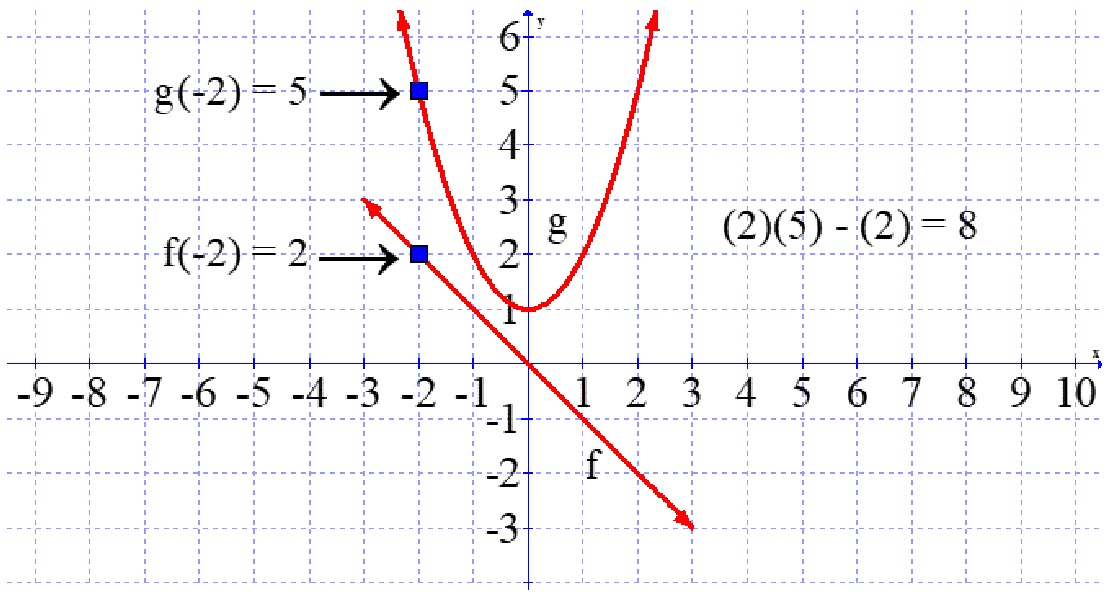
Name ______________

Function Operations and Rational Functions Litmus Test Solutions

Part A Place the correct answer on the sheet provided. Each question is worth one mark.

Use the following information to answer the first question.

The graph below displays 2 functions, y = f(x) and y = g(x). A new function is given by, h(x) = [f(x)g(x)] – f(x).



1. The value of h(-2) is

A) 2 B) 4 C) 6 D) 8

2. The domain and range for both linear functions, $f(x) = 5x + 2$, and $g(x) = x - 3$, is $x \in R$ and $y \in R$. For which operation applied to these functions will the domain remain the same, but the range will change?

   A) $h(x) = f(x) + g(x)$
   B) $h(x) = f(x) - g(x)$
   C) $h(x) = f(x)\,g(x)$
   D) $h(x) = \left(\frac{f}{g}\right)(x)$

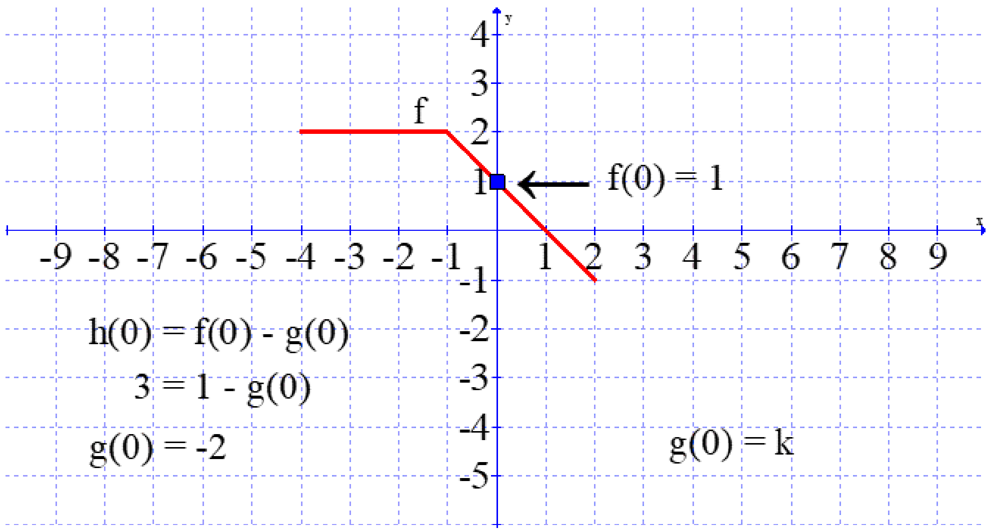
Solution

When 2 linear functions are added or subtracted, as with the case of options a) and b) above, the result is a linear function. Both the domain and range will remain the same.

In option d) above, dividing 2 linear functions results in a rational function. Both the domain and the range will **not** be $x \in R$.

The answer is c). Multiplying these 2 linear functions results in a quadratic function. The domain will be $x \in R$, but the range will change, depending on the location of the minimum or maximum point.

Use the following information to answer the next question.



