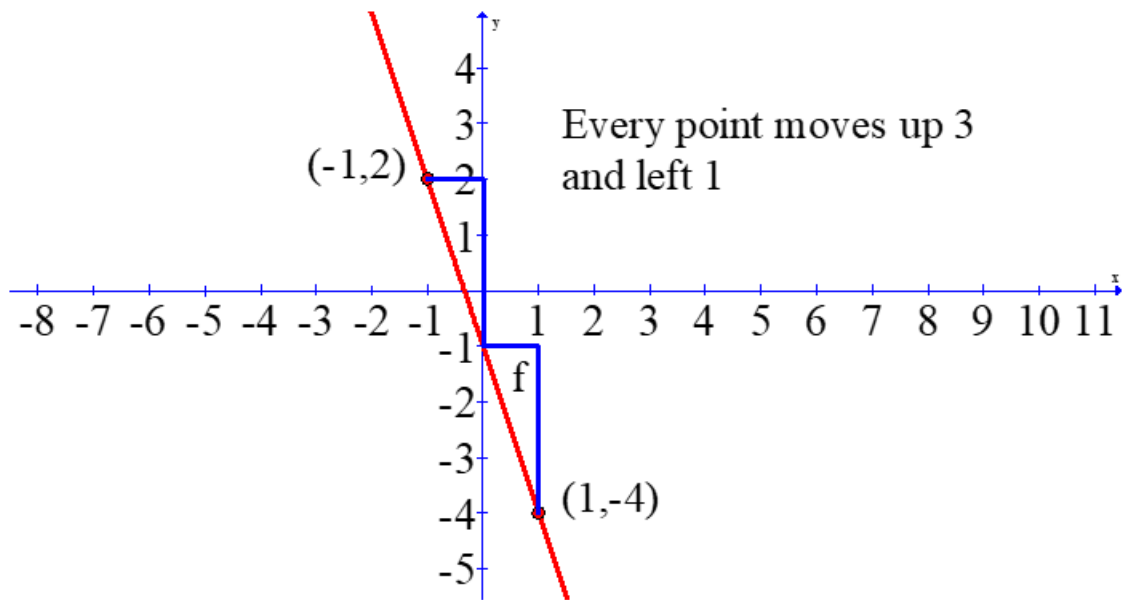


Name _____

Litmus Test For Transformations **Solutions**

Part A Place the correct answer on the sheet provided. Each question is worth 1 mark.

1. The graph of $y = f(x)$ is shown below. Which of the following transformations of $y = f(x)$ will produce an identical graph?



- A) $y - 3 = f(x - 1)$
B) $y - 3 = f(x + 1)$
C) $y + 1 = f(x - 3)$
D) $y - 1 = f(x + 3)$

2. The function $f(x) = |x + 3| + 2$ is transformed into the function $g(x) = |x - 1| + 5$. The transformations that will transform $y = f(x)$ into $y = g(x)$ are a translation 4 units right and a translation 3 units up. Using the chart below, what is a potential code to complete the sentence above?

Reference Number	Numerical Value		Reference Number	Translation Direction
1	1		5	up
2	2		6	down
3	3		7	left
4	4		8	right

