Math 30-1 Alberta Education Practice Test 2022 Solutions

Use the following information to answer numerical-response question 1.

The mapping notation $(x, y) \rightarrow (x - 2, \ -y + 5)$ is used to describe the transformation of the function $y = f(x)$ into the function $y = g(x)$.

Possible Transformations

1	Reflection in the x-axis
2	Reflection in the y-axis
3	Translation 5 units right
4	Translation 5 units left
5	Translation 2 units right
6	Translation 2 units left
7	Translation 5 units up
8	Translation 5 units down
9	Translation 2 units up
0	Translation 2 units down

Numerical Response

1. In correct order, the transformations above that would transform $y = f(x)$ into $y = g(x)$ are numbered ______, ______, and ______. (There is more than one correct answer.)

Solution

The mapping notation for x $\longrightarrow$ x – 2, means a translation 2 units left. [#6]

The mapping notation for y $\longrightarrow$ -y + 5, can mean two things, depending on what came first. This is because -y + 5, could also be thought of as – (y – 5). We could have a reflection in the x-axis [#1] and then a translation 5 units up [#7]; or a translation 5 units down [#8] and a reflection in the x-axis [#1].

If the reflection in the x-axis is first, the order of the two translations doesn't matter. Two possible answers are [1-6-7] and [1-7-6]

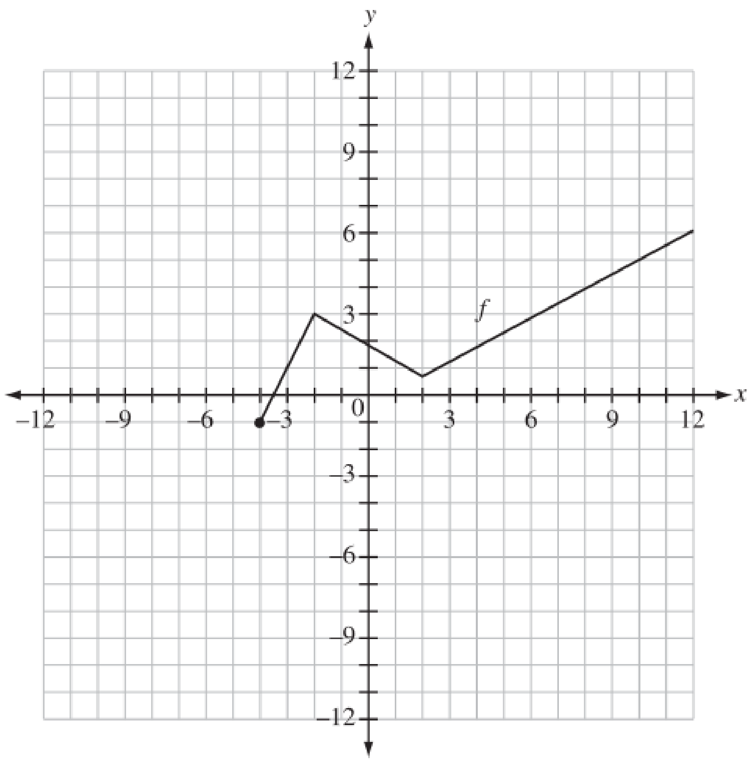
If the translation 5 down is first, the order of the other two transformations doesn't matter. Two possible answers are [8-1-6] and [8-6-1].

If the translation 2 units left is first, then we either have a reflection in the x-axis and a translation 5 units up [6-1-7] or a translation 5 units down and a reflection in the x-axis [6-8-1].

The total possible answers are [1-6-7], [1-7-6], [8-1-6], [8-6-1], [6-1-7], and [6-8-1].

Use the following information to answer question 1

The graph of $y = f(x)$, shown below, has the domain $\{x \mid x \geq -4, x \in R\}$.



1. The domain of the graph of $y = f(-x + 4)$ is

A. $\{x \mid x \leq 8, x \in R\}$

B. $\{x \mid x \geq -8, x \in R\}$

C. $\{x \mid x \leq 0, x \in R\}$

D. $\{x \mid x \geq 0, x \in R\}$

Solution

Factor out a negative 1 in the brackets from y = f (-x + 4), to y = f (-(x – 4). There is a reflection in the y-axis and a translation 4 units right.

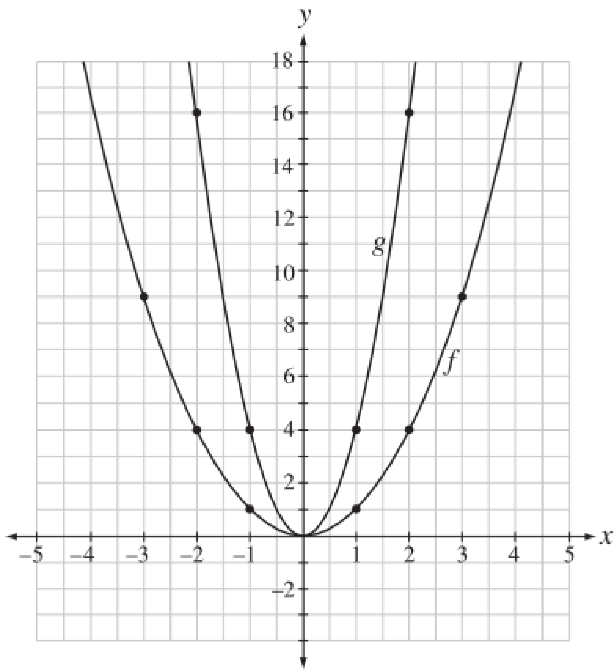
The key point of this graph is the one furthest to the left, which is (-4, -1). When this point is reflected in the y-axis and translated 4 units right, it will have the furthest x-coordinate to the right. Because of the reflection in the y-axis, all the points that were originally rising to the right in quadrant 1, are now rising to the left in quadrant 2. All points of this graph now have x-coordinates less than 8. This is because the original x-coordinate of -4 was reflected in the y-axis to +4, and then translated 4 units to the right.

The domain is $\{x \mid x \leq 8, x \in R\}$

The correct answer is A.

Use the following information to answer question 2.

The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$, as shown below.



2. A possible equation for $g(x)$ is

A. $g(x) = 2f(x)$

B. $g(x) = f(2x)$

C. $g(x) = \frac{1}{2}f(x)$

D. $g(x) = f\left(\frac{1}{2}x\right)$

Solution

Based on the given options, there is either a vertical stretch or a horizontal stretch.

Consider the possibility of a vertical stretch. When x = 1, g (1) = 4 and f (1) = 1. This would potentially indicate a vertical stretch by a factor of 4. Confirming, when x = 2,

g (2) = 16 and f (2) = 4. This would also potentially indicate a vertical stretch by a factor of 4. The equation would be g(x) = 4 f(x). Since this is not one of the options, we are looking for a horizontal stretch.

When y = 4, the x-coordinate for g(x) is 1 and the x-coordinate for f(x) is 2. The stretch factor is ½, since 2 is multiplied by ½ to get 1.

In the equation, the 'b' value is the reciprocal of the stretch factor. The value of 'b' is 2. The equation is g(x) = f(2x).

The correct answer is B.

Use the following information to answer question 3.

The point $A(2,\ 1)$ is on the graph of $y = f(x)$. The graph is stretched horizontally about the y-axis and then translated so that the new graph passes through the corresponding point $A'(8,\ 1)$. The equation of the new function can be written in the form $y = f\big(m(x-2)\big)$.

