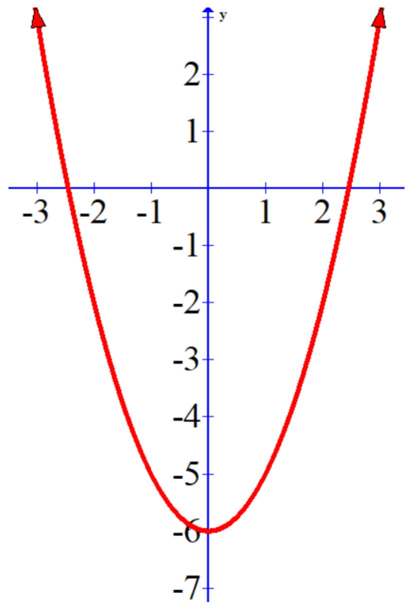
Restricting Domain so Inverse is a Function - Practice

Use the following graph to answer the first question.



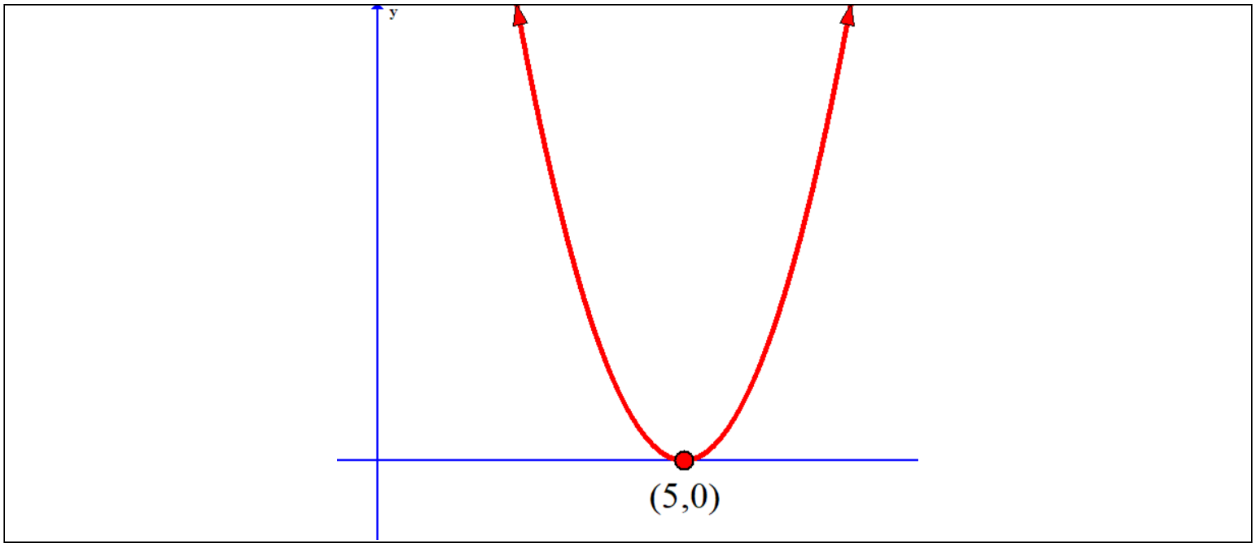
1. A restriction on the domain of the above graph, such that its inverse is also a function, could be

   A) $\{x \mid x \geq -6, x \in R\}$
   B) $\{x \mid x \leq 6, x \in R\}$
   C) $\{x \mid x \geq 0, x \in R\}$
   D) $\{x \mid x \geq -2.5, x \in R\}$

2. A restriction on the domain of $f(x) = -2(x - 3)^2 + 4$, such that its inverse is also a function, could be

   A) $\{x \mid x \leq 3, x \in R\}$
   B) $\{x \mid x \leq 4, x \in R\}$
   C) $\{x \mid x \geq -2, x \in R\}$
   D) $\{x \mid x \geq 0, x \in R\}$

Use the following graph to answer the next question.