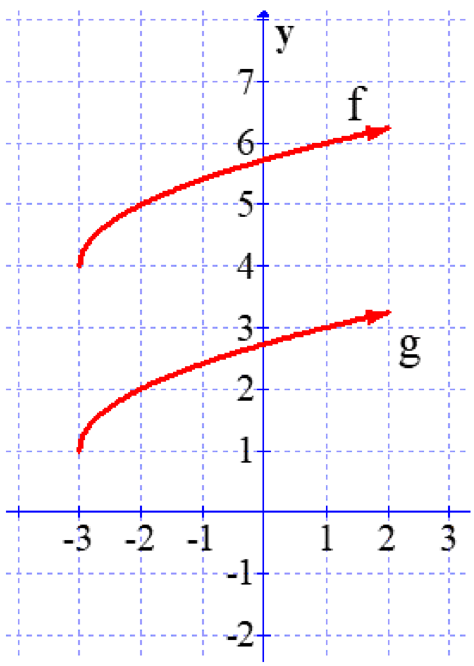
x	g(x)
-1	$\frac{-4}{3}$
0	**k**
2	undefined
3	4
4	2

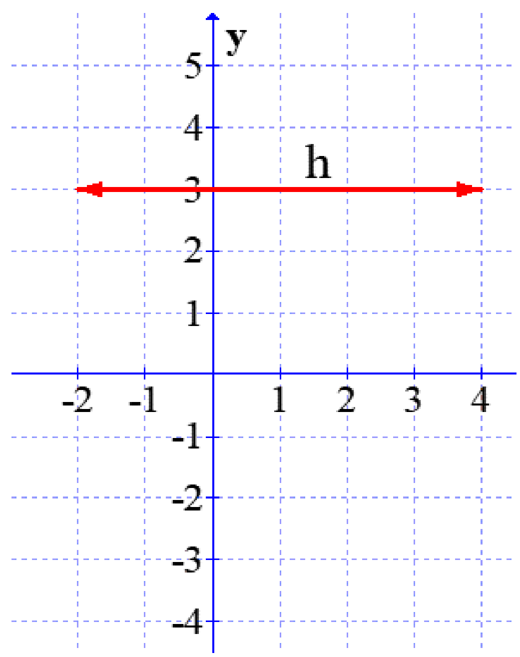
3. The graph above shows y = f(x) and the table of values above shows y = g(x). If h(x) = f(x) – g(x), and h(0) = 3, what is the value of **K** in the table of values above?

A) 1 B) 2 C) -1 D) -2

Use the following information to answer the next question.

The following graphs of y = f(x) and y = g(x) are shown on the left, and the graph of y = h(x) is shown on the right.





4. Which of the following statements is correct?

   A) h(x) = g(x) – f(x)
   B) h(x) = g(x) f(x)
   C) h(x) = g(x) + f(x)
   D) h(x) = f(x) – g(x)

Solution

For each value of x, when g(x) is subtracted from f(x), the value is always 3. For example, when x = -2, f(-2) = 4 and g(-2) = 1. The difference is 3. The correct answer is D.

5. Given the functions $f(x) = 2^x$; $g(x) = x^2$ and $h(x) = 2x$, a simplified expression for $k(x) = (h \circ g \circ f)(x)$ is

A) 2^{3x} B) 4^{2x} C) 4^{x+1} D) 2^{2x+1}

Solution

Begin by substituting function f into function g.

$g(2^x) = (2^x)^2$

$g(2^x) = 2^{2x}$

Now substitute this expression into function h.

$h(2^{2x}) = 2(2^{2x})$

$h(2^{2x}) = (2^1)(2^{2x})$

$h(2^{2x}) = 2^{2x+1}$

6. If $f(x) = 2x - 16$; $g(x) = \log_2 x$; and $h(x) = \frac{g(x)}{f(x)} + (f \circ g)(x)$,

then h(16) is

A) -7.75 B) -2.25 C) 5.5 D) 11.25

Solution

$$h(16) = \frac{g(16)}{f(16)} + (f \circ g)(16)$$

$$h(16) = \frac{\log_2 16}{2(16) - 16} + [2(\log_2 16) - 16]$$

h(16) = $\frac{4}{16} + -8$

h(16) = -7.75

Use the following information to answer the next question.

f(x) = $\frac{x+3}{x^2 - x - 12}$

7. In relation to the function f(x) above, which of the statements below is true?

   A) The equation for the vertical asymptote is x = 3. False. x = 4
   B) The equation for the horizontal asymptote is y = 0. True
   C) The point of discontinuity is (-3, $\frac{1}{7}$). False. (-3, -1/7)
   D) The y-intercept is -0.5. False. It is -0.25

The factored form of f(x) is $\frac{x+3}{(x+3)(x-4)} = \frac{1}{x-4}$

8. For the rational expression $\frac{(x+c)(2x-c)}{3m(x+c)}$, the point of discontinuity, in terms of c and m is

A) $\left(-c, \frac{3c}{m}\right)$

B) $\left(c, \frac{-c}{m}\right)$

C) $\left(-c, \frac{-c}{m}\right)$ Answer

D) $\left(c, \frac{3c}{m}\right)$

Solution

After simplifying by dividing out the common binomial, the expression is:

$$\frac{(2x-c)}{3m}$$

x ≠ -c

Substitute –c for x, to find y.