3. The value of ***m*** is

A. $\frac{1}{5}$

B. $\frac{1}{3}$

C. 3

D. 5

Solution

The value of 'm' in the equation y = f (m (x – 2)) relates to the horizontal stretch. It is important to remember that this value in the equation and the stretch factor are reciprocals of each other. And, given the component, (x – 2), it indicates that all the x-coordinates of y = f(x) are translated 2 units right after the stretch. From a mapping notation point of view, the x-coordinates get larger.

Set up an equation to describe the movement of the x-coordinates, starting with 2 and ending at 8.

$$2\left(\frac{1}{m}\right) + 2 = 8$$

$$\left(\frac{2}{m}\right) = 6$$

2=6m

$m = \frac{2}{6} or \frac{1}{3}$

The correct answer is B.

4. Given the function $f(x) = 3^{(x-a)} + b,\ a \neq 0,$ the domain of the inverse of $f(x)$ is

A. $\{x \mid x > 0, x \in R\}$

B. $\{x \mid x > 3, x \in R\}$

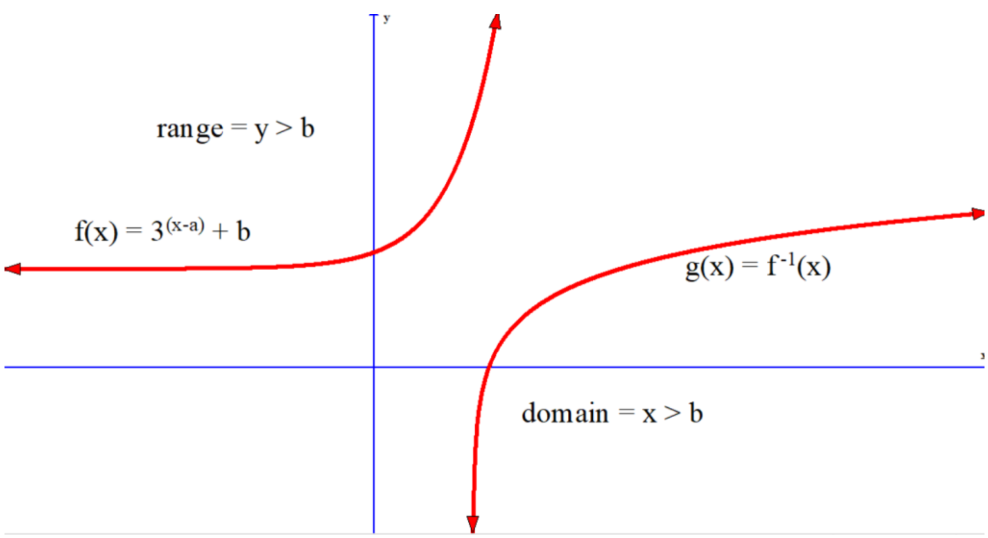
C. $\{x \mid x > a, x \in R\}$

D. $\{x \mid x > b, x \in R\}$

Solution

The function y = f(x) is an exponential function, and its inverse is the logarithmic function. For the exponential function, the parameter ‘b’ represents the horizontal asymptote, which in turn affects the range.

For the inverse of this function, the horizontal asymptote becomes the vertical asymptote, which in turn affects the domain.



The correct answer is D.

Numerical Response

2. Given that $C = B^2$ and $B^2 = A^5$, where $A, B, C > 0$, $A \neq 1$, the value of $\log_A(BC)$, to the nearest tenth, is __________.

Solution

One strategy would be to get the logarithmic expression in terms of the same letter.

Given $B^2 = A^5$, we could raise both sides of the equal sign to an exponent of $\frac{1}{5}$.

$$B^{2\left(\frac{1}{5}\right)} = A^{5\left(\frac{1}{5}\right)}$$

$$B^{\left(\frac{2}{5}\right)} = A$$

Substitute values into the logarithmic expression.

$$log_{B^{\left(\frac{2}{5}\right)}}((B)(B^2))$$

$$log_{B^{\left(\frac{2}{5}\right)}}B^3$$

Set the expression equal to ‘x’ and convert to exponential form.

$$log_{B^{\left(\frac{2}{5}\right)}}B^3 = x$$

$$B^{\left(\left(\frac{2}{5}\right)^{(x)}\right)} = B^3$$

$$B^{\frac{2x}{5}} = B^3$$

Since the bases are the same, set the exponents to be equal.

$$\frac{2x}{5} = 3$$

2x = 15

x = 7.5

5. An expression equivalent to $-\log x - \log y$ is

A. $\dfrac{-\log x}{\log y}$

B. $-\log\left(\dfrac{x}{y}\right)$

C. $\log\left(\dfrac{1}{xy}\right)$

D. $\dfrac{1}{\log x \log y}$

Solution

My first thought was to divide (-1) out of each term, to get, – (log x + log y). Then using the product law of logarithms, simplify to – (log (xy)). But this is not one of the options.

I could use the power law to move the number in front of the log (i.e. -1) to the exponential position.

(log $(xy)^{-1}$).

Making the exponent positive, the equivalent expression would be $log\left(\frac{1}{xy}\right)$

The correct answer is C.

[NOTE: An alternative would be to substitute convenient values for x and y; for example, x = 1000 and y = 100. Now which of the options would give an equivalent value?]

Numerical Response

3. The expression $\log_b b^2 - 5\log_b c + 4\log_b\left(bc^3\right)$, where $b > 1$ and $c > 1$, can be written in the form $m + n\log_b c$.

The single-digit value of ***m*** is __________. (Record in the **first** column)

The single-digit value of ***n*** is __________. (Record in the **second** column)

Solution

Since we are adding or subtracting logarithmic terms with the same base, applying one or more of the product, quotient, and power laws is likely.

Usually, numbers in front of the logs are moved to the exponential position using the power law first.

$$log_b b^2 + log_b c^{-5} + log_b (bc^3)^4$$

$$log_b b^2 + log_b \left(\frac{1}{c^5}\right) + log_b (b^4 c^{12})$$

Keep the base and multiply the values of the powers, or the arguments.

$$log_b (b^2)\left(\frac{1}{c^5}\right)(b^4 c^{12})$$

$$log_b (b^6)(c^7)$$

Write the equivalent expression as the sum of two logs.

$$log_b b^6 + log_b c^7$$

= 6 + 7$\log_b$c

The value of m is 6 and the value of n is 7.

Use the following information to answer question 6.

Three of the following functions have the same vertical asymptote.

1 $f(x) = \log_2(x - 3) + 6$

2 $g(x) = \log_2(2x - 6) + 3$

3 $h(x) = \dfrac{2x^2 + 6x}{x^2 - 3x}$

4 $k(x) = \dfrac{x - 3}{2x^2 - 6x}$

6. The function whose graph does **not** have the same vertical asymptote as the other graphs is Function

A. 1
B. 2
C. 3
D. 4

Solution

For function #1, the equation for the vertical asymptote is determined by setting the part in the backets, i.e. (x – 3), to zero and solving.

x – 3 = 0

x = 3

The equation of the vertical asymptote is x = 3.

Similarly, for function #2, 2x – 6 = 0.

2x = 6

x = 3

The equation of the vertical asymptote is x = 3.

For function #3, factor the numerator and the denominator.

$$h(x) = \frac{2x^2 + 6x}{x^2 - 3x} = \frac{2x(x+3)}{x(x-3)} = \frac{2(x+3)}{(x-3)}$$

The equation of the vertical asymptote is determined by setting the denominator equal to zero and solving.

x – 3 = 0

x = 3

The equation of the vertical asymptote is x = 3.

For function #4, factor the denominator.