3. In order for the inverse of the graph above to be a function, two possible restrictions are $\{x \mid x \geq M, x \in R\}$ and $\{x \mid x \leq M, \in R\}$. The value of M is ____.

4. Given the function $f(x) = a(x - p)^2 + q$, which of the following restrictions below would guarantee that the inverse would be a function?

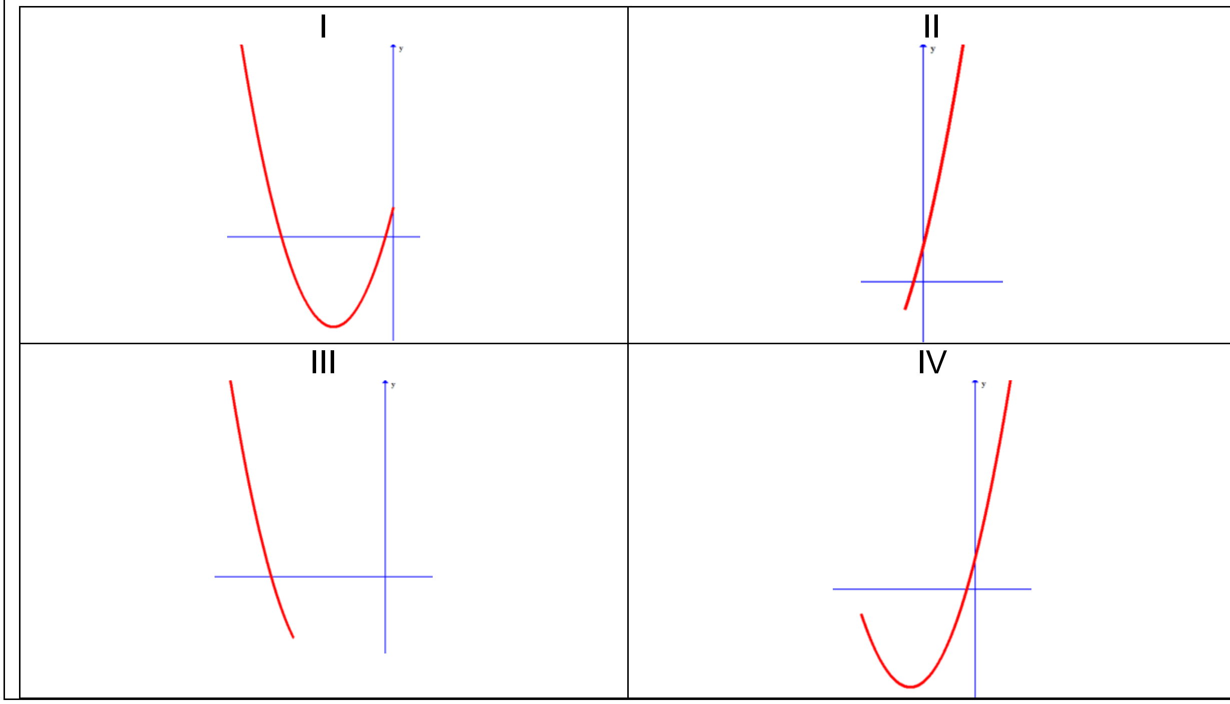
   A) $x \geq p$
   B) $x \geq q$
   C) $x \leq a$
   D) $x \leq 0$

5. The graph of $f(x) = (x + 9)^2 + 1$ is shifted 9 units to the right to become $y = g(x)$. A restriction on $g(x)$ such that its inverse is a function is

   A) $\{x \mid x \leq 1, x \in R\}$
   B) $\{x \mid x \leq 9, x \in R\}$
   C) $\{x \mid x \leq 0, x \in R\}$
   D) $\{x \mid x \geq -1, x \in R\}$

Use the following information to answer the next question.

Jared applied several restrictions to the graph of $f(x) = (x + 2)^2 - 3$. The results of his restrictions are shown below.

