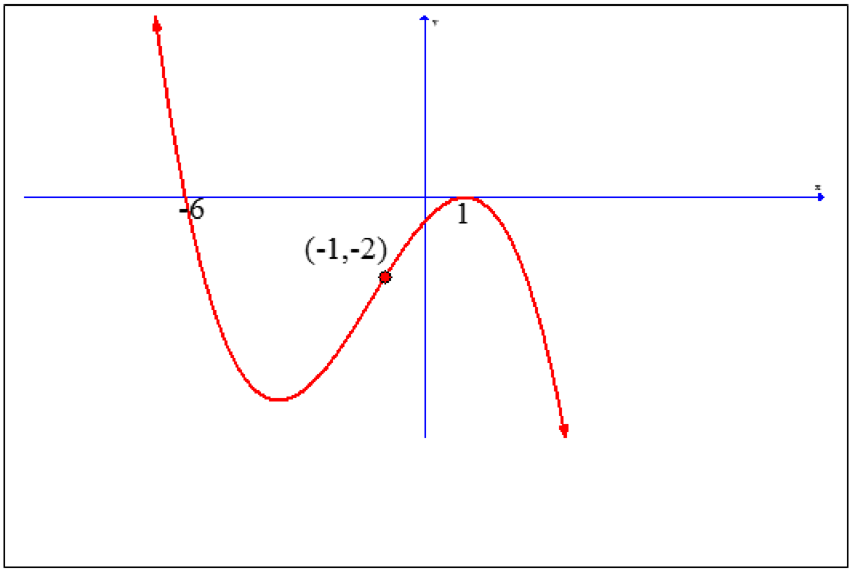
Characteristics of Polynomial Functions Solutions

Use the following graph to answer the first 3 questions.



1. The polynomial function above can be written in the form $y = a(x + m)(x - n)^2$. The values of m and n respectively, are

   a) 1 and -6 b) 1- and -2 c) 6 and 1 d) -6 and 1

The root of 1 has a multiplicity of 2. Therefore, the value of n is 1.

The root of -6 has a multiplicity of 1. Therefore, the value of m is 6.

2. The polynomial function above can be written in the form $y = a(x + m)(x - n)^2$. The value of a is

   a) $\frac{-1}{10}$ Ans. b) $\frac{1}{10}$ c) 10 d) -10

Substitute the point (1,-2) for 'x' and 'y', as well as the values for 'm' and 'n' and solve the equation for 'a'.

$-2 = a((-1) + 6)((-1) - 1)^2$

$-2 = a(5)(-2)^2$

$-2 = 20a$

$a = -\frac{1}{10}$

3. The value of the y-intercept is

a) $\frac{3}{5}$ b) $\frac{5}{3}$ c) $\frac{-5}{3}$ d) $\frac{-3}{5}$

To find the y-intercept, set x = 0 and solve for y.

$y = -1/10\ (0 + 6)\ (0 - 1)^2$

$y = -1/10\ (6)$

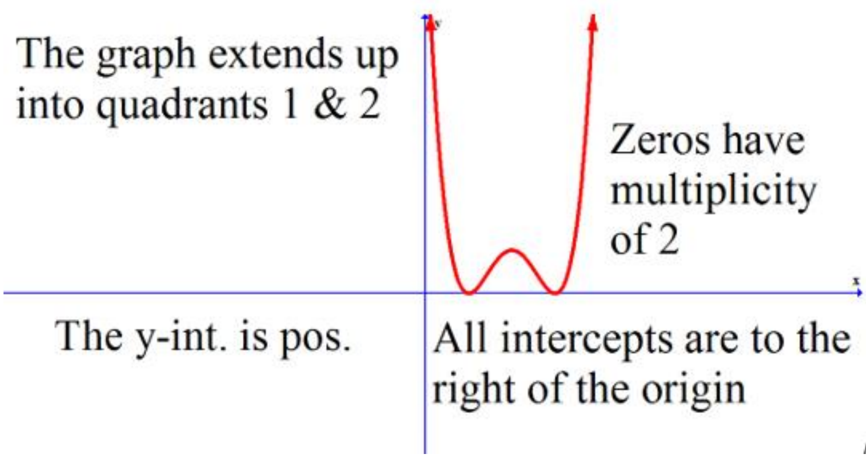
$y = -6/10$ or $-3/5$

Use the following information to answer the next question.

