Binomial Theorem With Non-Linear Terms Practice

Use the following information to answer the first question.

When determing the constant term in the expansion of

$$\left(5x^2 + \frac{1}{x}\right)^9$$

The work for the first step in finding K is shown below.

Step 1	$(5x^2 + (x^{-1}))^9$
Step 2	$((x^2)^{9-k})(x^{-k}) = x^0$
Step 3	$(x^{18-k})(x^{-k}) = x^0$
Step 4	$(x^{18-2k}) = x^0$
Step 5	18 – 2k = 0
Step 6	k = 9

1. Unfortunately an error was made. The step where the first error occurred **and** the correct value of k are

   A) Step 3 **and** k = 4
   B) Step 3 **and** k = 6
   C) Step 4 **and** k = 4
   D) Step 4 **and** k = 6

2. The constant term in the expansion of $\left(x - \frac{4}{x^2}\right)^{12}$ is

   A) 495 B) 1020 C) 50 680 D) 126 720

3. Given $\left(m + \frac{1}{m}\right)^n$ with k = 2, the value of the constant term is ____.

Use the following information to answer the next question.

Consider the following expressions.

A. $(x + 3y)^3$	**B.** $(x + \frac{1}{x})^4$
C. $(2x - y)^5$	**D.** $(x^3 - \frac{3}{x^2})^7$

4. When expanded, which expression will have a constant term?

A) A B) B C) C D) D

5. The constant term in the expansion of $\left(\frac{3}{x} + 2x^3\right)^{12}$ is a number over 34 million that can be written in the form 34 KMN 080, where K, M, and N are integers. The values for K, M, and N respectively, are

A) 531
B) 642
C) 814
D) 702

6. The constant term in the expansion of $\left(\frac{x^2}{2} + \frac{m}{x}\right)^6$ is 60. Determine possible values of *c*. Show all work.

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1. Unfortunately an error was made. The step where the first error occurred **and** the correct value of k are

A) Step 3 **and** k = 4
B) Step 3 **and** k = 6
C) Step 4 **and** k = 4
D) Step 4 **and** k = 6

Solution

The error was made in step 3. In the first power, k was not multiplied by 2.
Instead of $(x^{18-k})(x^{-k}) = x^0$, it should be $(x^{18-2k})(x^{-k}) = x^0$.

The next step should be $(x^{18-3k}) = x^0$, followed by

18 – 3k = 0.

18 = 3k

6 = k.

The correct answer is B.

2. The constant term in the expansion of $\left(x - \frac{4}{x^2}\right)^{12}$ is

A) 495 B) 1020 C) 50 680 D) 126 720

Solution

For the first step in finding k, rewrite letters in the denominator in an equivalent form and then remove any coefficients.

$(x - 4x^{-2})^{12}$

$((x)^{12-k})(x^{-2k}) = x^0$

12 – 3k = 0

12 = 3k

4 = k

Now use the general term to determine the constant term.

$t_{k+1} = {}_nC_k\, x^{n-k}\, y^k$

$t_{4+1} = {}_{12}C_4\, (x)^8 \left(-\frac{4}{x^2}\right)^4$

$t_5 = (495)\,(x^8)\left(\frac{256}{x^8}\right)$

$t_5 = 126\ 720$

The correct answer is D.

3. Given $\left(m+\frac{1}{m}\right)^n$ with k = 2, the value of the constant term is _6__.

Solution

Rewrite the second term in the binomial in an equivalent form.

$(\mathrm{m}-\mathrm{m}^{-1})^{\mathrm{n}}$

Determine the value of k.

$((m)^{n-k})(m^{-k})=m^0$

Substitute k = 2

$(m^{n-2})(m^{-2})=m^0$

$(m^{n-4})=m^0$

Since the bases are the same, set the exponents equal.

n – 4 = 0

n = 4

$t_{k+1}={}_nC_k\, x^{n-k}\, y^k$

$t_{2+1}={}_4C_2\,(m)^2\left(\frac{1}{m}\right)^2$

$t_3=(6)\,(m^2)\left(\frac{1}{m^2}\right)$

$t_3=6$

The value of the constant term is 6.

Use the following information to answer the next question.

Consider the following expressions.	
A. $(x + 3y)^3$	**B.** $(x + \frac{1}{x})^4$
C. $(2x - y)^5$	**D.** $(x^3 - \frac{3}{x^2})^7$

4. When expanded, which expression will have a constant term?

A) A B) B C) C D) D

Solution

Options A and C both have two separate letters, so every term in the expansion will have at least one letter, i.e. no constant.

For options B and D, check to see if there is an integer value for k. If so, there would be a constant.

The value of k in option B is 2. The value of k in option D is 4.2.

The correct answer is B.

5. The constant term in the expansion of $\left(\frac{3}{x} + 2x^3\right)^{12}$ is a number over 34 million that can be written in the form 34 KMN 080, where K, M, and N are integers. The values for K, M, and N respectively, are

A) 531
B) 642
C) 814
D) 702

Solution

Step 1 – Determine the value of k.

$(3\mathrm{x}^{-1} + 2\mathrm{x}^{3})^{12}$

$((x^{-1})^{12-k})(x^{3k}) = x^{0}$

$(x^{-12+k})(x^{3k}) = x^{0}$

$(x^{-12+4k}) = x^{0}$

$-12 + 4k = 0$

$4k = 12$

$k = 3$

Step 2 – Use the general term

$t_{k+1} = {}_{n}C_{k}\ x^{n-k}\ y^{k}$

$t_{3+1} = {}_{12}C_{3}\ (\frac{3}{x})^{9}\ (2x^{3})^{3}$

$t_{4} = (220)\ (\frac{3^{9}}{x^{9}})\ \left((2^{3})(x^{9})\right)$

$t_{4} = 34\ 642\ 080$

The correct answer is B.

6. The constant term in the expansion of $\left(\frac{x^2}{2} + \frac{m}{x}\right)^6$ is 60. Determine possible values of *c*. Show all work.

Solution

Rewrite without coefficients and with moving the variable in the denominator to the numerator.

$(x^2 + x^{-1})^6$

Find k.

$(x^2 + x^{-1})^6$

$((x^2)^{6-k})(x^{-k}) = x^0$

$(x^{12-2k})(x^{-k}) = x^0$

$(x^{12-3k}) = x^0$

$12 - 3k = 0$

$12 = 3k$

$k = 4$

Use the general term.

$60 = {}_6C_4 \left(\frac{x^2}{2}\right)^2 \left(\frac{m}{x}\right)^4$

$60 = {}_6C_4 \left(\frac{x^4}{4}\right) \left(\frac{m^4}{x^4}\right)$

$60 = (15) \left(\frac{m^4}{4}\right)$

$240 = (15)(m^4)$

$16 = m^4$

Take the 4th root of both sides.

$m = \pm 2$

The possible values of m are ± 2.