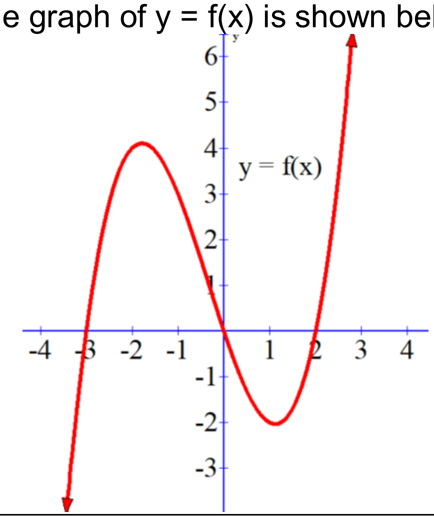


6. The inverse of which two graphs would **not** be a function?

A) I and II   B) III and IV   C) I and IV   D) II and III

Use the following graph to answer the next question.

The graph of y = f(x) is shown below.



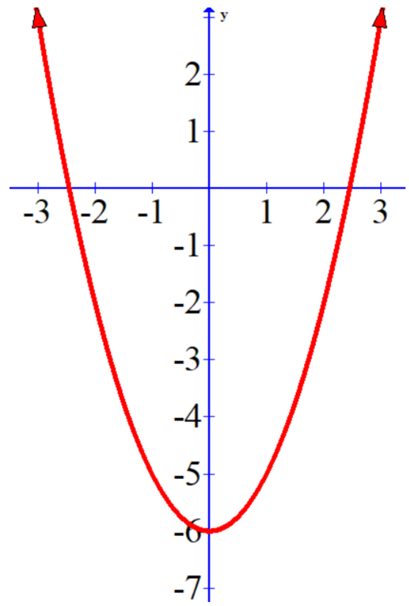
7. A restriction on $y = f(x)$ such that its inverse is also a function could be

   A) $\{x \mid x \le 2, x \in R\}$
   B) $\{x \mid x \le 0, x \in R\}$
   C) $\{x \mid x \ge 0, x \in R\}$
   D) $\{x \mid x \ge 2, x \in R\}$

8. Given the quadratic function $f(x) = (x + 5)^2 - 1$, Becky said that if the restriction is $\{x \mid x \le -3, x \in R\}$, then its inverse is a function. Lisa said that if that is the restriction, then its inverse is not a function. Who is correct? Explain.

Restricting Domain so Inverse is a Function – Practice Solutions

Use the following graph to answer the first question.

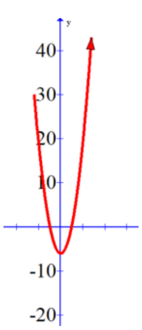


1. A restriction on the domain of the above graph, such that its inverse is also a function, could be

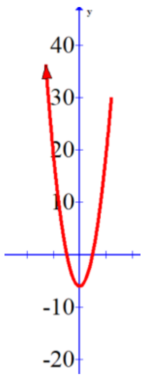
   A) {x | x ≥ -6, x ϵ R}
   B) {x | x ≤ 6, x ϵ R}
   C) {x | x ≥ 0, x ϵ R}
   D) {x | x ≥ -2.5, x ϵ R}

Solution

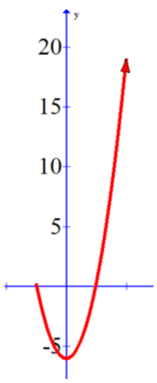
Option A cannot be correct because when the restriction is applied, there will be parts of two branches as shown below. When the vertical line test is used for the inverse, the vertical line will intersect the graph at more than one point.



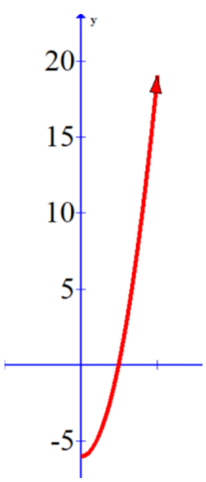
Option B cannot be correct for the same reasoning mentioned above. See the graph below.