Given $P(x) = a(x - b)^2 (x - c)^2$, where a, b, and c > 0, a student makes the following observations:

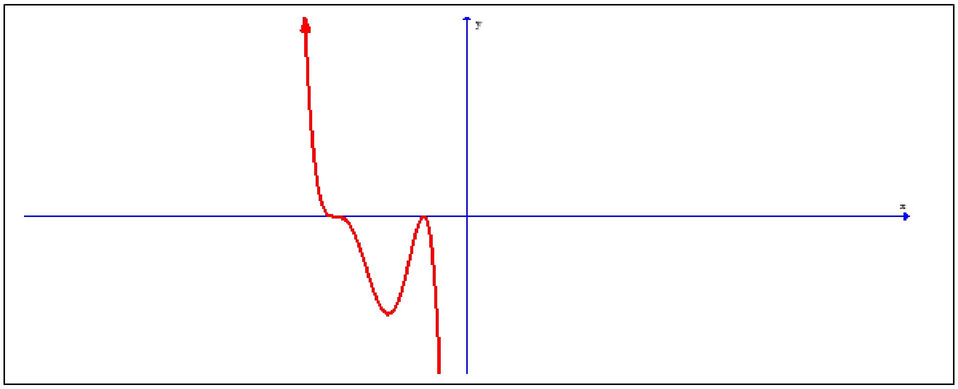
1) The graph extends down into quadrant 3 and up into quadrant 1.
2) All x-intercepts are to the right of the origin.
3) The zeros each have a multiplicity of 2.
4) The y-intercept is negative.

4. The two correct observations are _____ and _____.



The 2 correct observations are 2 and 3.

Use the graph and possible characteristics chart below to answer the next question.



Possible Characteristics

Equation	Sign of 'a'	Values of 'b' and 'c'
1. $y = ax(x - b)(x - c)^3$	**2.** Positive	**3.** $b < 0$ and $c < 0$
4. $y = a(x - b)^2(x - c)^3$	**5.** Negative	**6.** $b > 0$ and $c > 0$

5. The 3 numbers to represent a possible equation of the graph, the sign of 'a' and the signs of 'b' and 'c' are _4__, _5__, and _3__.

Since one zero has a multiplicity of 3 and the other zero has a multiplicity of 2, the minimum possible degree of this polynomial is 5.

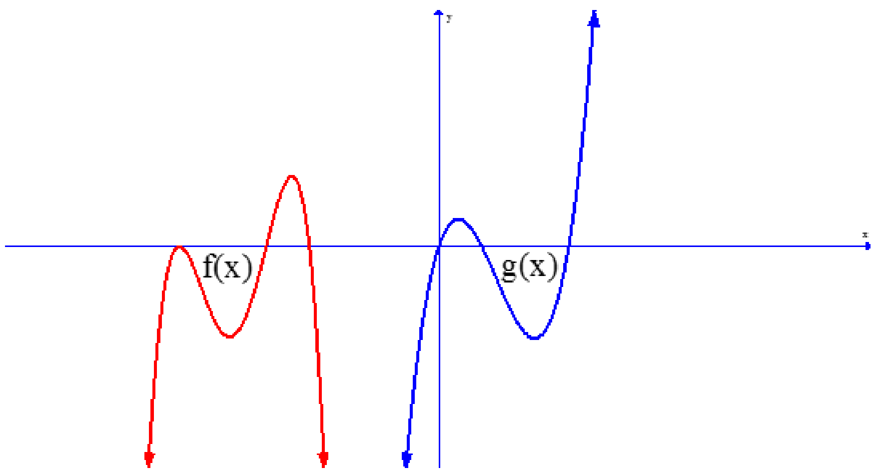
The equation choice is #4 because the multiplicities match with the exponents on the binomials.

For a 5^{th} degree polynomial, rising up into quadrant 2 indicates that the coefficient 'a' is negative.

Since both of the zeros are to the left of the origin, the values of both 'b' and 'c' are negative. Therefore, $b < 0$ and $c < 0$.

Use the graph below to answer the next question.

Two different polynomial functions, f(x) and g(x) are displayed in the graph. When answering the questions, use 1 for f(x) and 2 for g(x).



