Standard Deviation Practice Using Z-Scores

1. On a physics exam, the mean of all the scores was 67 and the standard deviation was 2.3. If Julianna's score was 72, what was her z-score?

   A) 2.17 B) 2.70 C) 1.11 D) 0.46

2. A café serves its customers with a mean time of 17 min with a standard deviation of 3 minutes. Assuming a normal distribution, what is the probability of being served in under 20 min?

   A) 0.1587 B) 0.3413 C) 0.8413 D) 0.9651

3. The life of a grass trimmer is normally distributed with a mean of 82 months. The manufacturer guarantees the trimmer for 33 months. If 2.5% of the trimmers are returned under this warranty, then the standard deviation, to the nearest whole month, of the life of the trimmer is ____.

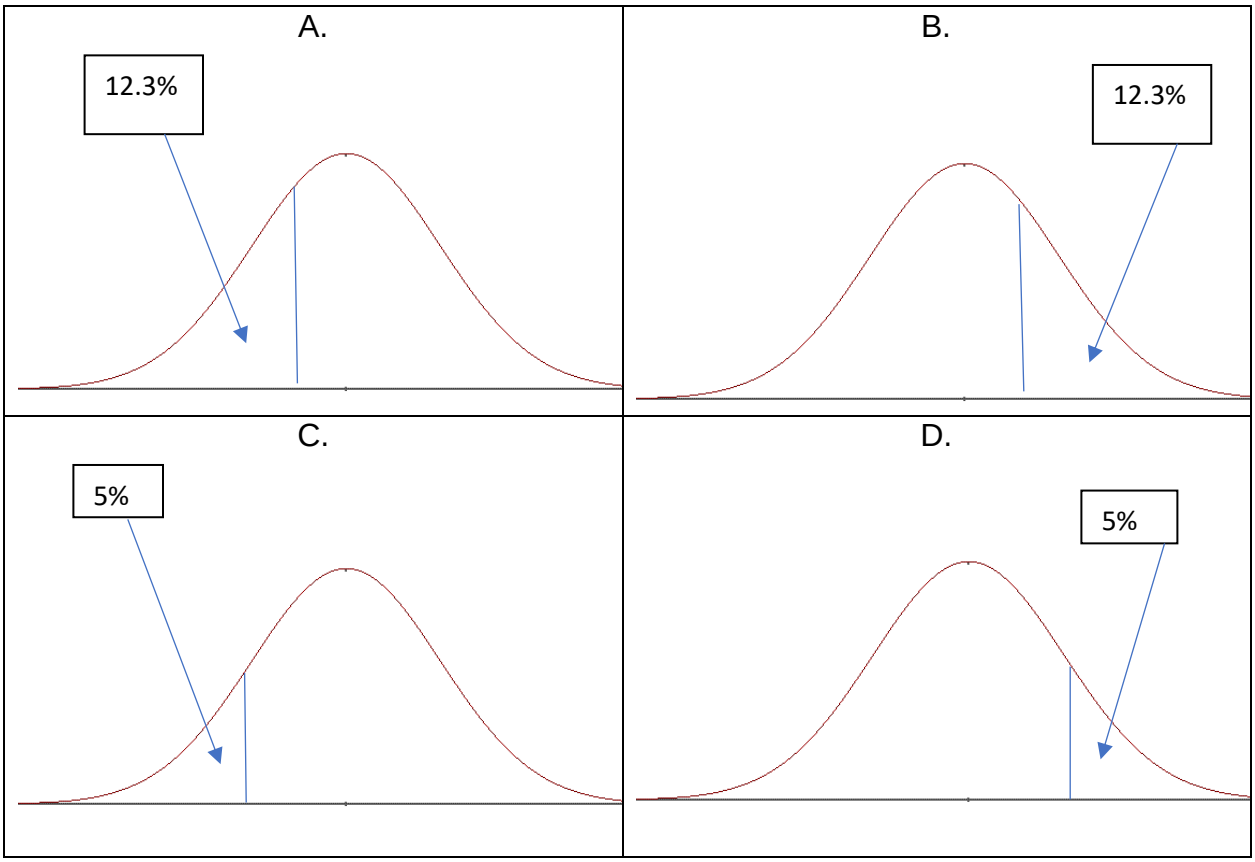
4. The time required for an athlete to run 1500 m is normally distributed with a mean of 4 minutes and a standard deviation of 4 seconds. To qualify for a track meet, a runner must run the 1500 m in less than 3 minutes and 57 seconds. If 200 athletes attempt to qualify, how many would we expect to be successful?