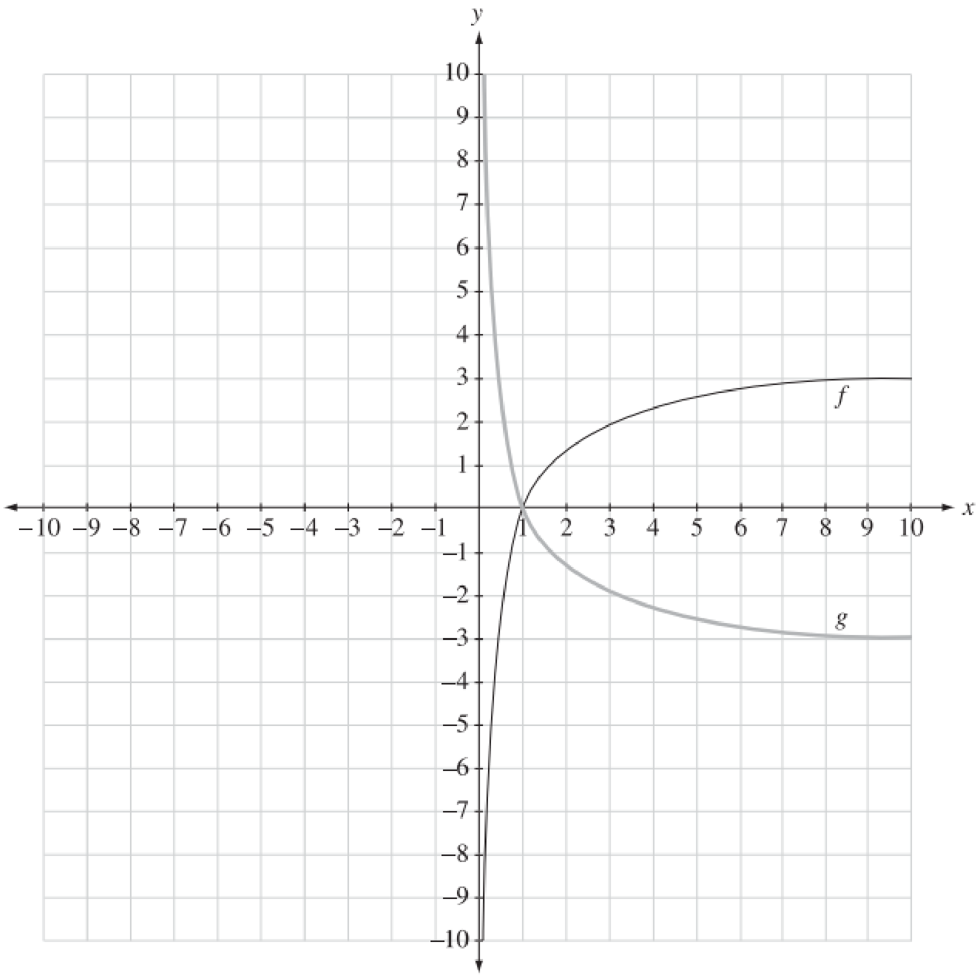
$$k(x) = \frac{x-3}{2x^2-6x} = \frac{x-3}{2x(x-3)} = \frac{1}{2x}$$

This function has a point of discontinuity at x = 3. There is no vertical asymptote.

The correct answer is D.

Use the following information to answer question 7.

The graphs of $f(x) = \log_a x$ and $g(x) = \log_b x$ are shown below.



7. When the equations of the above functions are compared, which of the following statements is true?

A. $a < b$

B. $b > 1$

C. $a > b$

D. $a < 1$

Solution

These two graphs are reflections in the x-axis. Given f(x) = $log_a x$, g(x) = -$log_a x$. Refer to the statement below regarding a logarithm with a fractional base.

Logarithm with a Fractional Base

Given a positive whole number b and $x > 0$, the following will always be true:

$$log_{\frac{1}{b}}(x) = -log_b(x)$$

When the reflection occurs, the base is reciprocated. Where f(x) has a base of ‘a’, g(x) has a base of $\frac{1}{a}$.

Thus, the relationship between bases ‘a’ and ‘b’ in this question is that ‘a’ > ‘b’.

The correct answer is C.

Use the following information to answer question 8.

Hala correctly solved the equation $\log_3(x+3) + \log_3(x-5) = 2$ using an algebraic process. She determined that one of the possible roots is extraneous.

8. The possible root that is extraneous is

A. $x = -6$

B. $x = -5$

C. $x = -4$

D. $x = -3$

Solution

Use the product law of logarithms to combine the two terms on the left side of the equal sign. $log_3(x+3)(x-5) = 2$

Convert to exponential form.

3^2 = (x + 3) (x – 5)

9 = x^2 - 2x -15

0 = x^2 – 2x – 24

0 = (x – 6) (x + 4)

x = 6 or x = -4

If x = -4, the logarithmic expressions would be undefined since it is not possible to take the log of a negative number.

Therefore, -4 is an extraneous solution.

The correct answer is C.

Use the following information to answer question 9.

The apparent brightness of stars is expressed in terms of magnitude, M, on a numerical scale that increases as the brightness decreases, as given by the formula

$$M = 6 - 2.5 \log\left(\frac{L}{L_0}\right)$$

where L is the light flux of a given star and L_0 is the light flux of the dimmest star visible to the unaided human eye.

9. How many times greater is the light flux from a star with a magnitude of –1.5 than the light flux from a star with a magnitude of 3.5?

A. 100
B. 1 000
C. 10 000
D. 100 000

Solution

Substitute the magnitude values of (-1.5) and then (3.5) to find the light flux.

-1.5 = 6 – 2.5 log L

2.5 log L = 6 + 1.5

2.5 log L = 7.5

log L = 3

10^3 = L

1000 = L

3.5 = 6 – 2.5 log L

$2.5 \log L = 6 – 3.5$

$2.5 \log L = 2.5$

$\log L = 1$

$10^1 = L$

$10 = L$

Comparing 1000 with 10, the light flux from a star with a magnitude of -1.5 is 100 times greater than the light flux from a star with a magnitude of 3.5.

The correct answer is A.

Use the following information to answer question 10.

Four statements about the polynomial function $P(x) = x^3 – 2x^2 – 13x – 10$ are shown below.

Statement 1	When $P(x)$ is divided by $(x + 1)$, the quotient is $x^2 – 3x – 10$.
Statement 2	$P(x)$ has a factor of $(x – 1)$.
Statement 3	$P(–2) = 0$, so $(x – 2)$ is a factor of $P(x)$.
Statement 4	$P(–3) = –16$, so $–16$ is the remainder when $P(x)$ is divided by $(x + 3)$.

10. The two statements about $P(x)$ that are correct are statements

A. 1 and 3
B. 1 and 4
C. 2 and 3
D. 2 and 4

Solution

For statement 1, use synthetic division.

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -13 & -10 \\ & & -1 & 3 & 10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

The quotient is $x^2 - 3x - 10$.

Statement 1 is true.

For statement 2, if P(x) has a factor of (x – 1), it would mean that substituting x = 1 into the equation would result in a remainder of 0.

$P(1) = (1)^3 - 2(1)^2 - 13(1) - 10$

$P(1) = 1 - 2 - 13 - 10$

$P(1) = -24$

Thus, (x – 1) is not a factor. Statement 2 is false.

For statement 3, if P (-2) = 0, then (x + 2) is a factor. Statement 3 is false.

Statement 4 is true.

The correct answer is B.

Use the following information to answer question 11.

Michael graphed the function $y = a(x - b)^2(x + c)$, where $a, b, c > 0$.