- A) 4835
 B) 4735
 C) 3826
 D) 3725

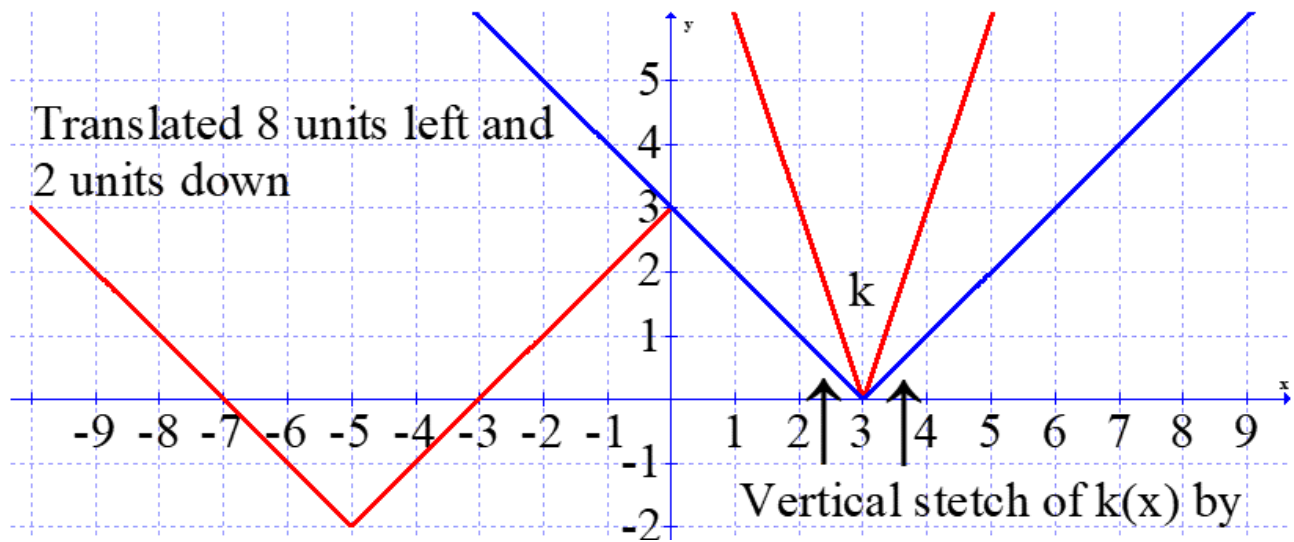
3. What transformations of the function $f(x) = x^2$ are described by the mapping notation $(x, y) \rightarrow (x + 3, y - 5)$?

- A) 3 units left and 5 units up
 B) 3 units right and 5 units down
 C) 3 units left and 5 units down
 D) 3 units right and 5 units up

Use the following information to answer the next question.

The graph of the function $y = k(x)$ is transformed to produce the graph of the function $y = m(x)$.

The graph of the function $y = k(x)$ is transformed to produce the graph of the function $y = m(x)$.



4. An equation for $m(x)$ in terms of $k(x)$ is

A) $m(x) = 3k(x + 8) - 2$

B) $m(x) + 2 = \frac{1}{3}k(x - 8)$

C) $m(x) - 2 = 3k(x - 8)$

D) $m(x) = \frac{1}{3}k(x + 8) - 2$

5. When Point M $(-3,4)$ on $y = f(x)$ is transformed by $y = \frac{1}{2} f(x + 9)$, Point M is now located at

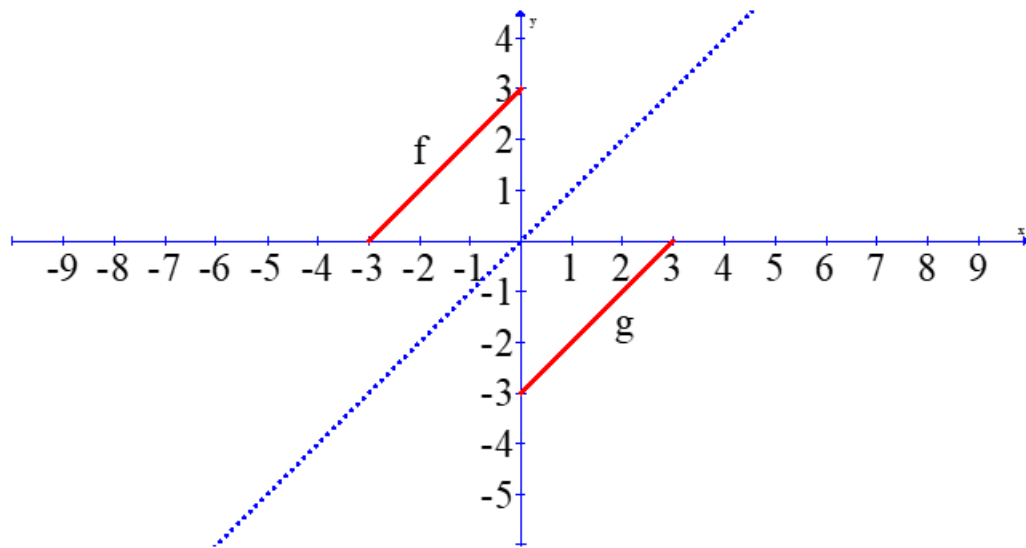
- A) $(-12,2)$
- B) $(6,2)$
- C) $(-6,8)$
- D) $(\frac{-3}{2}, 5)$

The x-coordinate, -3 , is moved 9 units left. The y-coordinate, 4 , is stretched vertically by a factor of $\frac{1}{2}$ about the x-axis.