   A) 30 B) 45 C) 55 D) 155

5. The standard deviation of a normally distributed exam was 10. Ray scored 51. If 6.68% of the scores were lower than his, then the mean of the exam is

   A) 69 B) 74 C) 66 D) 63

Use the following information to answer the next question.



6. Which diagram represents a z-score of -1.16?

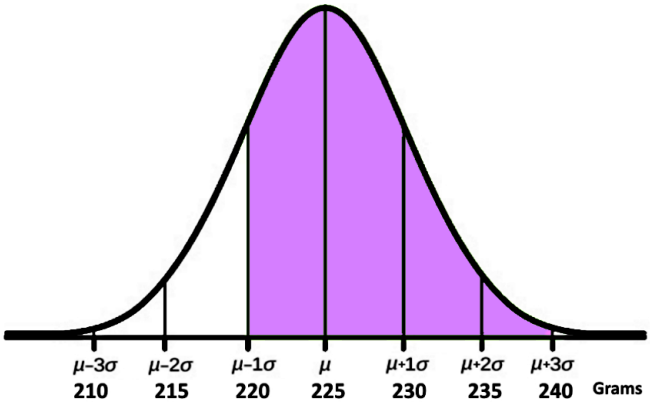
   A) A B) B C) C D) D

7. The mean weight of a group of students is 68 kg with a standard deviation of 8 kg. A z-score of -1.25 would correspond to a weight of _____.

Use the following information to answer the next question.

Consider the following statements based on the normal curve shown below.

Statement 1	The standard deviation is 5.
Statement 2	The z-score for a data value of 232 is 1.4.
Statement 3	The z-score for a data value of 221 is -0.6.
Statement 4	The area of the shaded region under the curve is about 68%.



8. The two true statements are

   A) 1 and 2 B) 3 and 4 C) 1 and 3 D) 2 and 4

9. The diameter of a golf ball must be within 1.78 standard deviations of the mean. Any ball outside of these limits are rejected. If 3000 golf balls are produced and their size is normally distributed, the number of balls that one would expect **not** to be rejected is ____.

Standard Deviation Practice Using Z-Scores Solutions

1. On a physics exam, the mean of all the scores was 67 and the standard deviation was 2.3. If Julianna's score was 72, what was her z-score?

A) 2.17 B) 2.70 C) 1.11 D) 0.46

Solution

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{72 - 67}{2.3}$$

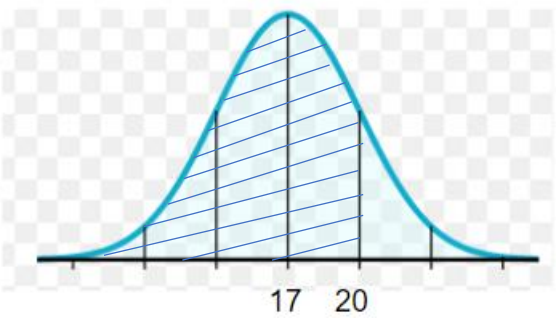
z = 2.17

The correct answer is A.

2. A café serves its customers with a mean time of 17 min with a standard deviation of 3 minutes. Assuming a normal distribution, what is the probability of being served in under 20 min?

A) 0.1587 B) 0.3413 C) 0.8413 D) 0.9651

Solution

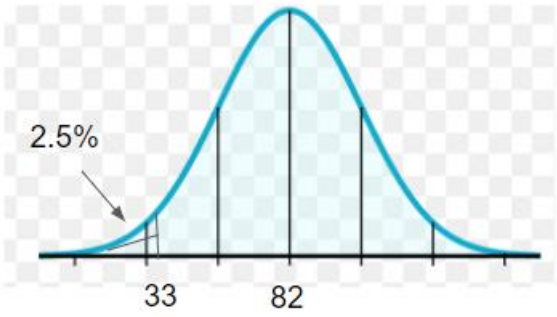


The data score of 20 represents a z-score of 1. From the z-score tables, the area to the left of a z-score of 1 is 0.8413.

The correct answer is C.