11. If Michael changes the function so that $a < 0$, and then compares the new graph to the graph of the original function, then there will be a change in the

A. domain

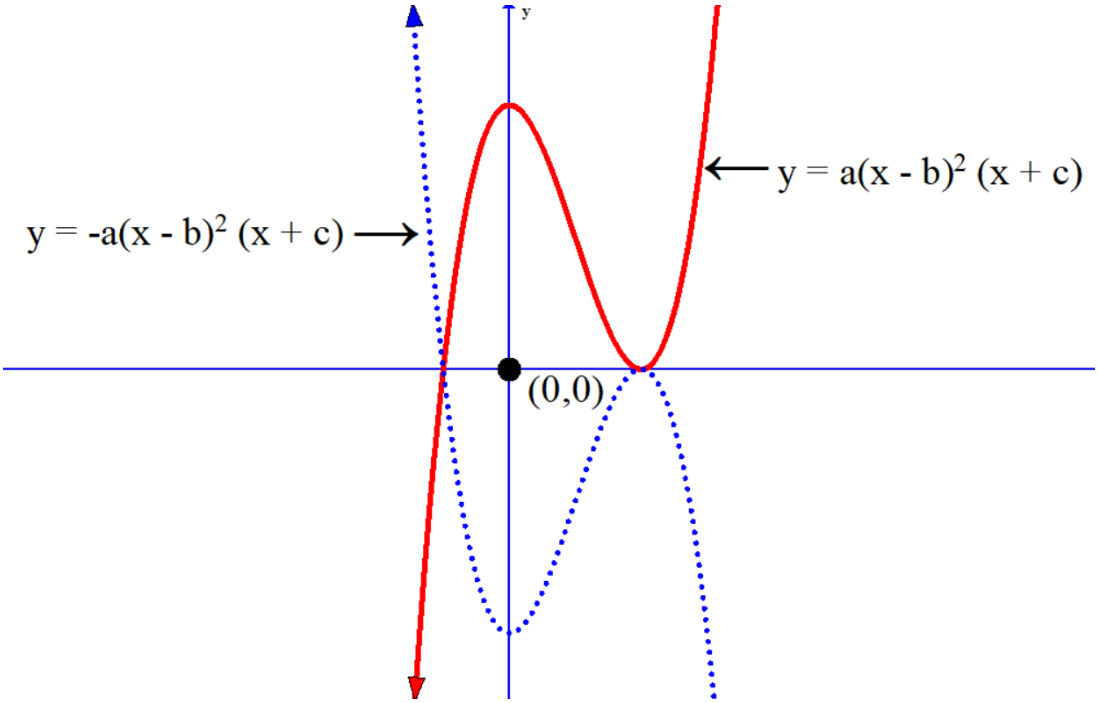
B. range

C. x-intercepts

D. y-intercept

Solution

Select some appropriate values for a, b, and c and draw a sketch to see the function on a graph.



Make the ‘a’ value negative and draw a new sketch (blue dotted).

For both graphs, the domain and range are the set of real numbers.

The x-intercepts will remain the same.

The y-intercept changes.

The correct answer is D.

Use the following information to answer question 12.

The graph of the function $P(x) = a(x - r)(x - 1)^2(x - 4)$ passes through the point (0, 6).

12. The value of $\boldsymbol{a}$ in terms of $\boldsymbol{r}$ is

A. $a = \frac{3}{2r}$

B. $a = -\frac{3}{2r}$

C. $a = \frac{2}{3r}$

D. $a = -\frac{2}{3r}$

Solution

In terms of 'r' means that the answer will have an 'r' in it. We do not know what the value is for 'r', but we can still isolate 'a'.

If a graph passes through a point, it means that the point will satisfy the equation. Substitute the point (0,6) for 'x' and 'y' in the equation.

$6 = a\,((0) - r)\,((0) - 1)^2\,((0) - 4)$

$6 = a(-r)\,(1)\,(-4)$

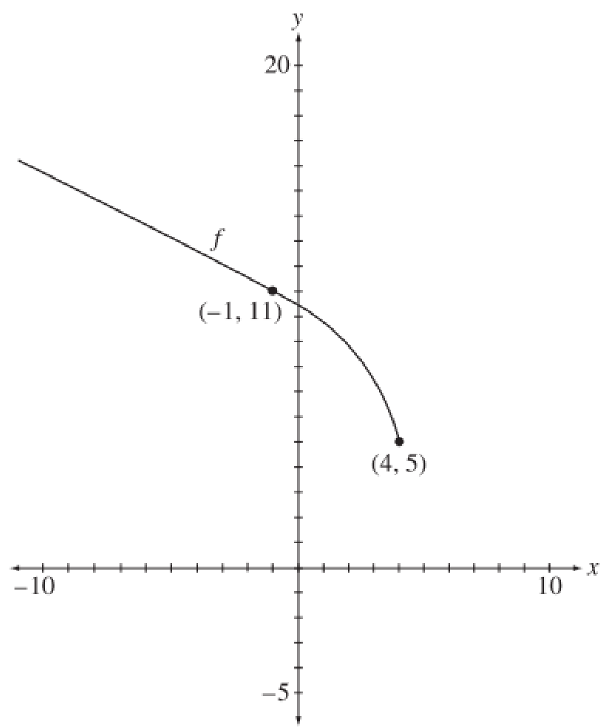
$6 = 4ar$

$$a = \frac{6}{4r} = \frac{3}{2r}$$

The correct answer is A.

Use the following information to answer numerical-response question 4.

The graph of $y = f(x)$ is shown below.



The equation for the graph of $y = f(x)$ can be written in the form $f(x) = a\sqrt{-(x-h)} + k$.

Numerical Response

4. The value of ***a***, to the nearest hundredth, is ___________.

Solution

Compared to the base function, $y = \sqrt{x}$, there is vertical stretch by a factor of 'a', a reflection in the y-axis, a translation 4 units right and a translation 5 units up. Use the given point (-1, 11) to determine the value of 'a'.

$$11 = a\sqrt{-((-1) - 4)} + 5$$

$$11 = a\sqrt{5} + 5$$

$$6 = a\sqrt{5}$$

$$a = \frac{6}{\sqrt{5}}$$

a = 2.683…

The value of a to the nearest hundredth is 2.68.

13. Given $f(x) = 8 - 3x$, $g(x) = \left|\frac{1}{2}x - 5\right|$, and $h(x) = \log_{\frac{1}{2}} x$, the value of $(f \circ g \circ h)(16)$ is

A. –13

B. –5

C. 29

D. 480

Solution

We are dealing with composite functions, where the output of one function becomes the input of the next function.

Begin with determining h (16).

$$h(16) = log_{\frac{1}{2}}(16)$$

$$h(16) = \frac{log16}{\log(0.5)}, [using\ change\ of\ base]$$

h (16) = -4

The output for function h is -4 and this becomes the input for function g.

$$g(-4) = \left|\frac{1}{2}(-4) - 5\right|$$

$$g(-4) = |-7|$$

$$g(-4) = 7$$

The output for function g is 7 and this becomes the input for function f.