Option D cannot be correct for the same reasoning mentioned above.



Option C works because the restriction will result in only one branch.



The correct answer is C.

2. A restriction on the domain of $f(x) = -2(x - 3)^2 + 4$, such that its inverse is also a function, could be

   A) $\{x \mid x \leq 3, x \in R\}$
   B) $\{x \mid x \leq 4, x \in R\}$
   C) $\{x \mid x \geq -2, x \in R\}$
   D) $\{x \mid x \geq 0, x \in R\}$

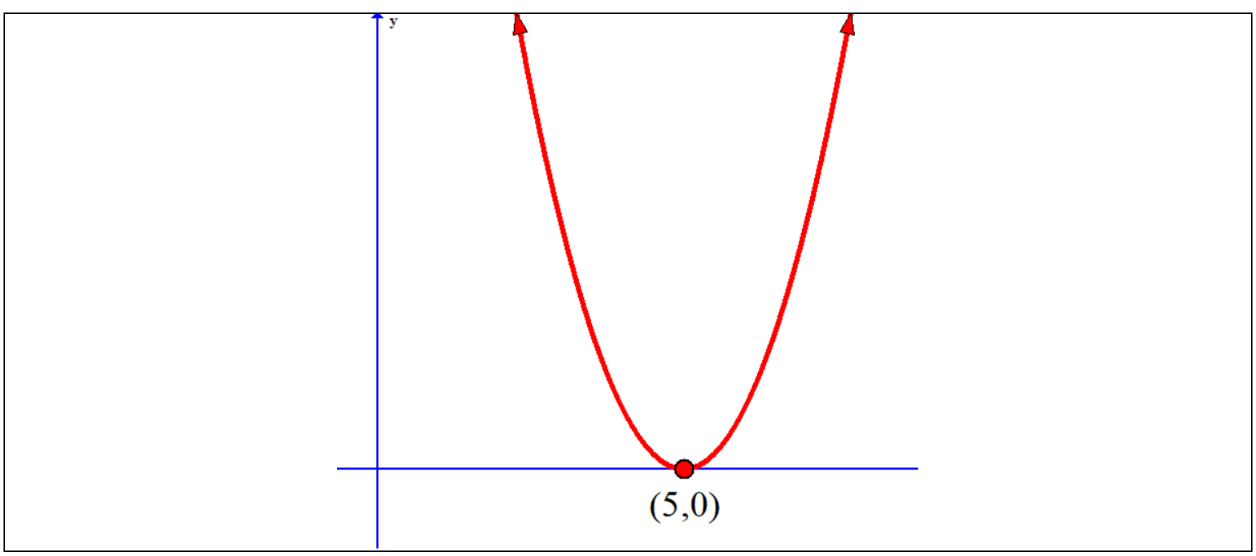
Solution

The vertex of the parabola is (3,4). Any restriction to the left or the right of the x-coordinate will result in only one branch of the parabola. Thus, the inverse will then be a function. Since there is only one answer, we know the correct answer is A.

All other options would produce parts of 2 branches of the parabola.

The correct answer is A.

Use the following graph to answer the next question.



3. In order for the inverse of the graph above to be a function, two possible restrictions are $\{x \mid x \geq M, x \in R\}$ and $\{x \mid x \leq M, \in R\}$. The value of M is _5__.

Solution

For any value of x other than the x-coordinate of the vertex, restricting the domain with "less than values" and then "greater than values", will result in one restriction creating an inverse that is a function and one restriction creating an inverse that is not a function. The only value of x that can create restrictions such that the "is less than values" and the "is greater than values" **both** create inverses that are functions is the x-coordinate of the vertex.

The value of M is 5.

4. Given the function $f(x) = a(x - p)^2 + q$, which of the following restrictions below would guarantee that the inverse would be a function?