3. The life of a grass trimmer is normally distributed with a mean of 82 months. The manufacturer guarantees the trimmer for 33 months. If 2.5% of the trimmers are returned under this warranty, then the standard deviation, to the nearest whole month, of the life of the trimmer is _25_.

Solution



Determine the z-score represented by the data value 33, by looking inside the z-score tables.

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233

The z-score is -1.96.

$$z = \frac{x - \mu}{\sigma}$$

$$-1.96 = \frac{33 - 82}{\sigma}$$

$$\sigma = \frac{33 - 82}{-1.96}$$

The standard deviation is 25.

4. The time required for an athlete to run 1500 m is normally distributed with a mean of 4 minutes and a standard deviation of 4 seconds. To qualify for a track meet, a runner must run the 1500 m in less than 3 minutes and 57 seconds. If 200 athletes attempt to qualify, how many would we expect to be successful?

A) 30 B) 45 C) 55 D) 155

Solution

Convert the times to seconds in order to make all times in the same unit.

In seconds, 4 minutes is 240 seconds.

In seconds, 3 minutes and 57 seconds is 237 seconds.

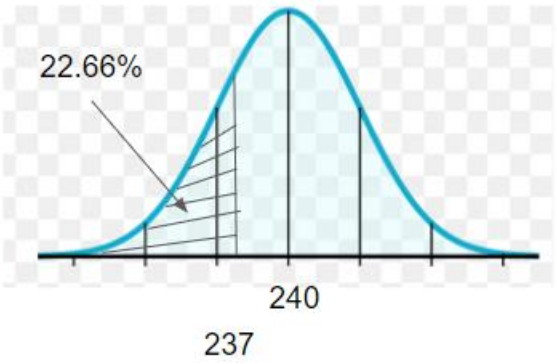
The standard deviation is 4 seconds.

Calculate the z-score for 237 seconds.

$$z = \frac{237 - 240}{4}$$

z = -0.75

The area to the left of a z-score of -0.75 is 22.66%.



Multiply this percentage by the total number of athletes.

(200) (0.2266) = 45.32

The correct answer is B.

5. The standard deviation of a normally distributed exam was 10. Ray scored 51. If 6.68% of the scores were lower than his, then the mean of the exam is

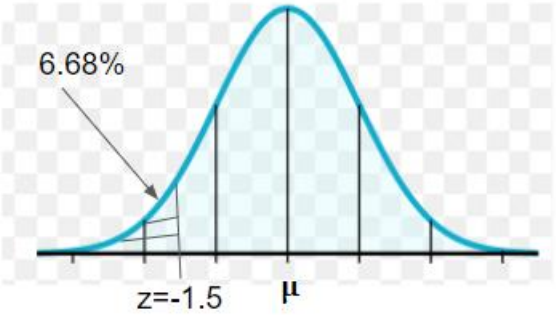
A) 69 B) 74 C) 66 D) 63

Solution

Look inside the z-score tables to find the z-score, such that 6.68% of the data is to the left of this z-score.

z	0	0.01	0.02	0.03
-0	.50000	.49601	.49202	.48803
-0.1	.46017	.45620	.45224	.44828
-0.2	.42074	.41683	.41294	.40905
-0.3	.38209	.37828	.37448	.37070
-0.4	.34458	.34090	.33724	.33360
-0.5	.30854	.30503	.30153	.29806
-0.6	.27425	.27093	.26763	.26435
-0.7	.24196	.23885	.23576	.23270
-0.8	.21186	.20897	.20611	.20327
-0.9	.18406	.18141	.17879	.17619
-1	.15866	.15625	.15386	.15151
-1.1	.13567	.13350	.13136	.12924
-1.2	.11507	.11314	.11123	.10935
-1.3	.09680	.09510	.09342	.09176
-1.4	.08076	.07927	.07780	.07636
-1.5	.06681	.06552	.06426	.06301
-1.6	.05480	.05370	.05262	.05155
-1.7	.04457	.04363	.04272	.04182

z = -1.5



Now use the z-score formula.