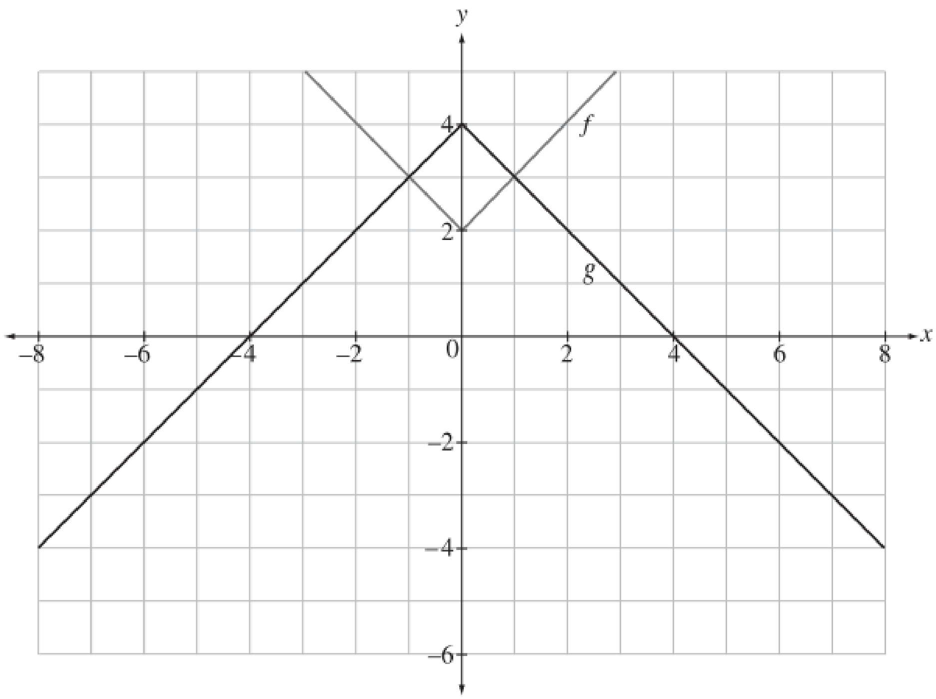
$$f(7) = 8 - 3(7)$$

$$f(7) = -13$$

The correct answer is A.

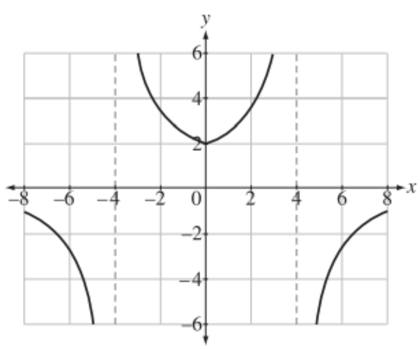
Use the following information to answer question 14.

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.

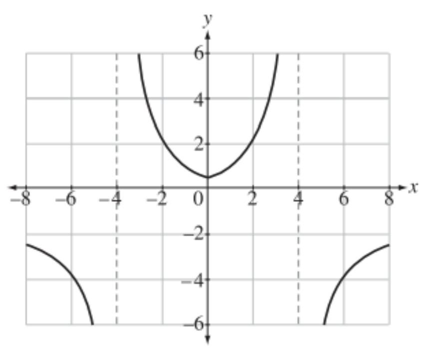


14. Which of the following graphs represents $y = \dfrac{f(x)}{g(x)}$?

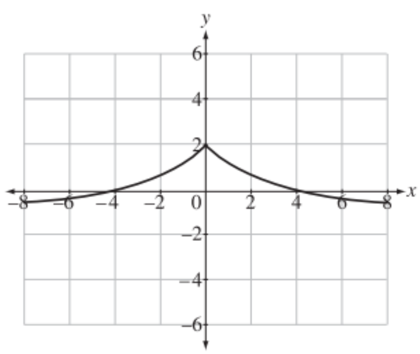
A.



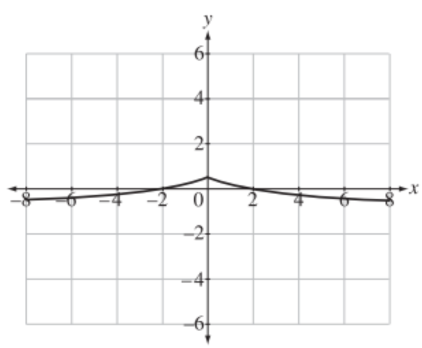
B.



C.



D.



Solution

As with all operations on functions, for a given 'x' value, perform the operation on the corresponding 'y' values.

For example, when x = 0, function f is 2 and function g is 4. Since this question involves quotients, $\frac{2}{4} = \frac{1}{2}$. The new function should have the point (0, ½).

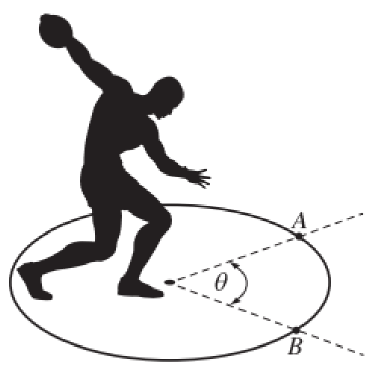
Options A and C are eliminated because they have the point (0,2).

On the original f(2) = 4 and g(2) = 2. Since $\frac{4}{2} = 2$, the point (2,2) should be on the new graph. Graph B shows this point.

The correct answer is B.

Use the following information to answer question 15.

Track and field athletes participating in the discus competition must stay in the throwing circle, which has a diameter of 2.5 m, and their discus must land within the boundary lines. The boundary lines are set from the centre of the circle through two points, *A* and *B*, on the circumference of the circle. The length of the arc between points *A* and *B* is 87 cm.



Note: The diagram is **not** drawn to scale.

15. Angle θ, to the nearest degree, is

A. 20°

B. 35°

C. 40°

D. 82°

Solution

This question involves a radius, a measure of an angle in radians, and an arc length. We will determine the measure of the angle in radians, and then convert to degrees.

Use the relationship, $radian = \frac{arc\ length}{radius}$

With a diameter of 2.5 m, the radius is 1.25 m. The arc length is 87 cm.

Converting so that we can use the same units, the radius is 125 cm, and the arc length is 87 cm.

$$radian = \frac{87}{125}$$

The angle measure is 0.696 radians.

Multiply this number by $\frac{180}{\pi}$

$$0.069\left(\frac{180}{\pi}\right) = 39.877 \ldots$$

Angle θ to the nearest degree is 40^0.

The correct answer is C.

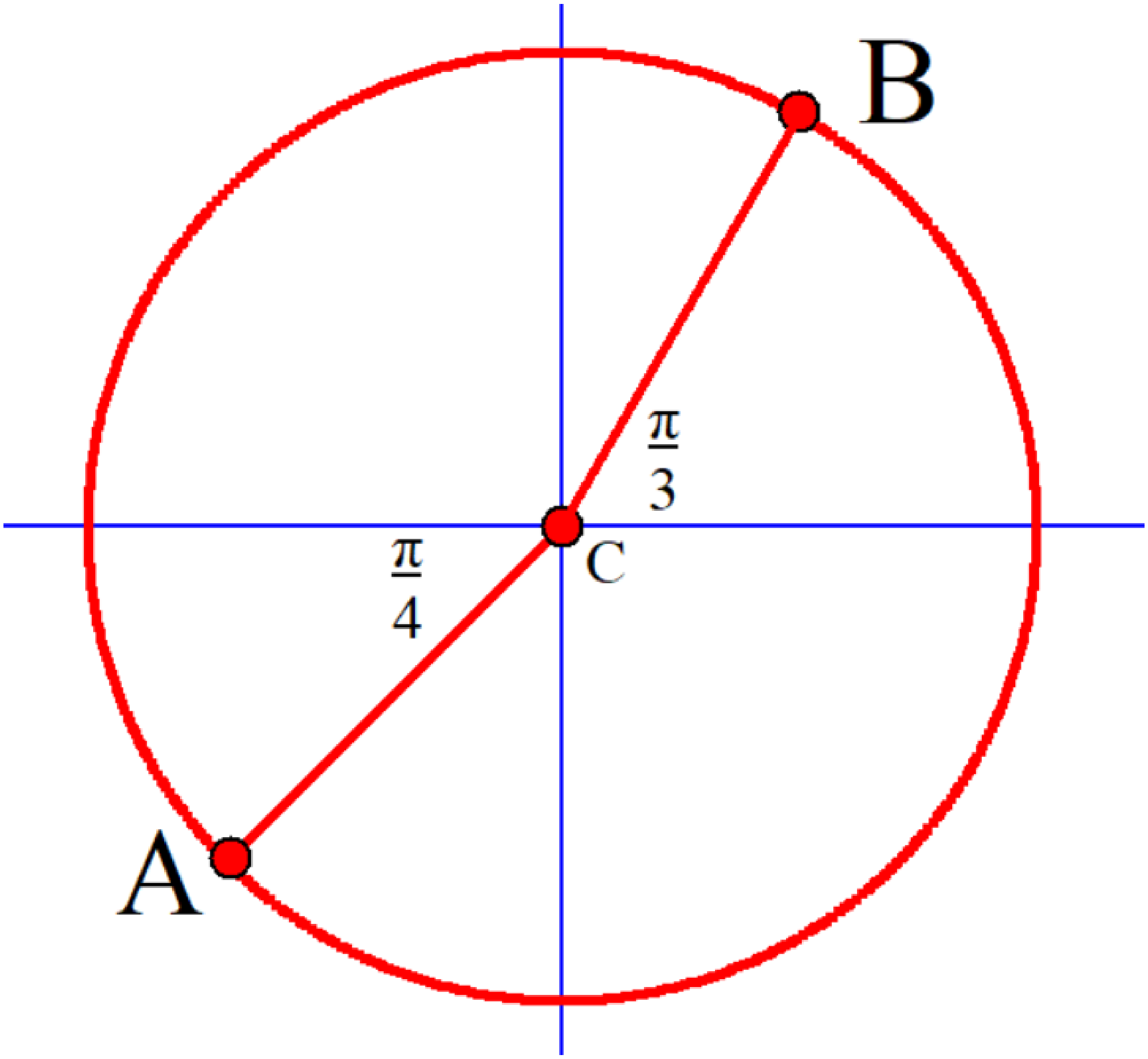
Use the following information to answer numerical-response question 5.