6. If (m,n) is a point on the graph of $y = f(x)$, which of the following points is on the graph of $y + 3 = -f(x - 1)$?

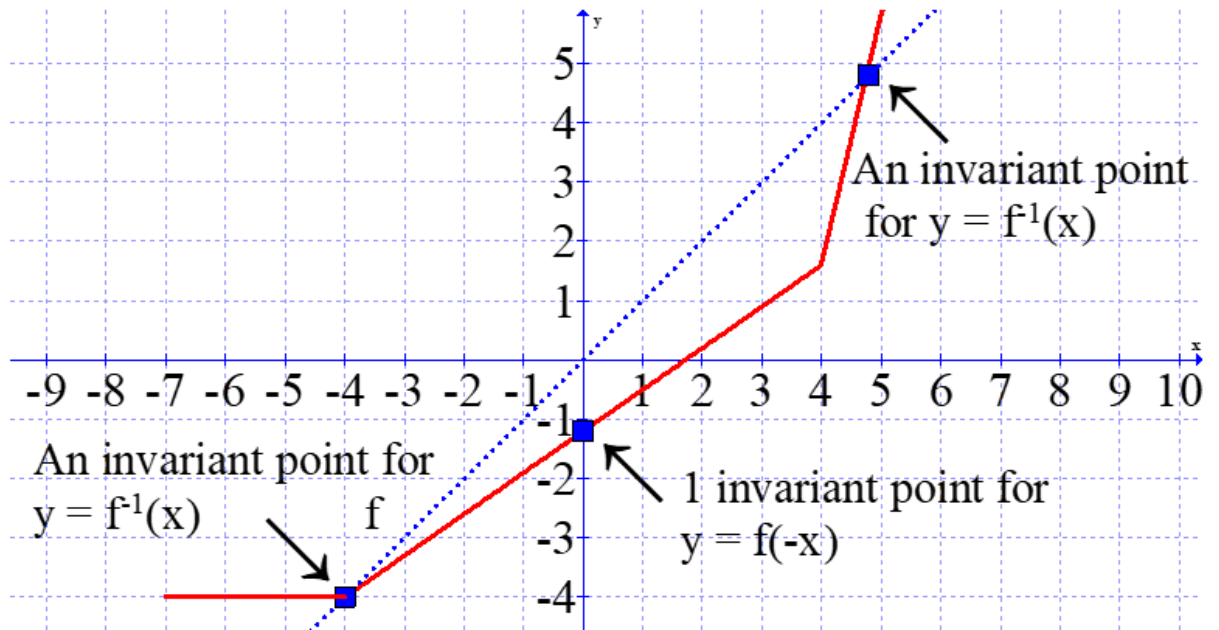
- A) $(m - 1, -n + 3)$
- B) $(m - 3, n - 1)$
- C) $(m + 3, -n + 1)$
- D) $(m + 1, -n - 3)$

7. The graph of the function $y = f(x)$ is shown below in quadrant 2; what transformation will produce $y = g(x)$ shown below in quadrant 4?