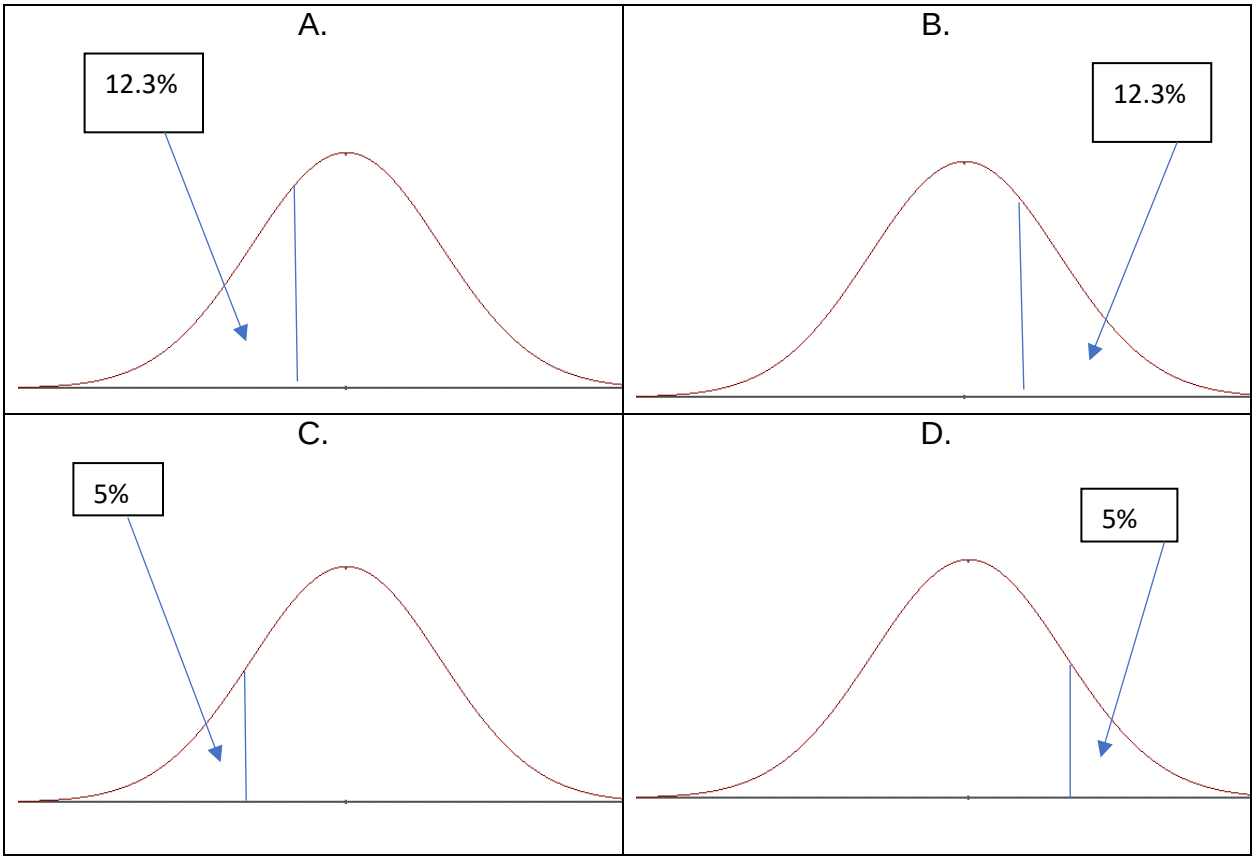
$$-1.5 = \frac{51 - \mu}{10}$$

$-15 = 51 - \mu$

$\mu = 66$

The correct answer is C.

Use the following information to answer the next question.



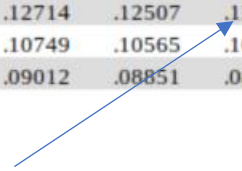
6. Which diagram represents a z-score of -1.16?

A) A B) B C) C D) D

Solution

The areas for B and D are to the right of the mean; which means that the z-scores must be positive. Thus, the correct answer will be A or C.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226



A z-score of -1.16 will yield an area of 0.123 to the left.

The correct answer is A.

7. The mean weight of a group of students is 68 kg with a standard deviation of 8 kg. A z-score of -1.25 would correspond to a weight of _58__.