y = $\frac{(2(-c)-c)}{3m}$

y = $\frac{-3c}{3m}$

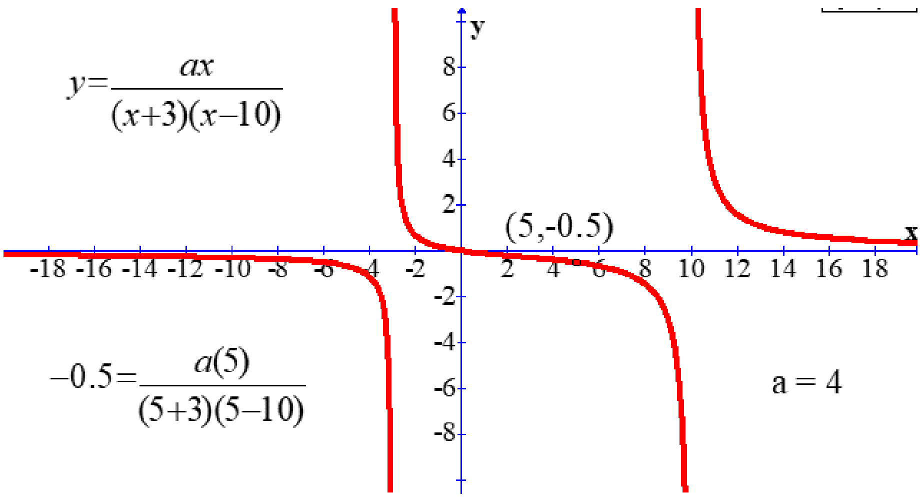
y = $\frac{-c}{m}$

Use the information below to answer the next 2 questions.

The graph of the function below can be expressed in the form

$y = \dfrac{ax}{x^2 + bx + c}$ The domain is {x | x ≠ -3, 10, x ϵ R}

$$y = \frac{ax}{(x+3)(x-10)}$$



$$-0.5 = \frac{a(5)}{(5+3)(5-10)}$$

$a = 4$

9. Determine the values of b and c.

A) b = 7 and c = 30

B) b = -3 and c = 10

C) b = 3 and c = -10

D) b = -7 and c = -30

Solution

Since the domain is x ≠ -3, 10, the corresponding binomials in the denominator are (x + 3) and (x – 10). We will find the values of b and c when multiplying these binomials.

$(x + 3)(x - 10) = x^2 - 7x - 30$

10. Determine the value of a.

A) 4

B) -4

C) 5

D) -5

Solution

Substitute the given point for x and y to find the value of a.

The correct answer is A.

11. When solving the rational equation, $\frac{4}{x} = 3 - \frac{5x}{x-2}$ graphically by finding the intersection points of y_1 and y_2, the approximate solution in quadrant 1 is