Points $A\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ and $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ lie on the unit circle.

Numerical Response

5. If Point C (0, 0) is the centre of the unit circle, then the measure, to the nearest hundredth of a radian, of the smaller Angle ACB is ____________ rad.

Solution



If point B rotates to π radians, it will have travelled $\frac{2\pi}{3}$ radians. Adding $\frac{2\pi}{3}$ to $\frac{\pi}{4}$ will determine the measure of the smallest angle ACB.

The sum of these two radian measures is 2.879…

To the nearest hundredth of a radian, the smaller angle ACB is 2.88.

2.88

Use the following information to answer question 16.

Angle θ is drawn in standard position on a unit circle. The point $P\left(x, -\frac{8}{17}\right)$, where $x > 0$, lies on the terminal arm of Angle θ.

16. What is the **exact** value of $\cot\theta$?

A. $-\frac{8}{\sqrt{353}}$

B. $-\frac{\sqrt{353}}{8}$

C. $-\frac{8}{15}$

D. $-\frac{15}{8}$

Solution

All points on the unit circle are (cosθ, sinθ). Thus, we know that $sin\theta = -\frac{8}{17}$. Since $cot\theta = \frac{cos\theta}{sin\theta}$, we need to find x, or cosθ.

Use the equation $sin^2\theta + cos^2\theta = 1$.

$$\left(-\frac{8}{17}\right)^2 + cos^2\theta = 1$$

$$cos^2\theta = 1 - \left(-\frac{8}{17}\right)^2$$

$$cos^2\theta = 1 - \left(\frac{64}{289}\right)$$

$$cos^2\theta = \left(\frac{289}{289}\right) - \left(\frac{64}{289}\right)$$

$$cos^2\theta = \left(\frac{225}{289}\right)$$

Take the square root of both sides.

$$cos\theta = \pm\left(\frac{15}{17}\right)$$

Since we are told in the question that x > 0, $cos\theta = \frac{15}{17}$

$$cot\theta = \frac{\frac{15}{17}}{-\frac{8}{17}} = \left(\frac{15}{17}\right)\left(-\frac{17}{8}\right) = -\frac{15}{8}.$$

The correct answer is D.

17. If $\csc\left(\frac{4\pi}{3}\right) + k = \cot\left(\frac{8\pi}{3}\right)$, then the **exact** value of k is

A. $-\sqrt{3}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Solution

Using the unit circle, $\left(\frac{4\pi}{3}\right)$ is in quadrant 3 and the reference angle is $\left(\frac{\pi}{3}\right)$ or 60^0. Using the special triangle, $30^0 - 60^0 - 90^0$, we know that $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$. Since we must be in quadrant 3 and sin is negative in this quadrant, $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$. Now take the reciprocal, and we get $\csc\left(\frac{4\pi}{3}\right) = -\frac{2}{\sqrt{3}}$.

Since 1 complete rotation in the unit circle is 2π radians, or $\frac{6\pi}{3}$ radians, for $\left(\frac{8\pi}{3}\right)$, we have rotated a full rotation, and then another $\left(\frac{2\pi}{3}\right)$, placing the terminal arm in quadrant 2, with a reference angle of $\left(\frac{\pi}{3}\right)$ or 60^0. Again, using this specific special triangle, $tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$. Since we want cot, reciprocate this ratio.

Thus, $cot\left(\frac{8\pi}{3}\right) = -\frac{1}{\sqrt{3}}$.

We now have an equation to solve K.

$-\frac{2}{\sqrt{3}} + K = -\frac{1}{\sqrt{3}}$.

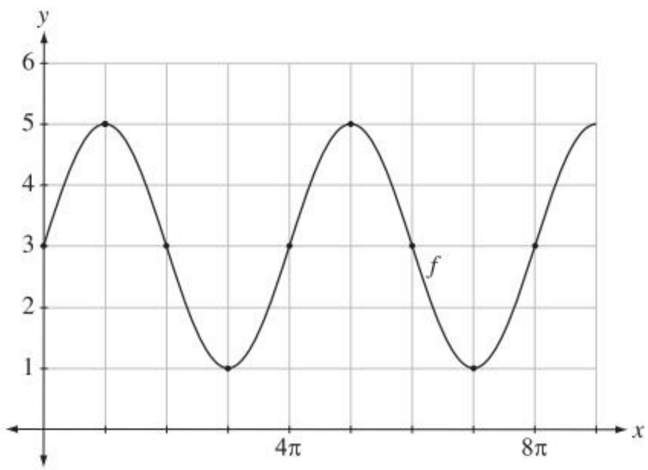
$K = -\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}.$

$$K = \frac{1}{\sqrt{3}}$$

The correct answer is C.

Use the following information to answer question 18.

The graph of the function $f(x) = a\cos[b(x-c)] + d$, where $a > 0$, is shown below.