- A) $f(-x)$
- B) $-f(x)$
- C) $f(x) - 3$
- D) $f^{-1}(x)$

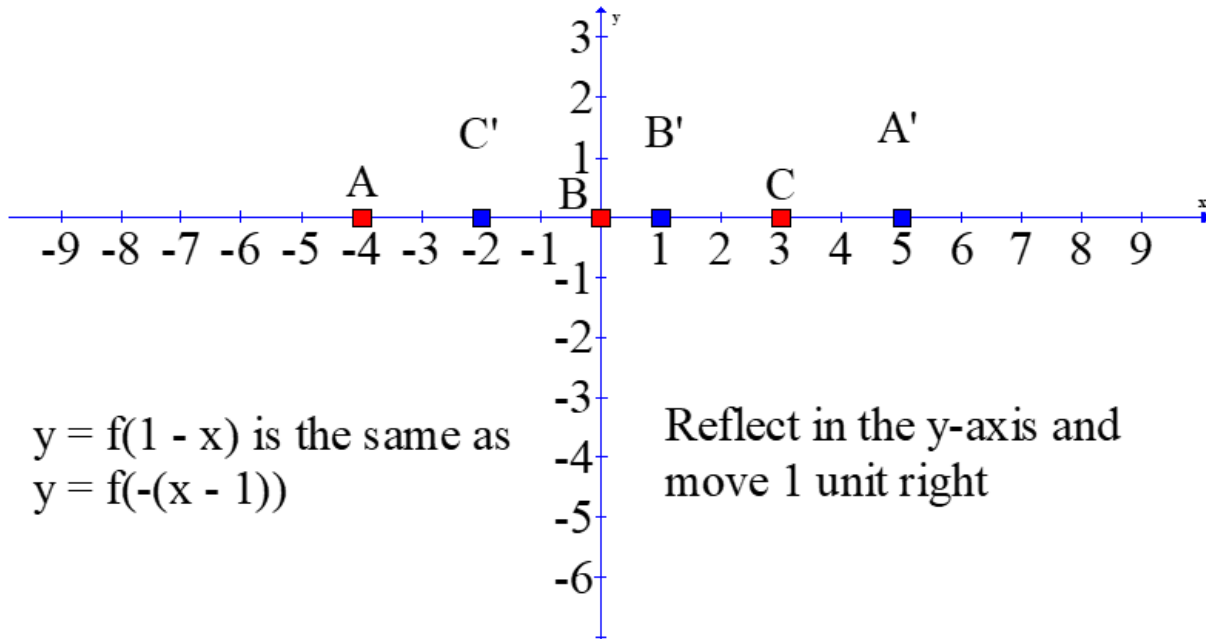
8. The graph of $y = f(x)$ is shown below. When the total number of invariant points for the transformation $y = f(-x)$ is added to the total number of invariant points for the transformation $y = f^{-1}(x)$, the sum is