A) 0.44

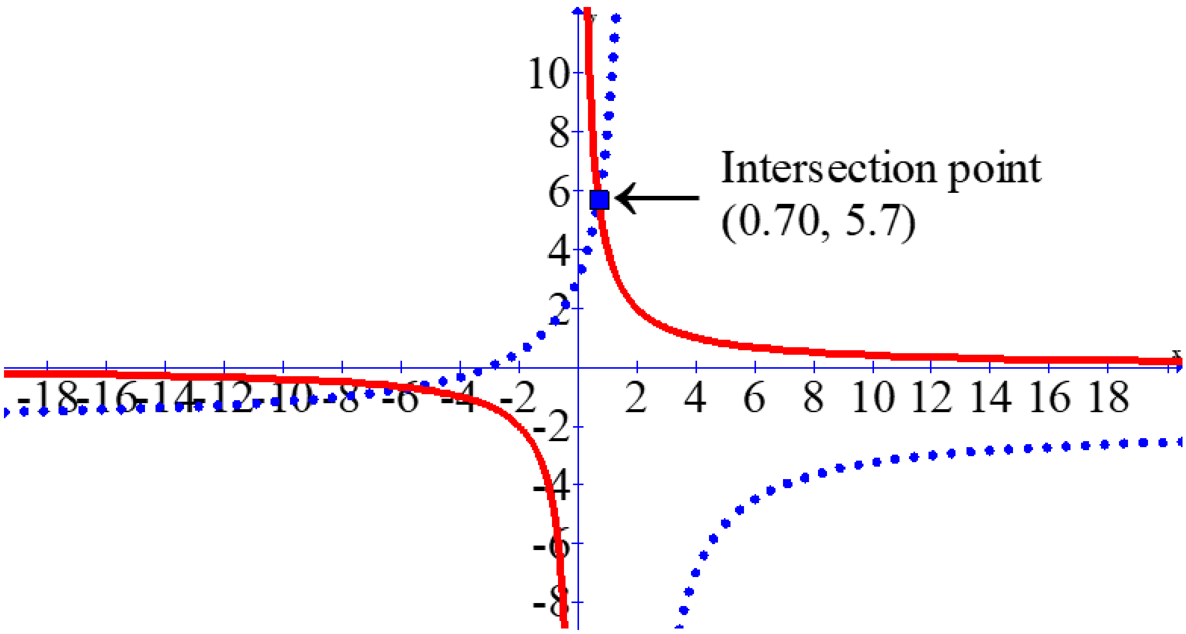
B) 0.52

C) 0.70

D)1.12

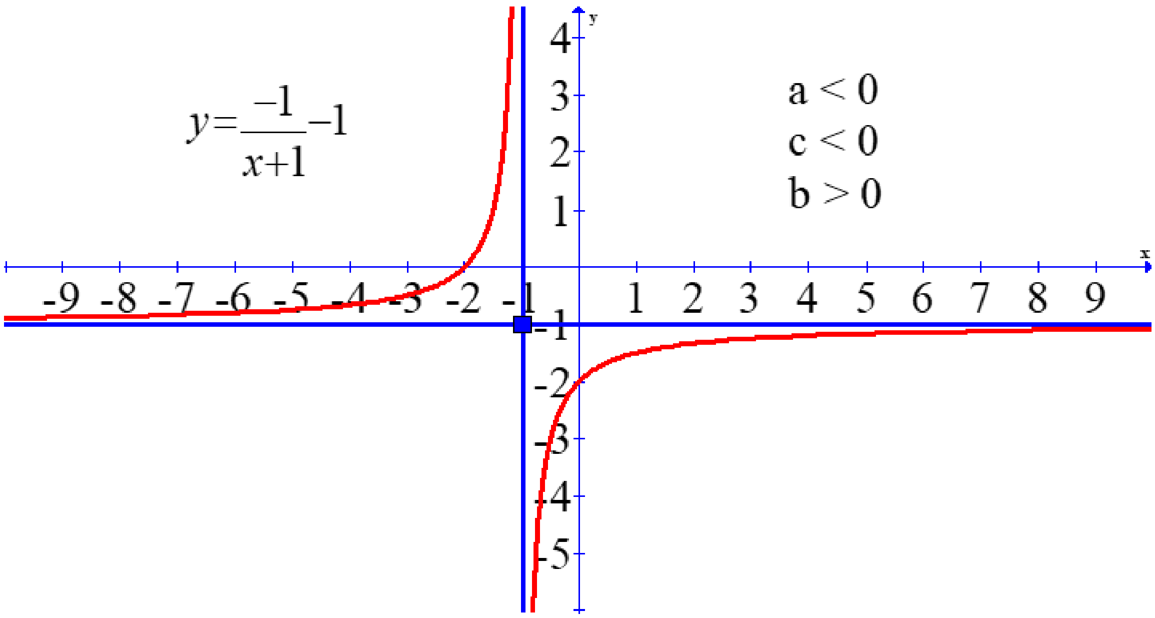
Solution

Check the graph below.



12. Given the rational equation, y = $\frac{a}{x+b}+c$, if a < 0, c < 0 and b > 0, the vertical and horizontal asymptotes will intersect in quadrant

A) 1 B) 2 C) 3 D) 4



Solution

By substituting sample values that align with the parameters given a < 0, c < 0 and b > 0, we see that the asymptotes will cross in quadrant 3.

Part B Place the correct answer in the space provided. Each correct answer is worth 1 mark.

Use the following information to answer the next question.

As he was wrapping up his lesson related to operations on functions, a Math 30-1 teacher gave this question to his students as an exit pass. Given $f(x) = -3x + 2$ and $g(x) = x + 4$, he asked them to consider various operations that could be performed:

1. $f(x) + g(x)$

2. $f(x) - g(x)$

3. $f(x)\,g(x)$

4. $\frac{f(x)}{g(x)}$

13. Using the numbers, 1, 2, 3, or 4 from the above numbered operations, submit three numbers, in order from left to right, which would satisfy the following: The **first** would have a range of $y \le 16.3$; the **second** would have a domain of $x \neq -4$; and the **third** would have a y-intercept of -2.

3	4	2	

Solution

1. $f(x) + g(x) =$ $(-3x + 2) + (x + 4) =$ $-2x + 6$

   This is a linear function. The domain and range are real numbers. The y-int. is 4.
2. $f(x) - g(x) =$ $(-3x + 2) - (x + 4) =$ $-4x - 2$

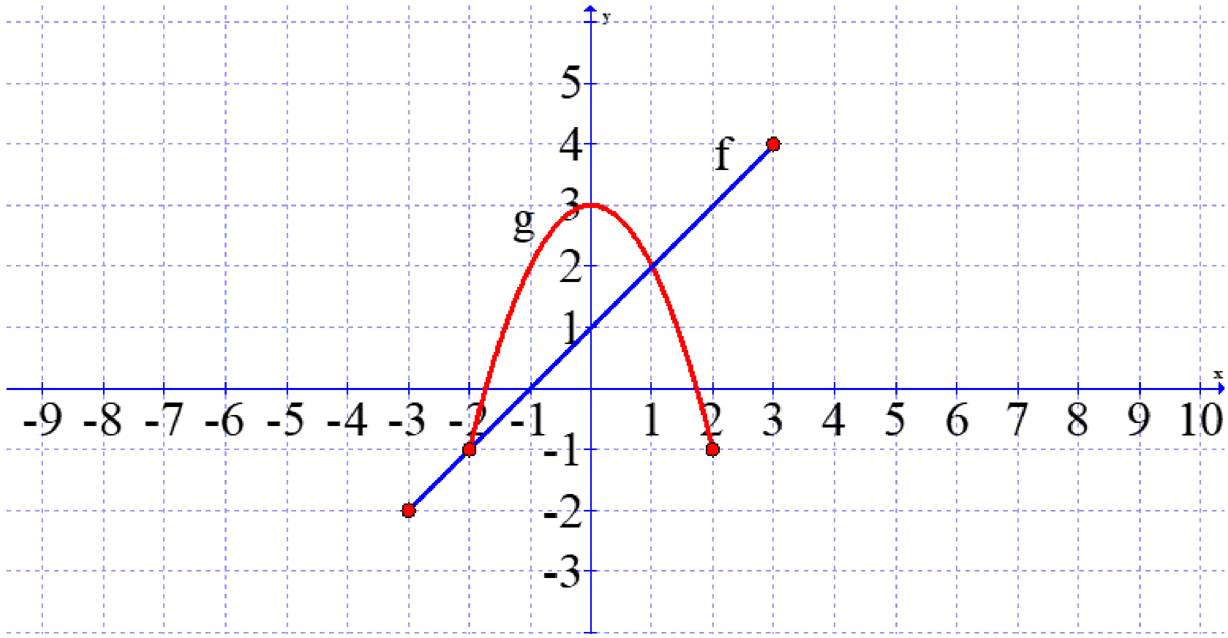
   This is a linear function. The domain and range are real numbers. **The y-int. is -2.**
3. $f(x)\ g(x) =$ $(-3x + 2)(x + 4) =$ $-3x^2 - 10x + 8$.