6. a) Which graph could be a degree of 4? __1__

The graph of f(x) has 2 zeros with multiplicity of 1, and 1 zero with a multiplicity of 2. The sum of these multiplicities is 4, meaning the graph could be a 4^{th} degree polynomial. The graph of g(x) has 3 zeros each with a multiplicity of 1.

b) Which graph has a positive leading coefficient? __2__

c) Which graph has a zero with a multiplicity other than 1? __1__

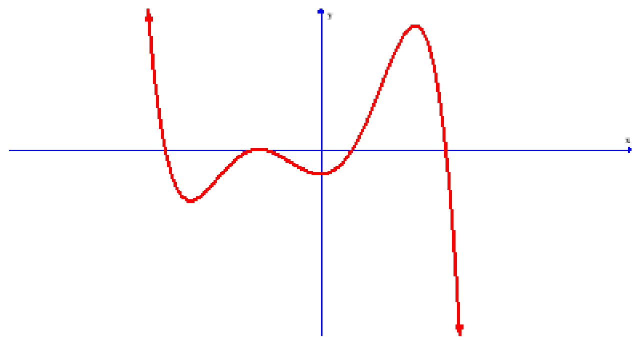
d) Which graph has the largest y-intercept? __2__

e) Which graph has the smallest x-intercept? __1__

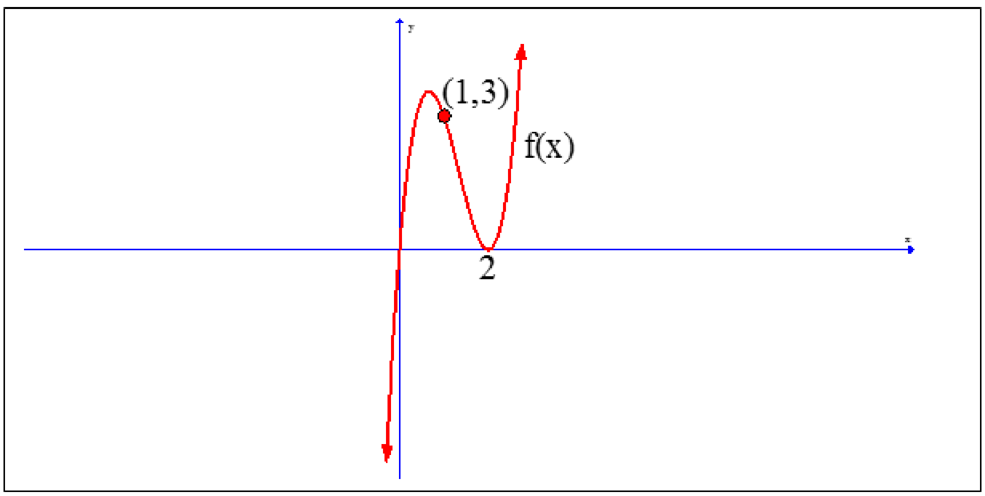
f) Which graph has a domain different from its range? __1__

The domain and range of g(x) are both the set of real numbers. The domain of f(x) is the set of real numbers, and the range all y values less than the maximum value.

7. Sketch a 5^{th} degree polynomial, with 1 zero having a multiplicity of 2 and a negative leading coefficient.



Use the graph below to answer the next question.



8. The graph of y = f(x) above can be written in the form $y = ax(x - m)^2$.
A)What are the values of a and m?

This is a 3^{rd} degree polynomial, with a positive leading coefficient. The zeros are 2 and 0. Since the zero of 2 has a multiplicity of 2, the value of m is 2.

Substitute the point (1,3) to find the value of 'a'.

$3 = a(1)((1) - 2)^2$

$3 = a(1)$

$a = 3$

The value of 'a' is 3 and the value of 'm' is 2.

b) When f(x) is expanded to the form $y = ax^3 + bx^2 + cx + d$, what is the value of both c, and the constant?

$y = 3x(x - 2)(x - 2)$

$y = 3x(x^2 - 4x + 4)$

$y = 3x^3 - 12x^2 + 12x$

The value of c is 12 and the value of the constant is 0.

9. Which of the following is not an example of a polynomial? Explain.

$f(x) = -5x^3 - 7x + 1$

$g(x) = 2x^{-2} + 6x - 9$

The function g(x) is not a polynomial because the exponent cannot be negative.