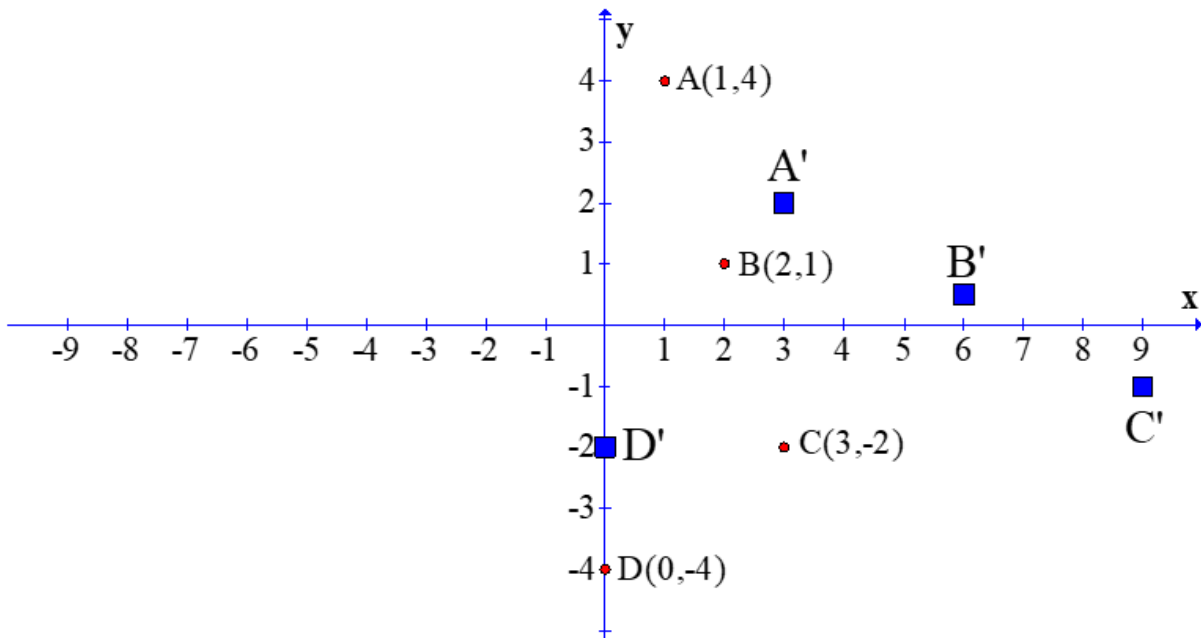
- A) 1 B) 2 C) 3 D) 4

9. The zeros of a function $y = f(x)$ are $-4, 0$ and 3 . Determine the zeros of $y = f(1 - x)$.

- A) $-2, 1, 5$ B) $-1, 2, 5$ C) $-5, -1, 2$ D) $-2, -1, 4$



10. The following four points, $A(1,4)$, $B(2,1)$, $C(3,-2)$ and $D(0,-4)$ are on the graph of a function, $f(x)$, shown below. When $f(x)$ is stretched vertically by a factor of $\frac{1}{2}$ about the x-axis and stretched horizontally by a factor of 3 about the y-axis, which of the four points is now furthest from the origin?



A) A

B) B

C) C

D) D

11. The y-intercept of $y = f(x)$ is $A(0,6)$. Which of the following series of transformations will move point A so that it becomes an x-intercept of $y = f(x)$?

A) $y = -f\left(\frac{1}{3}x\right) + 5$

B) $y = \frac{1}{3}f(-x) - 1$

C) $y = f(x - 3) - 7$

D) $y = -2f(x + 1) + 12$

When $(0,6)$ is reflected in the x-axis and stretched vertically by a factor of 2, it moves to $(0, -12)$. When the point is horizontally translated 1 unit left, it moves to $(-1,-12)$. When the point is vertically translated 12 units up, it moves to $(-1,0)$. It is now an x-intercept.

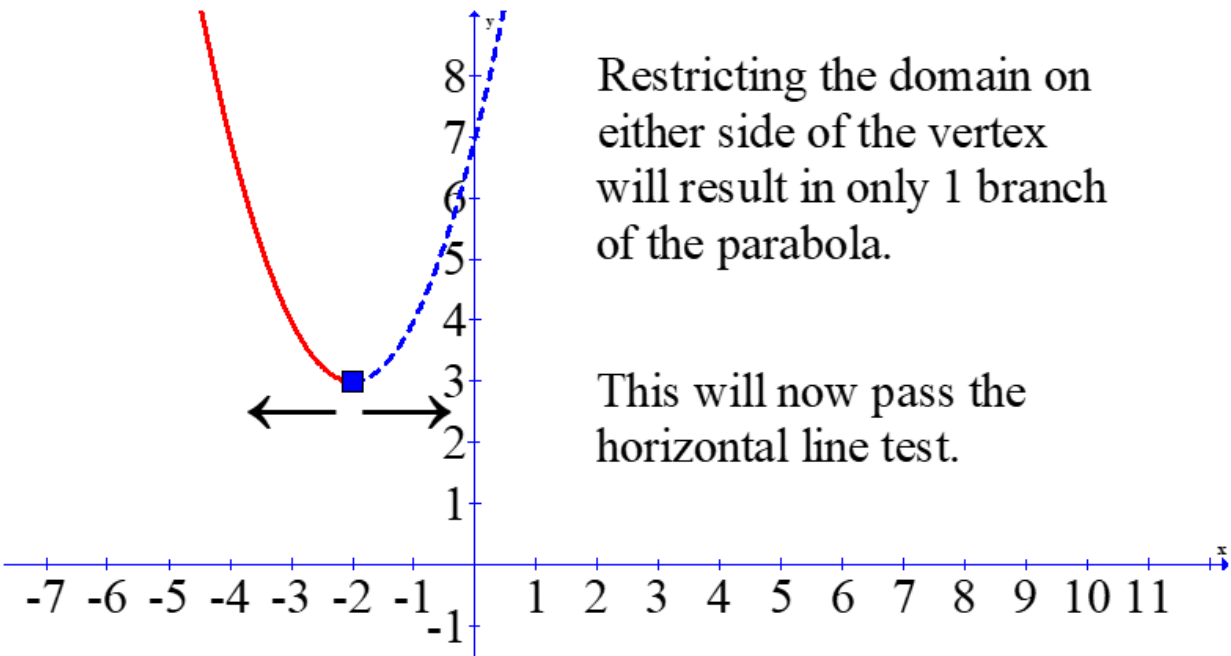
12. Given the equation, $f(x) = (x + 2)^2 + 3$, a restriction on the domain of $f(x)$ such that its inverse is also a function could be:

A) $x \leq 3$

B) $x \geq -3$

C) $x \geq -2$

D) $x \leq 0$



Restricting the domain on either side of the vertex will result in only 1 branch of the parabola.