   This is a quadratic function. The domain is the set of real numbers. **The range is $y \le 16.3$**
4. $\frac{f(x)}{g(x)} =$ $\frac{-3x+2}{x+4}$

   This is a rational function. **The domain is $x \neq -4$**

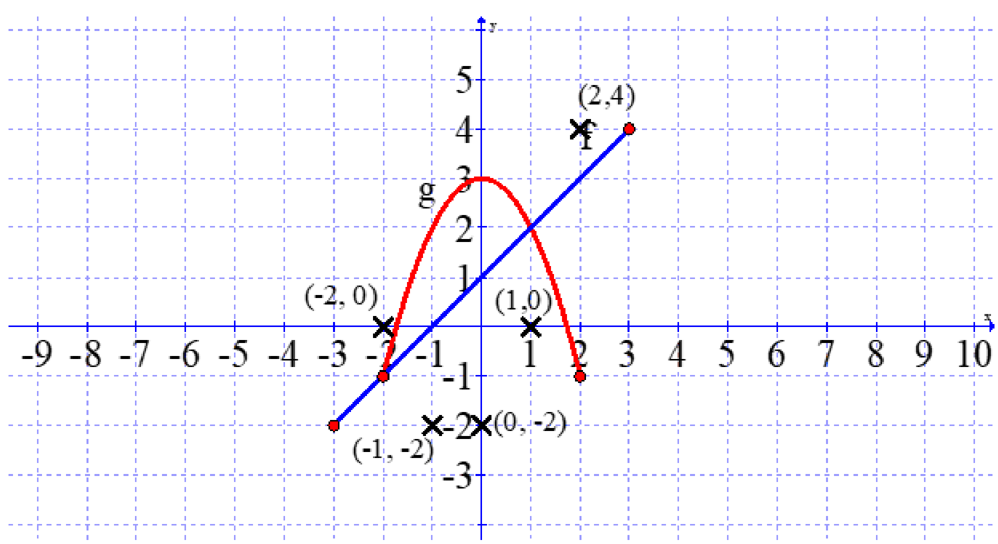
Use the following information to answer the next question.

The partial graphs of a linear function, y = f(x), and a quadratic function, y = g(x) are shown below. The domain of f(x) is [-3,3] and the domain of g(x) is [-2,2].



14. The range of h(x) = f(x) – g(x) can be written in the form [-m,n]. Rounding to the nearest integer, the values of m and n respectively are __2__ and __4__.

Solution



The domain of h(x) will be the over-lap, or the common domain, of the two functions. The over-lap is [-2,2].

For each value of x, determine the y values for the respective functions and subtract these values.

When x = 2, f (2) = 3 and g (2) = -1. Thus, f(x) – g(x) = 4. The point (2,4) is on h(x).

When x = 1, f (1) = 2 and g (1) = 2. Thus, f(x) – g(x) = 0. The point (1,0) is on h(x).

When x = 0, f (0) = 1 and g (0) = 3. Thus, f(x) – g(x) = -2. The point (0, -2) is on h(x).

When x = -1, f (-1) = 0 and g (-1) = 2. Thus, f(x) – g(x) = -2. The point (0, -2) is on h(x).

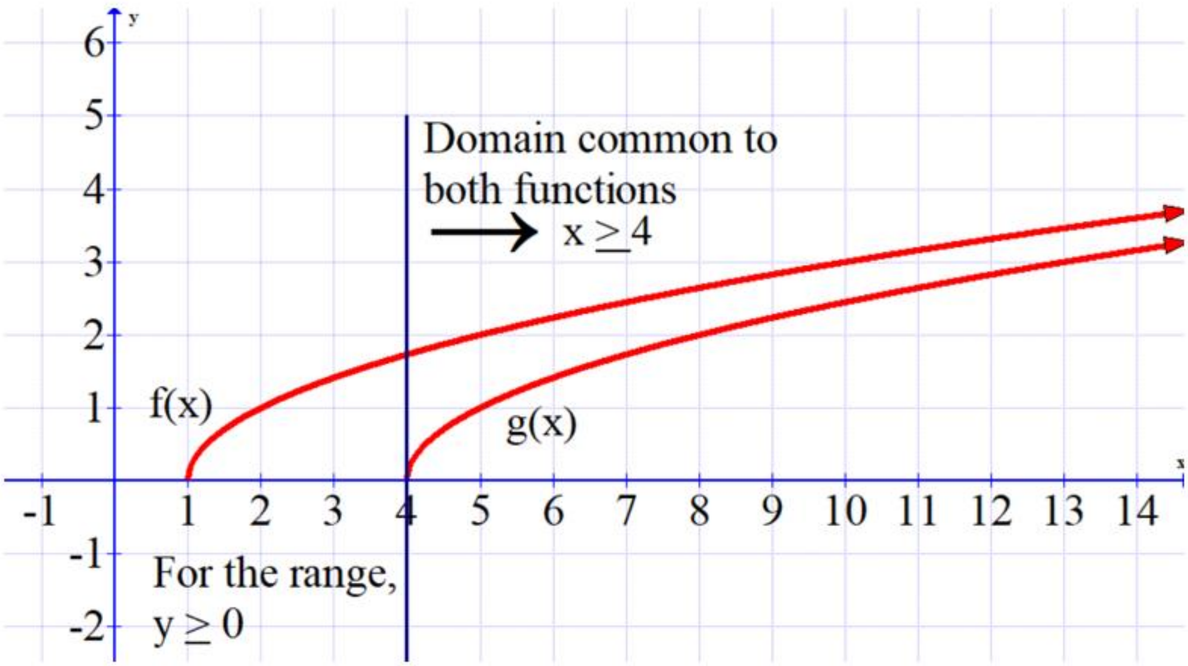
When x = -2, f (-2) = -1 and g (-2) = -1. Thus, f(x) – g(x) = 0. The point (-2, 0) is on h(x).

