Name ____________

Litmus Test For Combinatorics Solutions

Part A Place the correct answer on the sheet provided. Each question is worth 1 mark.

1. An expression for the number of arrangements of the word PROJECT that start and end with a consonant is

A) (5)(4)(5!) B) (5)(4)(4!) C) (4)(3)(5!) D) (5)(5)(5!)

There are 2 vowels and 5 consonants.

_5_ X ___ X ___ X ___ X ___ X ___ X _4_

_5_ X _5_ X _4_ X _3_ X _2_ X _1_ X _4_

= (5) (4) (5!)

2. The first three questions on a quiz are True/False, while the next nine questions are multiple choice, each with possible answers, a, b, c, and d. How many possible ways can students answer these twelve questions?

A) $(3^2)(9^4)$ B) $(2^3)(4^9)$ C) $(4^3)(2^9)$ D) $(9^2)(3^4)$

True/False and Multiple Choice

__ __ __ __ __ __ __ __ __ __ __ __

2 X 2 X 2 and 4 X 4 X 4 X 4 X 4 X 4 X 4 X 4 X 4

2^3 X 4^9

3. How many arrangements of the word TENNESSEE are possible, if it must begin with at least 3 e's?

Case 1 – Begin with 3 e's + Case 2 – Begin with 4 e's

$$\frac{4X3X2X5X5X4X3X2X1}{4!2!2!} + \frac{4X3X2X1X5!}{4!2!2!}$$

$$\frac{14400}{96} + \frac{2880}{96}$$

150 + 30

A) 120 B) 150 C) 180 D) 220

Use the following information to answer the next question.

Besides teaching math, Mr. Smith is also the high school football coach. On his offensive line, he has 5 positions: Left Tackle, Left Guard, Centre, Right Guard and Right Tackle. He really wants to keep his 3 biggest and strongest offensive linemen together because he feels his team could effectively run the ball with these 3 blocking as a unit. Each of these 3 particular linemen are capable of playing any position. There are only 5 players available to play these 5 positions.

4. How many different ways can coach Smith arrange his offensive line so that his 3 biggest and strongest players are positioned next to each other?

A) 18 B) 24 C) 30 D) 36

Put the 3 biggest strongest offensive linemen together in one stage.

There are now a total of 3 stages.

__1/2/3__ ___ ___

There are 3! ways to arrange these 3 stages.

Now we have to account for the fact that each of these 3 linemen in one stage can be arranged in 3! ways.

The total number of ways is 3! X 3! Or 36.

5. The local high school is planning to form a committee of 3 people to organize the athletic awards. How many committees are possible if they are selecting from a group of 4 teachers and 6 students and having at most 1 student?

   A) $({}_4C_2)({}_6C_1)$

   B) $({}_4P_2)({}_6P_1)$

   C) $({}_4P_3) + ({}_4P_2)({}_6P_1)$

   D) $({}_4C_3) + ({}_4C_2)({}_6C_1)$

At most means, no students or 1 student.

Case 1 – No students	+	Case 2 – 1 student
${}_4C_3$	+	$({}_4C_2)\ ({}_6C_1)$

6. There are 6 girls and 4 boys auditioning for 3 female roles and 2 male roles for the school play. If after 1 week, 1 girl withdraws due to illness, how many **fewer** ways are there now to pick their roles?

   A) 10 B) 32 C) 44 D) 60

Before the withdrawl:

$({}_6C_3)\ ({}_4C_2) = 120$

$120 - 60 = 60$

After the withdrawl:

$({}_5C_3)\ ({}_4C_2) = 60$

7. If there are 13 points on a circle, how many triangles can be formed?

A) 78 B) 128 C) 286 D) 304

${}_{13}C_3 = 286$

Use the following information to answer the next question.

A school has 2 different Art 30 classes. From class A, teachers are asked to vote for the top 3 paintings from a selection of 10. From class B, teachers are asked to vote for the top 4 paintings, from a selection of 15. The 7 winners will then be displayed in a horizontal line on a wall at the entrance to the school.

8. How many ways can the 7 winners be displayed on the wall?

A) $({}_{10}P_3)\ ({}_{15}P_4)\ (7!)$

B) $({}_{10}P_3) + ({}_{15}P_4) + (7!)$

C) $({}_{10}C_3)\ ({}_{15}C_4)\ (7!)$

D) $({}_{10}C_3) + ({}_{15}C_4) + (7!)$

9. At a meeting, every person shakes hands with every other person twice, once at the beginning to introduce themselves, and once at the end to say farewell. If there are 182 handshakes in total, which statement below best describes the number of people attending the meeting?