This will now pass the horizontal line test.

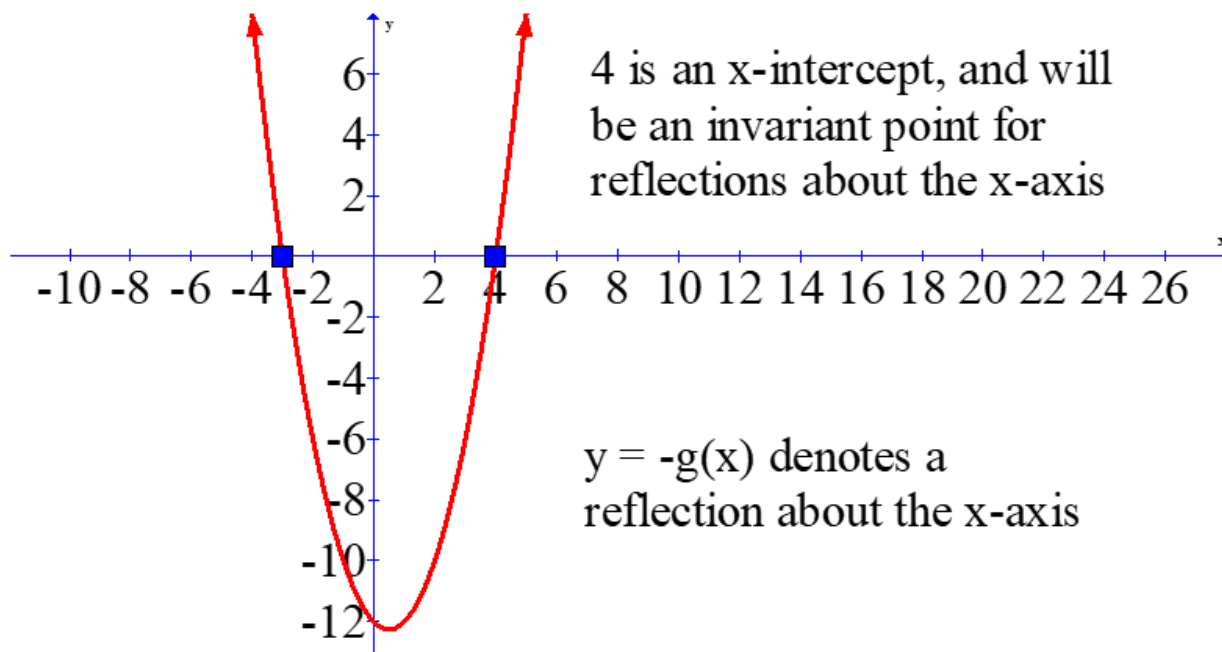
13. The point (a,b) is on $y = f(x)$. The corresponding point on $y = f^{-1}(x) + 7$ is:

- A) $(b + 7, a)$ B) $(-a, b + 7)$ C) $(b, a + 7)$ D) $(a, b - 7)$

For the inverse function, interchange the coordinates, to become (b, a) . Now add 7 to the second coordinate.

14. The function $g(x) = x^2 - x - 12$ is graphed. The point where $x = 4$ is invariant for which of the following transformations?

- A) $y = g(-x)$ B) $y = g^{-1}(x)$ C) $y = -g(x)$ D) $y = g(x) + 4$



Part B Place the correct answer in the space provided. Each correct answer is worth 1 mark.

Use the following information to answer the next question.

The ordered pairs below represent possible transformations of Point K (m, n) on the graph of the function $y = f(x)$.

Point 1 $(m, 6n)$	Point 3 $(-m, n)$	Point 5 $(6m, n)$
Point 2 $(\frac{m}{6}, n)$	Point 4 $(m, \frac{n}{6})$	Point 6 $(m, -n)$

15. If $y = f(x)$ undergoes the following single transformations, identify the coordinates of the corresponding Point K on the new graph.