Of these 5 points shown on the graph below (X), the smallest y value is -2 and the largest y value is 4. The range is [-2, 4].

Rounding to the nearest integer, the values of m and n respectively are 2 and 4.

15. If f(x) = $\sqrt{x-1}$ and g(x) = $\sqrt{x-4}$, then h(x) = f(x) g(x). The domain of h(x) can be written in the form [m, ∞) and the range of h(x) can be written in the form [n, ∞). The values of m and n respectively, are _<u>4</u>_ and _<u>0</u>_.

Solution



The domain is [4, ∞) and the range is [0,∞).

The values of m and n respectively, are 4 and 0.

Use the following information to answer the next question.

$f(x) = 2^x$

$g(x) = \dfrac{-5}{x-8}$

$h(x) = (g \circ f)(x)$,

16. The domain of h(x) can be written in the form, $x \neq k$. The value of k is _<u>3</u>___.

Solution

The function h(x) is a composite function. The complete function f(x) is substituted for x in g(x).

$$g(x) = \frac{-5}{\big(f(x)\big) - 8} \; or \; g(x) = \frac{-5}{(2^x) - 8}$$

Since the denominator of a rational function cannot be equal to 0, we know that 2^x cannot be equal to 8. The value of x to make 2^x equal to 8 is 3.

The value of k is 3.

17. Given $\frac{(2x-1)(x+w)}{(2x-1)(x-2)}$, there is a point of discontinuity at $\left(\frac{1}{2},-1\right)$. The value of w is _1_.

Solution

When the common binomial (2x – 1) is divided out of the numerator and the denominator, we know that $x \neq \frac{1}{2}$ in the simplified expression.

To find the value of y, we would then substitute this value of x into the simplified expression. Since we are given the value of y (i.e., -1), we can use this information to determine the missing variable, w.

$$\frac{x+w}{x-2} = -1$$

$$\frac{\left(\frac{1}{2}\right) + w}{\left(\frac{1}{2}\right) - 2} = -1$$

$$\frac{\left(\frac{1}{2}\right) + w}{\left(\frac{1}{2}\right) - \left(\frac{4}{2}\right)} = -1$$

$$\frac{\left(\frac{1}{2}\right)+w}{\left(\frac{-3}{2}\right)}=-1$$

$$\left(\frac{1}{2}\right)+w=\left(\frac{3}{2}\right)$$

$$w=\left(\frac{3}{2}\right)-\left(\frac{1}{2}\right)$$

$$w=\left(\frac{2}{2}\right)$$

w = 1

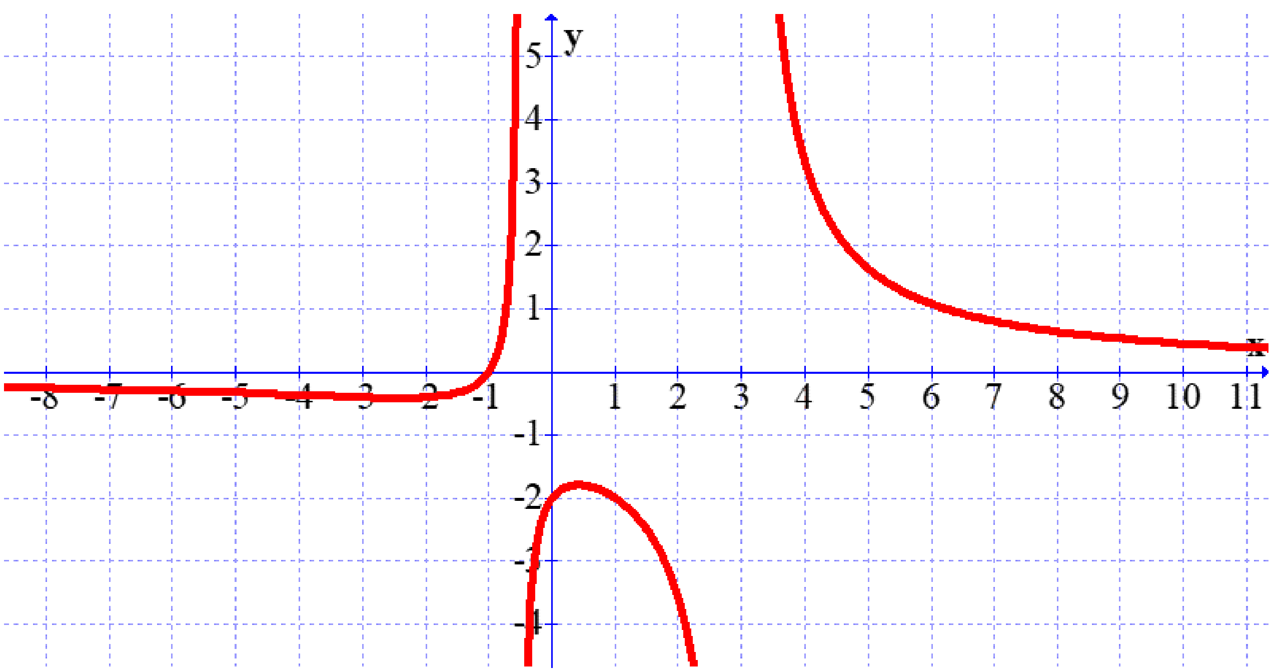
The value of w is 1.