A) There was between 6 and 10 people in attendance.

B) There was between 11 and 15 people in attendance.

C) There was between 16 and 20 people in attendance.

D) There was between 21 and 25 people in attendance.

$2({}_nC_2) = 182$

${}_nC_2 = 91$

$$\frac{n!}{(n-2)!2!} = 91$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 182$$

$n(n - 1) = 182$

$n^2 - n - 182 = 0$

$(n - 14)(n + 13) = 0$

$n = 14$

Use the following information to answer the next 2 questions.

James was asked to solve for n algebraically: $({}_9P_4)({}_nC_2) = 635\ 040$. The steps to his work are shown below.

Step I $(3024)({}_nC_2) = 635\ 040$

Step II $\frac{n!}{(n-2)!} = 210$

Step III $\frac{n(n-1)(n-2)!}{(n-2)!} = 210$

Step IV $n^2 - n - 210 = 0$

Step V $(n - 15)(n + 14) = 0$

Step VI $n = 15$

10. Unfortunately James made an error. Which statement best describes when he made the error?

A) The error was made from Step I to Step II.

B) The error was made from Step II to Step III.

C) The error was made from Step III to Step IV.

D) The error was made from Step IV to Step V.

Step II should be: $\frac{n!}{(n-2)!2!}$ = 210

11. What is the correct answer and the extraneous root?

A) The correct answer is 14 and the extraneous root is -12.

B) The correct answer is 14 and the extraneous root is -15.

C) The correct answer is 21 and the extraneous root is -20.

D) The correct answer is 21 and the extraneous root is -22

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 420$$

$n^2 - n - 420 = 0$

$(n - 21)(n + 20) = 0$

12. The coefficient of the middle term of $(2x - 3)^{12}$ can be written as an 8 digit number in the form, 43<u>ABCD</u>44. The values of ABCD respectively are:

A) 2305

B) 4692

C) 1101

D) 7314

There are 13 terms and the middle term is the 7^{th} term. The value of 6 is 6.

$t_{k+1} = {}_{12}C_6\,(2x)^6\,(-3)^6$

$= (924)\,(64x^6)\,(729)$

= 43**1101**44

13. One term in the expansion of $\left(3k + \sqrt{m}\right)^4$ is $270k^2$. The value of m is a number within the interval

A) $1 \le m \le 3$

B) $4 \le m \le 6$

C) $7 \le m \le 9$

D) $10 \le m \le 11$

In order to get a term with a k^2, the exponent on the 3k must be 2. Thus, the exponent on the $\sqrt{m}$ must be 2, as the sum of these exponents must be 4. Thus, k = 2.

${}_4C_2\,(3k)^2\,(\sqrt{m})^2 = 270k^2$

$(6)\,(9k^2)\,(m) = 270k^2$

$54m = 270$

$m = 5$

Part B Place the correct answer in the space provided. Each correct answer is worth 1 mark.

Use the following information to answer the next question.

The following statements are made regarding $(4x^2 - y)^{5r}$	
Statement 1	There are 11 terms, thus r = 2. True
Statement 2	The coefficient of the 9^{th} term is 180. False
	The coefficient is 720.
Statement 3	For the term, $-40x^m y^9$, the value of m is 1. False
	${}_{10}C_9\,(4x^2)^1\,(-y)^9$, or $-40x^2y^9$
Statement 4	The first term is $4^{10}x^{20}$. True

14. The 2 correct statements are _1__ and _4__.

15. How many 3 or 4 digit codes can be made using only the numbers **5**, **6**, **7**, and **8**, with no repetitions allowed?

3 Digit	+	4 Digit	
4 X 3 X 2	+	4 X 3 X 2 X 1	
24	+	24	= 48

The number of codes is __48__.

16. A family of 5 went on-line to book their airline seats for their flight to Hawaii. If there were 9 available seats, how many ways could this family select their seats?

$_9P_5$

_15120__

17. How many arrangements of the letters of the word RUGBY are possible if the U and the Y are not together?

Not together = All – together

= 5! – (4!)(2!)

= 120 – 48

= 72

__72__

18. A bag contains 6 red balls and 5 purple balls. How many ways can 2 balls of the same color be drawn?

All red	or	All purple
$_6C_2$	+	$_5C_2$
15	+	10

__25__

19. Twelve University students, including a married couple, are eligible to attend a biochemical conference. Six students can attend, and the married couple will only go as a pair. How many different possibilities are possible?