   A) $x \geq p$
   B) $x \geq q$
   C) $x \leq a$
   D) $x \leq 0$

Solution

The key point here is the vertex, which is (p, q), and the key coordinate is the x-coordinate, which is p. Any restriction to the left (≤) or to the right (≥) of this coordinate, will produce only one branch of the parabola. Thus, the inverse will then be a function.

All the other options are possible, depending on the values of a. p. and q. The only one that would guarantee the appropriate restriction is $x \geq p$.

The correct answer is A.

5. The graph of $f(x) = (x + 9)^2 + 1$ is shifted 9 units to the right to become y = g(x). A restriction on g(x) such that its inverse is a function is

   A) $\{x \mid x \leq 1, x \in R\}$
   B) $\{x \mid x \leq 9, x \in R\}$
   C) $\{x \mid x \leq 0, x \in R\}$
   D) $\{x \mid x \geq -1, x \in R\}$

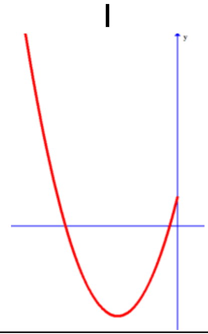
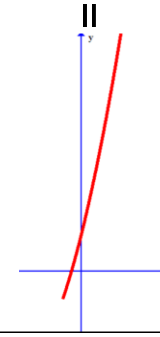
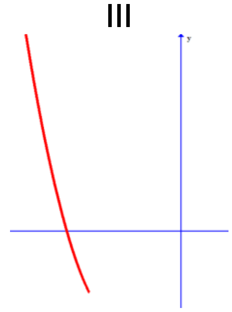
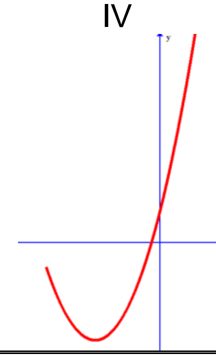
Solution

The vertex is currently at (-9,1). Shifting 9 units to the right will result in a vertex of (0,1). This point is on the y-axis, and any restriction to the left (≤) or to the right (≥) will result in only one branch of the parabola.

The correct answer is C.

Use the following information to answer the next question.

Jared applied several restrictions to the graph of $f(x) = (x + 2)^2 – 3$. The results of his restrictions are shown below.

I	II
	
III	IV
	

6. The inverse of which two graphs would **not** be a function?

A) I and II B) III and IV C) I and IV D) II and III

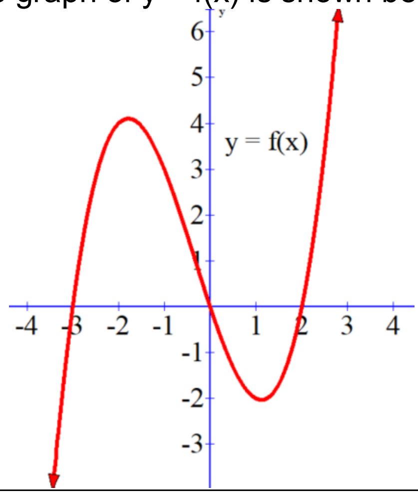
Solution

Restrictions I and IV create parabolas having parts of two branches. Thus, the inverses of these two options will **not** be functions.

The correct answer is C.

Use the following graph to answer the next question.

The graph of y = f(x) is shown below.



7. A restriction on y = f(x) such that its inverse is also a function could be

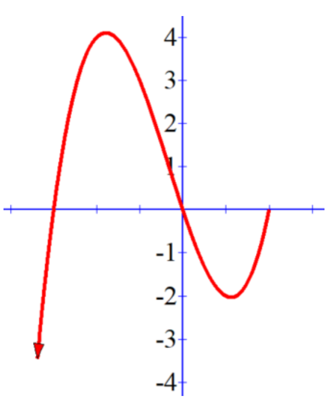
   A) {x | x ≤ 2, x ϵ R}
   B) {x | x ≤ 0, x ϵ R}
   C) {x | x ≥ 0, x ϵ R}
   D) {x | x ≥ 2, x ϵ R}

Solutions

Although we didn't specifically talk about cubic functions, and it is very unlikely that you would see a question like this, the same principle applies. We cannot have more than 1 branch of a parabola or a cubic function after the restriction is applied. Let's look at each restriction separately.