Solution

Use the z-score formula.

$$z = \frac{x - \mu}{\sigma}$$

$$-1.25 = \frac{x - 68}{8}$$

-10 = x – 68

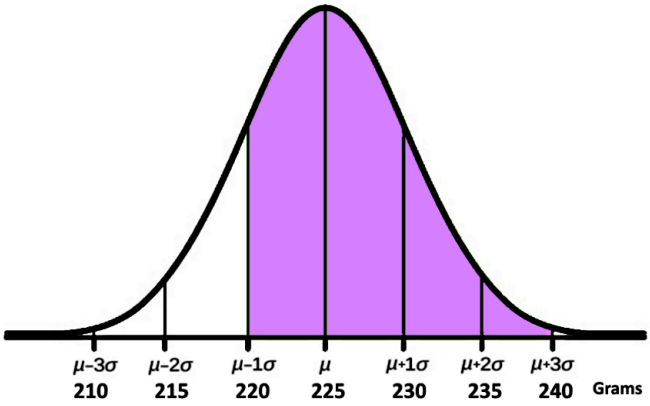
58 = x

A z-score of -1.25 would correspond to a weight of 58 kg.

Use the following information to answer the next question.

Consider the following statements based on the normal curve shown below.

Statement 1	The standard deviation is 5.
Statement 2	The z-score for a data value of 232 is 1.4.
Statement 3	The z-score for a data value of 221 is -0.6.
Statement 4	The area of the shaded region under the curve is about 68%.



8. The two true statements are

A) 1 and 2 B) 3 and 4 C) 1 and 3 D) 2 and 4

Solution

Statement 1

The mean is 225 and the data value at one standard deviation above the mean is 230. The difference between these two values is the standard deviation. The standard deviation is 5. This statement is **true.**

Statement 2

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{232 - 225}{5}$$

$$z = \frac{232 - 225}{5}$$

z= 1.4

This statement is **true.**

Statement 3

$$z = \frac{221 - 225}{5}$$

z = -0.8

Since statement 3 states that the z-score is -0.6, this statement is **false.**

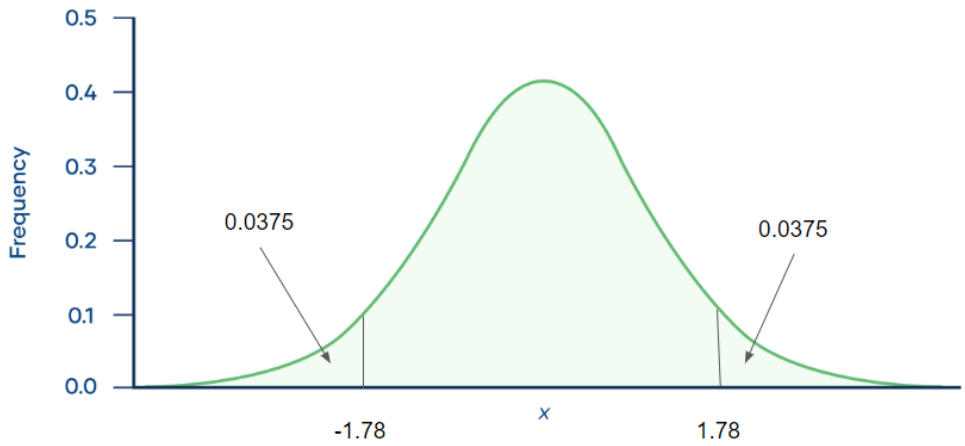
Statement 4

The shaded area is about 84%, not 68%. This statement is **false.**

The correct answer is A.

9. The diameter of a golf ball must be within 1.78 standard deviations of the mean. Any ball outside of these limits are rejected. If 3000 golf balls are produced and their size is normally distributed, the number of balls that one would expect **not** to be rejected is _2775_.

Solution



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-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
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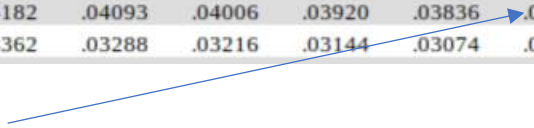
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938



The area to the left of -1.78 is 0.0375. Since the graph is symmetrical, the area to the right of 1.78 is 0.0375.

The percent in between the standard deviations is 1 – (0.0375 + 0.0375), or 0.925. This area represents the golf balls that are accepted, i.e., **not** rejected.

Multiply this percentage by the total of 3000 golf balls.

(0.925) (3000) = 2775.

If 3000 golf balls are produced and their size is normally distributed, the number of balls that one would expect not to be rejected is 2775.