Case 1 – If couple doesn't attend + Case 2 – If couple attends

$_{10}C_6$ + $_{10}C_4$

210 + 210

__420__

20. What is the value of the constant term in the expansion of

$$\left(x^3 + \frac{4}{x}\right)^4 ?$$

For a constant term, the value of the exponent on the variable is zero.

Find the value of k, and deal only with the letters.

$(x^3)^{4-k}\,(x^{-1})^k$

$= \ (x^{12-3k})\,(x^{-k})$

When multiplying powers with the same base, add exponents.

$= x^{12-4k}$

The value of this exponent is zero.

$12 - 4k = 0$

$12 = 4k$

$k = 3$

$_4C_3(x^3)^1(4/x)^3$

$4(x^3)\,(64/x^3)$

The x^3 terms will divide out and we are left with a constant of 256.

__256__

Part C Show all work and provide all explanations to receive full marks in this section.

Use the following information to answer the next question.

In 1954, Alberta introduced two-letter series licence plates, with each series initially followed by 3 digits. Only the letters B, C, E, H, J, K ,L ,N, R, T, X, and Z were used for the first letter. In 1960, the number of digits increased to 4.

21. In 1960, how many more licence plates were available in Alberta?

1954

12 X 26 X 10 X 10 X 10

= 312 000

1960

12 X 26 X 10 X 10 X 10 X 10

= 3 120 000

3 120 000 - 312 000 = 2 808 000

22. Solve for n. ${}_nC_2 + 170 = {}_{n-1}P_2$

$$\frac{n!}{(n-2)!2!}+170=\frac{(n-1)!}{(n-1-2)!}$$

$$\frac{n(n-1)(n-2)!}{(n-2)!2}+170=\frac{(n-1)!}{(n-3)!}$$

$$\frac{n(n-1)}{2}+170=\frac{(n-1)(n-2)(n-3)!}{(n-3)!}$$

$n(n - 1) + 340 = 2(n - 1)(n - 2)$

$n^2 - n + 340 = 2(n^2 - 3n + 2)$

$n^2 - n + 340 = 2n^2 - 6n + 4$

$n^2 - 5n - 336 = 0$

$(n - 21)(n + 16) = 0$

$N = 21$

Use the following information to answer the next question.

A library group of parents, whose mandate is to encourage all children in the community to read, is composed of 10 moms and 3 dads. The library board is requesting that a committee of 5 be formed from this group to make decisions on how to spend a recent grant from the municipal government.

23. If the committee is to be comprised of at least 2 dads, how many committees are possible?

<u>2 Dads</u>	+	<u>3 Dads</u>		
$({}_{10}C_3)\,({}_3C_2)$	+	$({}_{10}C_2)\,({}_3C_3)$		
360	+	45	=	405

24. Find the middle term of $\left(x - \frac{y^2}{2}\right)^8$.

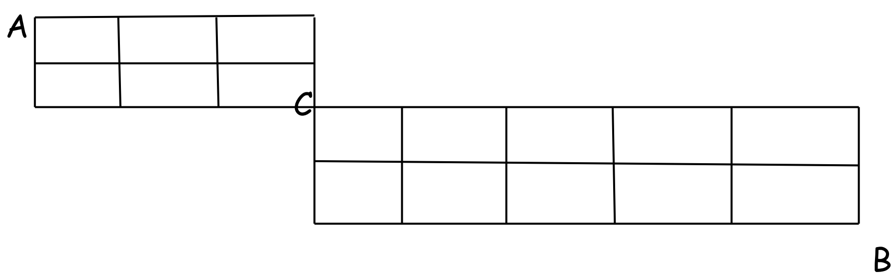
There are 9 terms. The middle term is the 5^{th}. Thus, $k = 4$.

$$_8C_4(x)^4\left(\frac{-y^2}{2}\right)^4$$

$$=\quad 70(x)^4\left(\frac{y^8}{16}\right)$$

$$=\quad \frac{35x^4y^8}{8}$$

Use the diagram below to answer the next question.



25. Ricky says that the number of pathways from A to B, going only down and right, is found using the expression $\frac{12!}{3!2!5!2!}$. However, his cousin Ronny says that the number of pathways are found by using the expression $\frac{7!}{3!2!2!}$. Is either person correct? If so, which one? Explain your reasoning.

From A to C | From C to B

$$\frac{5!}{2!3!} \times \frac{7!}{5!2!} = \frac{7!}{3!2!2!}$$

Ronny is correct as his expression gives the product of the ways from A to C and the ways from C to B.