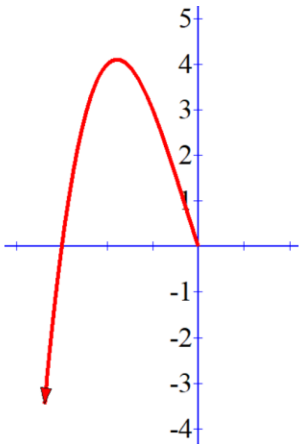
Option A

{x| x ≤ 2, x ϵ R}



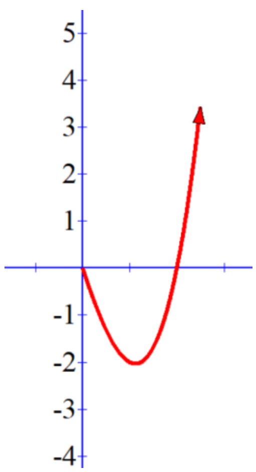
Option B

$\{x|\ x \le 0,\ x \in R\}$



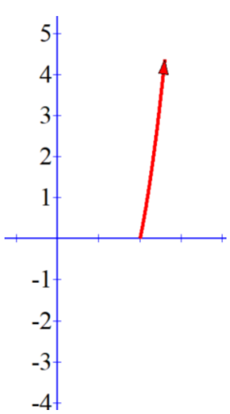
Option C

$\{x|\ x \ge 0,\ x \in R\}$



Option D

$\{x|\ x \ge 2,\ x \in R\}$

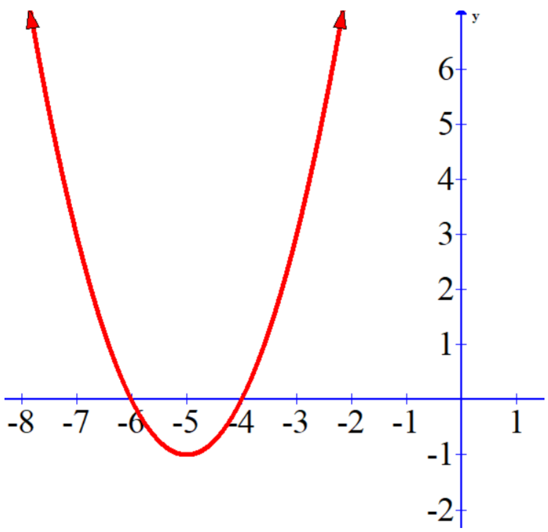


The correct answer is D.

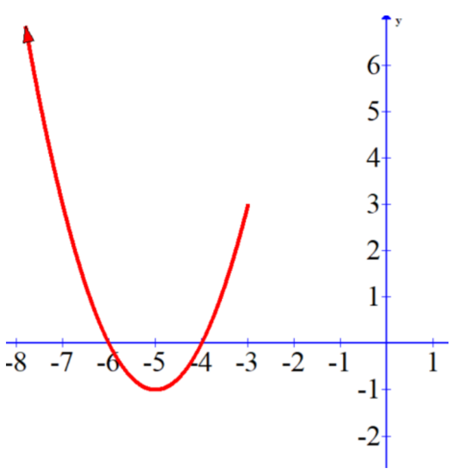
8. Given the quadratic function $f(x) = (x + 5)^2 - 1$, Becky said that if the restriction is $\{x \mid x \le -3, x \in R\}$, then its inverse is a function. Lisa said that if that is the restriction, then its inverse is not a function. Who is correct? Explain.

Solution

The initial graph $y = f(x)$ is shown below.



The restriction of $\{x \mid x \le -3, x \in R\}$ would create a graph that looks like:



Given this restriction, the inverse of the function would not be a function because there are parts of both branches.
Lisa is correct.