Use the following information to answer the next question.

The graph below has two vertical asymptotes having the equations:

x = $\frac{-1}{2}$, and x = 3. This graph can be written in the form:

$$y=\frac{a(x+1)}{2x^2-bx-c}$$



18. If the y-intercept of the graph is (0,-2), determine the values of a, b, and c and place them respectively in the box below.

6	5	3	

Solution

With an equation of a vertical asymptote being x = $-\frac{1}{2}$, we can determine the corresponding binomial by setting the equation equal to 0. Multiply both sides of the equal sign by 2 and add 1 to both sides. (2x + 1) is the corresponding binomial.

With an equation of a vertical asymptote being x = 3, we can determine the corresponding binomial by setting the equation equal to 0. Subtract 3 from both sides. The binomial is (x – 3). These binomials are in the denominator of the rational equation.

$$y = \frac{a(x+1)}{(2x+1)(x-3)}$$

Now substitute the given point on the graph (0, -2) to find the missing parameter 'a'.

$$(-2) = \frac{a((0)+1)}{(2(0)+1)((0)-3)}$$

$$-2 = \frac{a(1)}{(1)(-3)}$$

Cross multiply. (-2) (1) (-3) = a(1)

a = 6

Multiply the two binomials to determine the values of b and c.

(2x + 1) (x – 3) = $2x^2$ – 5x – 3

The value of a is 6, b is 5 and c is 3.

19. Carl is practicing for an up-coming curling tournament. On a particular day of practice, his focus in on drawing to the house, in particular to the 4 foot rings. So far he has been successful on 34 of 55 attempts. If he tries x attempts from now on and is successful on 75% of them, how many attempts will it take before his average is above 70%?

The number of attempts is __90___.

Solution

$$0.7 = \frac{34 + 0.75x}{55 + x}$$

$0.7(55 + x) = 34 + 0.75x$

$38.5 + 0.7x = 34 + 0.75x$

$4.5 = 0.05x$

Divide both sides of the equation by 0.05.

$x = 90$

The number of attempts is 90.

Part C Provide all work and necessary explanations to receive full marks.

Use the following information to answer the next question.

Consider the following list of functions, where b > 1.

Function 1 $y = x^2 + b$

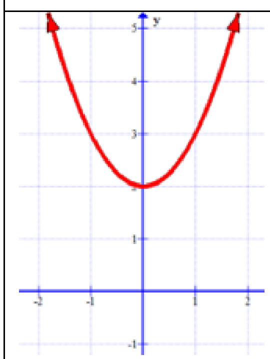
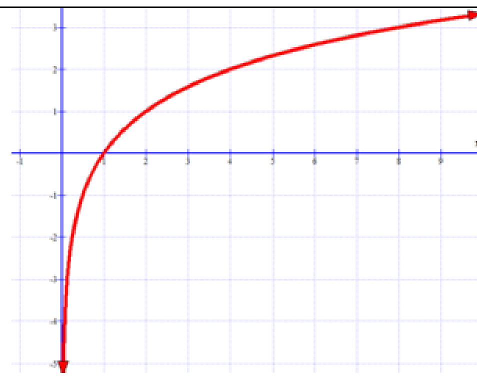
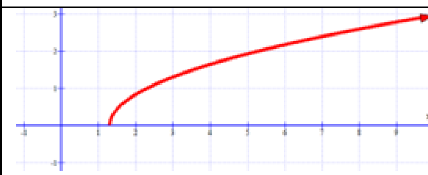
Function 2 $y = \log_b x$

Function 3 $y = \sqrt{x - b}$

20. A new function, h(x) , which is the quotient of 2 different functions from the above list is determined and the domain of h(x) is {x | x > 0, x ϵ R}. If h(x) = $\frac{f(x)}{g(x)}$, then what 2 functions should be selected for f(x) and g(x)? Explain. Include a diagram sketch of the graphs.

Solution

Let's compare possible images for each function.

Function 1: y = x^2 + b	Function 2: y = $\log_b x$	Function 3: y = $\sqrt{x-b}$
		
Domain: x ϵ R	Domain: x > 0	Domain: x $\geq$ b

The domain of the new function, y = h(x) is x > 0, which needs to be the common domain of two of these functions. Since we are told that b > 1, function 3 cannot have any values between 0 and 1. Thus, its domain does not overlap with either of functions 1 or 2 to create a domain of x > 0. We must eliminate function 3 as a possible answer.

The question is now which graph is in the numerator and which is in the denominator.

The logarithmic function (2) must be in the numerator. This function contains the point (1, 0). If it were in the denominator, the expression would be undefined if x = 1.

f(x) is function 2 : y = $\log_b x$

g(x) is function 1: y = $x^2 + b$

$$h(x) = \frac{log_b x}{x^2 + b}$$

21. Given the functions, f(x) = $x^2 - 4x$, g(x) = $\frac{1}{x+3}$, and h(x) = g(x) – (f ∘ g)(x), find the value of h(-2).