18. An equation for $f(x)$ with the correct values for the parameters $\boldsymbol{b}$ and $\boldsymbol{c}$ is

A. $f(x) = a\cos\left[\frac{1}{2}(x-\pi)\right] + d$

B. $f(x) = a\cos\left[\frac{1}{2}\left(x-\frac{\pi}{2}\right)\right] + d$

C. $f(x) = a\cos[2(x-\pi)] + d$

D. $f(x) = a\cos\left[2\left(x-\frac{\pi}{2}\right)\right] + d$

Solution

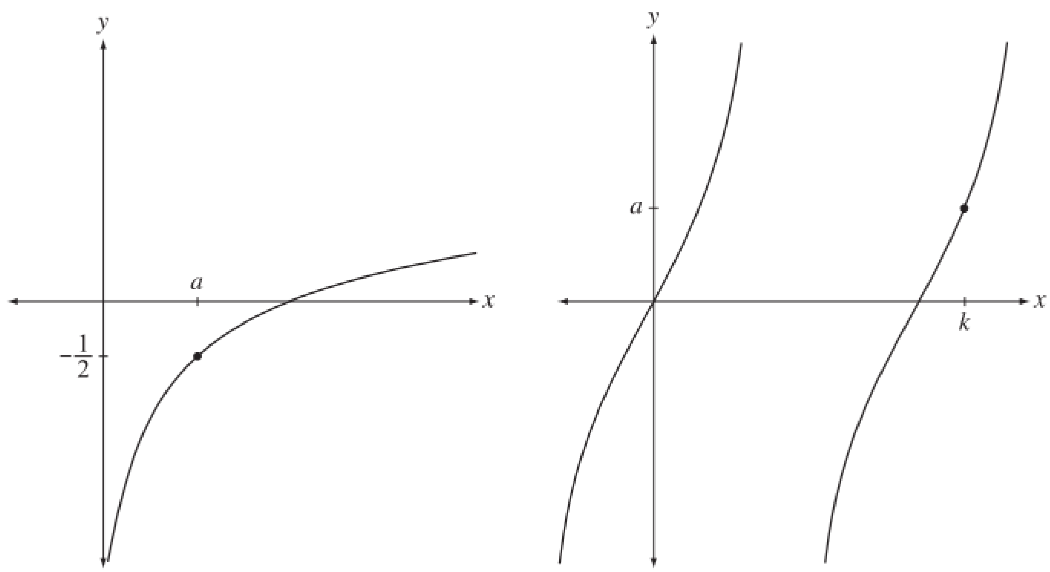
The parameter ‘b’ is related to the period. By looking at the graph, we can pick two distinguishable points, such as (0,3) and (4π,3), that show the beginning and ending of one complete cycle. We read the period off the x-axis. The period is 4π. The 'b' value in the equation is found by, $b = \frac{2\pi}{period} = \frac{2\pi}{4\pi} = \frac{1}{2}$. The answer must be A or B.

The phase shift, c, of a cosine graph is determined in relation to the maximum value. If the maximum value is on the y-axis, there is no phase shift. In this question, the closest maximum value to the y-axis, is π units to the right. Showing this in the equation, would be (x – π).

The correct answer is A.

Use the following information to answer question 19.

The graphs of $y = \log_3 x$ and $y = \tan x$ are shown below. The point $\left(a, -\frac{1}{2}\right)$ is marked on the graph of $y = \log_3 x$, and the point (k, a) is marked on the graph of $y = \tan x$.



19. The value of ***k*** is

A. 120°

B. 150°

C. 210°

D. 240°

Solution

Use the first graph to determine the value of ‘a’, where ‘a’ represents the x coordinate on the graph. Substitute into the equation.

$y = \log_3 x$

$$-\frac{1}{2} = log_3 x$$

Re-write in exponential form.

$$3^{-\frac{1}{2}} = x$$

$$x = \frac{1}{3^{\frac{1}{2}}} = \frac{1}{\sqrt{3}}$$

Going to the second graph, y = tan x, the given point is (k, a), where ‘k’ represents the x- coordinate and ‘a’ represents the y-coordinate.

Substitute into the equation.

y = tan x

$$\frac{1}{\sqrt{3}} = \tan x$$

What value of x, in degrees, will yield a tan ratio of $\frac{1}{\sqrt{3}}$?

From the graph, we can see that the value can’t be in quadrant 1 because of the asymptote at 90^0. The only other quadrant where tangent is positive is in quadrant 3. Using special triangles, the reference angle in this quadrant must be 30^0. Therefore, the rotation angle is 210^0.

The correct answer is C.

Use the following information to answer numerical-response question 6.

Students are solving an equation of the form $(2\sin x - 2)(a\sin x + b) = 0$, where $a > 0$, $b > 0$, and $a > b$.

Numerical Response

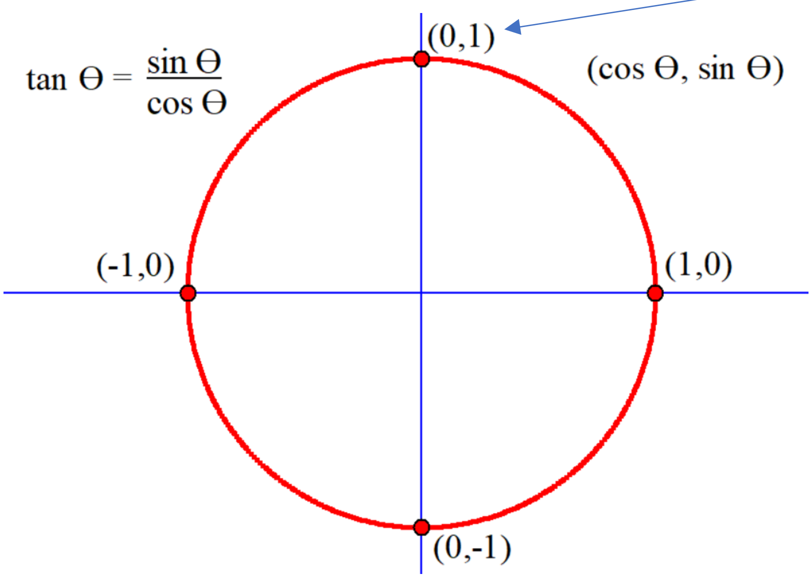
6. If the domain is $0 \le x < 2\pi$, then the **number** of solutions to this equation is __________.

Solution

When dealing with the first binomial, (2 sin x – 2), set it equal to zero and isolate sin x.

sin x = 1.

Using this unit circle tool, we can see that the **only** solution in the domain is $\frac{\pi}{2}$ radians.



When dealing with the second binomial, (a sin x + b), we can't determine the specific solutions because we do not know the values of 'a' and 'b'. But since we know that

a > b, when setting this binomial equal to zero and isolating sin x, the ratio will be between 0 and 1. Being that both letters are positive, using the CAST rule tells us that there are two positive sine ratios (and thus **2 solutions**) in quadrants 1 and 2.

The number of solutions in the domain is 3.

Use the following information to answer question 20.

Four students wrote the general solution to the equation $10\cos^2\theta = 5$ as shown below.

Sandy $\theta = \frac{\pi}{4} + n\pi,\ n \in I$

Noah $\theta = \frac{\pi}{4} + \frac{n\pi}{2},\ n \in I$

Luke $\theta = \frac{\pi}{4} + 2n\pi,\ n \in I,$ and $\theta = \frac{7\pi}{4} + 2n\pi,\ n \in I$

Jane $\theta = \frac{\pi}{4} + n\pi,\ n \in I,$ and $\theta = \frac{3\pi}{4} + n\pi,\ n \in I$

20. The two students who have stated a correct general solution are

- **A.** Sandy and Luke
- **B.** Sandy and Jane
- **C.** Noah and Luke
- **D.** Noah and Jane

