two stages are Jacks and No Jacks]

Solution

In this context, **at most** 1 means 0 or 1. There are 2 cases.

There is a total of 4 Jacks, and a total of 20 that are not Jacks.

Case 1		Case 2
0 Jacks and 5 Non-Jacks		1 Jack and 4 Non-Jacks
$({}_4C_0)\ ({}_{20}C_5)$	+	$({}_4C_1)\ ({}_{20}C_4)$
(1) (15 504)	+	(4) (4845)
15 504	+	19 380

= 34 884

The correct answer is A.

8. A volleyball squad of 12 players is to be chosen from 19 available players. In how many ways can this be done if Alisha or Carmen cannot both be selected? Explain and show all work.

Solution

Think of 4 possible options with respect to Alisha and Carmen.

1. Both on the team
2. Both not on the team
3. Only Alisha on the team
4. Only Carmen on the team

For the criteria in this specific question, we will consider 3 of these 4 as our cases.

Case 1		Case 2		Case 3
Both not on the team		Only Alisha on the team		Only Carmen on the team
$({}_{17}C_{12})$	+	$({}_1C_1)\ ({}_{17}C_{11})$	+	$({}_1C_1)\ ({}_{17}C_{11})$
6188	+	12 376	+	12 376

= 30 940