Solution

h(-2) = g(-2) - (f ∘ g)(-2),

$$g(-2) = \frac{1}{(-2) + 3}$$

$$g(-2) = 1$$

Now deal with the composite function. The value of g(-2), or 1, is substituted for x in function f.

f(1) = $(1)^2$ – 4(1)

f(1) = 1 – 4

f(1) = -3

h(-2) = (1) – (-3)

h(-2) = 4

22. Convert y = $\frac{-3x+1}{x-1}$ to the form y = $\frac{a}{x-h} + k$. Sketch the graph below and identify the equations of all asymptotes, state the domain and range and state all intercepts

Solution

The task is to create a common binomial in the numerator and the denominator. We are looking to add and subtract a number (i.e., insert zero) to the numerator which will yield a common (x – 1).

Adding and subtracting 3 will allow us to factor out a common (-3) in the numerator to create a common binomial (x – 1).

$$y = \frac{-3x + 3 - 3 + 1}{x - 1}$$

$$y = \frac{-3(x - 1) - 3 + 1}{x - 1}$$

$$y = \frac{-3(x - 1) - 2}{x - 1}$$

$$y = \frac{-3(x - 1)}{(x - 1)} - \frac{2}{(x - 1)}$$

$$y = -3 - \frac{2}{(x - 1)}$$

$$y = -\frac{2}{(x - 1)} - 3$$

The equation for the vertical asymptote is x = 1.

The equation for the horizontal asymptote is y = -3.

The domain is x ≠ 1.

The range is y ≠ -3.

To find the y-intercept, set x = 0 and solve for y.

$$y = \frac{-2}{\left((0) - 1\right)} - 3$$

y = 2 – 3

y = -1

The y-intercept is -1.

To find the x-intercept, set y = 0 and solve for x.

$$(0) = -\frac{2}{(x-1)} - 3$$

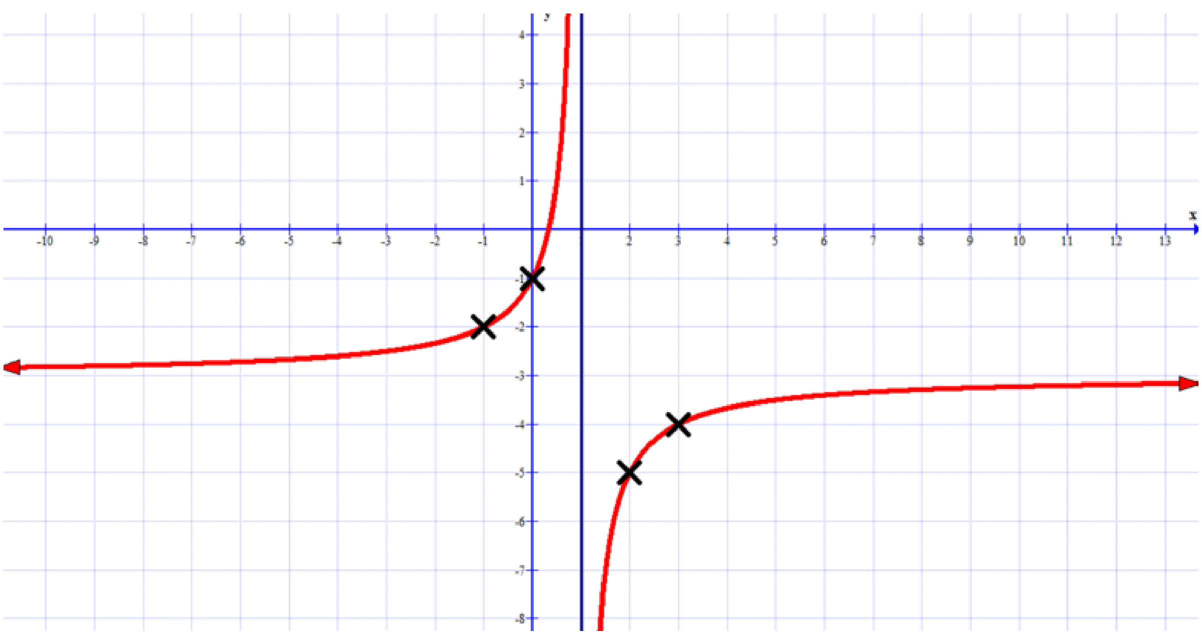
$$3 = -\frac{2}{(x-1)}$$

3x – 3 = -2

3x = 1

$$x = \frac{1}{3}$$

The x-intercept is $\frac{1}{3}$.



23. a) When solving the rational equation, $3x + 2 = \frac{15 - 2x}{x - 4} + 12$, algebraically, a quadratic equation, in the form, $ax^2 + bx + c = 0$, will be part of the process leading to the solution. What are the values of a, b, and c?

Solution

Multiply each term by $(x - 4)$

$$(x-4)\left[3x + 2 = \frac{15 - 2x}{x - 4} + 12\right]$$

$3x(x-4) + 2(x - 4) = 15 - 2x + 12(x - 4)$

$3x^2 - 12x + 2x - 8 = 15 - 2x + 12x - 48$

$3x^2 - 10x - 8 = 10x - 33$

$3x^2 - 20x + 25 = 0$

The value of a is 3, b is -20 and c is 25.

b) Explain how to solve the rational equation above by a graphical method.

Solution

Input $y_1 = 3x + 2$ and $y_2 = \frac{15-2x}{x-4} + 12$ into the calculator. Determine the x-coordinate of the intersection point of the two graphs.

c) Of the 2 roots to the rational equation above, what is the non-integer root, to an exact answer?

Solution

The factored form of $3x^2 - 20x + 25$ is $(x - 5)(3x - 5)$. The roots are 5 and $\frac{5}{3}$.

The non-integer root is $\frac{5}{3}$.