Solution

$$10cos^2\theta = 5$$

$$cos^2\theta = \frac{1}{2}$$

$$cos\theta = \pm\frac{1}{\sqrt{2}}$$

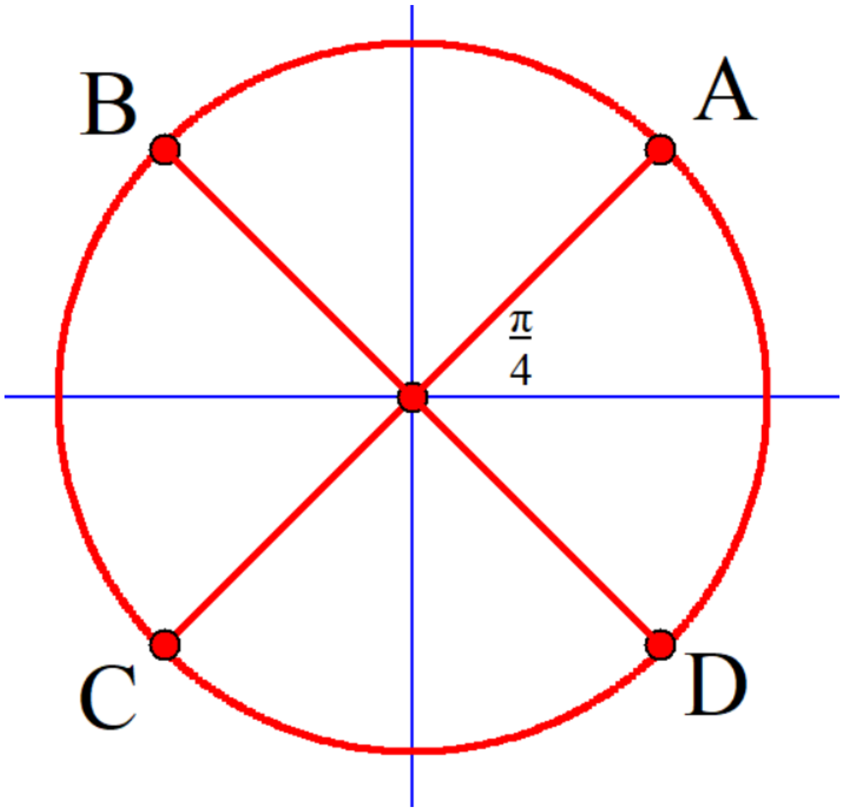
The reference angle is $\frac{\pi}{4}$ and because there are two positive and two negative ratios, there is a terminal arm in each of the 4 quadrants.

Referring to the diagram below, Sandy cannot be correct because her solution starts with $\frac{\pi}{4}$ at Point A, and then adding π will get to Point C and will continue to just include these two solutions. It misses the solutions in quadrants 2 and 4.

Luke can also not be correct. He starts at $\frac{\pi}{4}$ and $\frac{7\pi}{4}$, which are both valid solutions, but by adding or subtracting multiples of 2π, the other two solutions will not be included.

Noah's solution is correct. He says to start at $\frac{\pi}{4}$, and then with a $\frac{\pi}{2}$ rotation, we will consistently arrive at the next possible solution.

Jane's solution is also correct. She has two starting points, $\frac{\pi}{4}$ and $\frac{3\pi}{4}$. Adding π to each of these solutions, will consistently include all 4 of the possible solutions.



The correct answer is D.

Use the following information to answer question 21.

A student incorrectly simplified the expression $\left(\dfrac{\frac{2\cot x}{\csc^2 x}}{\cos^2 x+\sin^2 x}\right)$. The steps of the student's work are shown below.

Step 1 $\left(\dfrac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}}\right)\left(\dfrac{1}{\cos^2 x+\sin^2 x}\right)$

Step 2 $\left(\dfrac{2\cos x}{\sin x}\cdot\dfrac{\sin^2 x}{1}\right)\left(\dfrac{1}{1+\sin^2 x+\sin^2 x}\right)$

Step 3 $(2\cos x\sin x)\left(\dfrac{1}{1+2\sin^2 x}\right)$

Step 4 $\dfrac{\sin(2x)}{\cos(2x)}$

Step 5 $\tan(2x)$

21. The student's **first** error was recorded in Step

A. 1

B. 2

C. 3

D. 4

Solution

In step 2, the student substituted $(1 + \sin^2 x)$ for $\cos^2 x$. It should have been $(1 - \sin^2 x)$

On the formula sheet, the identity, $\sin^2 x + \cos^2 x = 1$, can be written in two equivalent forms.

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

The correct answer is B.

Use the following information to answer numerical-response question 7.

Eden is booking a vacation package. She has the option of travelling by train or by plane. The train has only one type of ticket. If she takes the plane, she must choose a regular ticket or a first-class ticket. Regardless of whether she travels by train or by plane, she must make a meal choice during travel of chicken, beef, or vegetarian. At her destination, she must choose one of three different hotels.

Numerical Response

7. The number of different vacation packages that Eden can select is __________.

Solution

Use the Fundamental Counting Principle.

There are 3 stages:

Type of Ticket		Meal Choice		Hotels
3	X	3	X	3

The product is 27.

The number of different vacation packages that Eden can select is 27.

22. The number of distinct arrangements using all 10 of the letters in the word **ACCESSIBLE** if the vowels, AEIE, must all be together is

A. 60 480
B. 15 120
C. 8 640
D. 2 160

Solution

Place the 4 vowels together in one stage. With the 6 remaining letters, there is a total of 7 stages.

4 Vowels	letter	letter	letter	letter	letter	letter

For the first part of the answer, we will not worry about the ordering of the vowels in their specific stage. We will just be concerned about the 4 vowels being together in any one of the 7 stages.

As this is done, we must be concerned about the repetition within the 6 remaining letters. There are two Cs and two Ss to go along with the B and the L.

This first part of the answer will be determined by $\left(\frac{7!}{2!2!}\right)$ or 1260.

For the second part, we will now focus on the ordering of the 4 vowels within their unique stage. We will account for the repetition of the two Es. This part of the answer is determined by $\left(\frac{4!}{2!}\right)$ or 12.

The answer is the product of (1260) (12).

The correct answer is B.

Use the following information to answer question 23.

Josef is selecting 5 songs to be randomly played between games at a basketball tournament. Each of the 10 players on the team suggested a different song. The coach suggested 4 songs that are all different from those suggested by the players.

23. The number of different possible selections of songs that Josef can make that use exactly 2 of the coach's suggestions is

A. 8 640
B. 2 002
C. 720
D. 126

Solution

We can break this question into two stages.

Songs Suggested By The Coach

There is a total of 4 and we want exactly 2.

Songs Suggested By The Players

There is a total of 10 and we want exactly 3 (to make the overall total of 5)

$_4C_2$ X $_{10}C_3$

(6) X (120)

= 720

The correct answer is C.

24. In the expansion of $(x - 2y)^8$, written in descending powers of x, the coefficient of the middle term is

A. 1 120
B. –1 120
C. 1 792
D. –1 792

Solution

There are 9 terms, and the middle term is 5th. K = 4

${}_8C_4\ (x)^4\ (-2y)^4$

$(70)\ (x^4)\ (16y^4)$

$1120x^4y^4$

The correct answer is A.

Use the following information to answer numerical-response question 8.

In the expansion of the binomial $(x + a)^b$ written in descending powers of x, given $a, b \in N$, the second and third terms are $24x^7$ and $252x^6$, respectively.

Numerical Response

8. The values of $\boldsymbol{a}$ and $\boldsymbol{b}$ are, respectively, _____ and _____.

Solution

Since the 2^{nd} term has an exponent of 7 on the variable, the 1^{st} term must have an exponent of 8. That would mean that b = 8.

For the 2^{nd} term, k = 1. Use the general term to find 'a'.

$t_{k+1} = {}_nC_K\ x^{n-k}\ y^k$

$t_{1+1} = {}_8C_1\ (x)^7 (a)^1$

$24x^7 = (8)\ (x^7)\ (a)$

$24x^7 = (8a)\ (x^7)$

$24 = 8a$

$3 = a$

Confirm the 3^{rd} term, $252x^6$, by using the binomial theorem to expand $(x + 3)^8$.

$t_3 = {}_8C_2\ (x)^6\ (3)^2$

$t_3 = (28)\ (x)^6\ (9)$

$t_3 = 252x^6$

The values of a and b respectively are 3 and 8.