The corresponding point on the function $6y = f(x)$ is point 4

The corresponding point on the function $y = -f(x)$ is point 6

The corresponding point on the function $y = f(6x)$ is point 2

The corresponding point on the function $y = f(-x)$ is point 3

Use the following information to answer the next question.

When the math teacher asked his students to transform $y = \sqrt{x}$ to $y - 8 = \sqrt{\frac{-1}{3}x + 2}$, he gave them the following statements to consider.

Statement 1 The graph is translated 8 units down. **F**

Statement 2 There is a horizontal stretch by a factor of 3 about the y-axis. **T**

Statement 3 The graph is translated 2 units left. **F**

Statement 4 The graph is reflected in the y-axis. **T**

16. The math teacher told his students that 2 of the statements are false. The false statements are 1 and 3.

Begin by re-writing the function, taking into account that the expression under the square root sign needs to be factored.

$$y = \sqrt{-\frac{1}{3}(x-6)} + 8$$

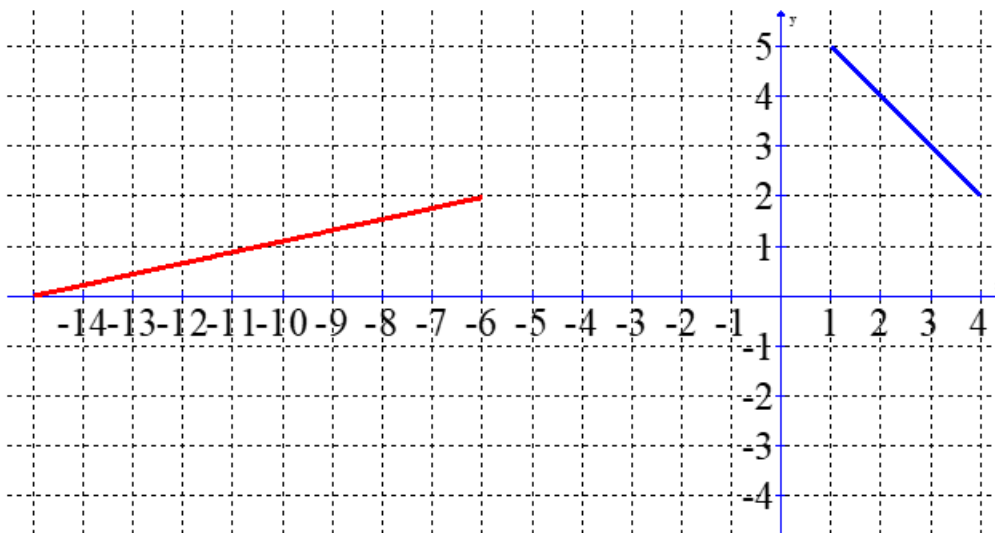
There is a reflection in the y-axis, a horizontal stretch by a factor of 3 about the y-axis, a horizontal translation 6 units right, and a vertical translation 8 units up.

17. The graph of $y = f(x)$ is transformed into the graph of $g(x) - 2 = [f(-3)(x + 1)]$. The domain and range of each graph is shown below.

	Domain	Range
Graph of $f(x)$	$[-15, -6]$	$[0, 2]$
Graph of $g(x)$	$[a, b]$	$[c, d]$

For the graph of $g(x)$, the values a , b , c , and d respectively are:

1, 4, 2, and 4.



The red line is a general visualization of showing a domain of a function between -15 and -6, and a range between 0 and 2.

If we take the far-left boundary of the domain (-15), reflect it in the y-axis, it moves to 15. Now horizontally stretch this by a factor of $\frac{1}{3}$ and it moves to 5. Lastly move it 1 unit left, and the far-right boundary is now at 4.

Thus $b = 4$.

Continuing with the domain, take the original far right boundary of -6 and reflect it in the y-axis. It moves to 6. When it is then stretched horizontally by a factor of $\frac{1}{3}$, it moves to 2. Lastly, move it 1 unit left, and the far-left boundary is now at 1.

Thus $a = 1$.

The only transformation for 'y' is a vertical translation 2 unit up. Moving the boundaries of 0 and 2 up 2, they now become 2 and 4.

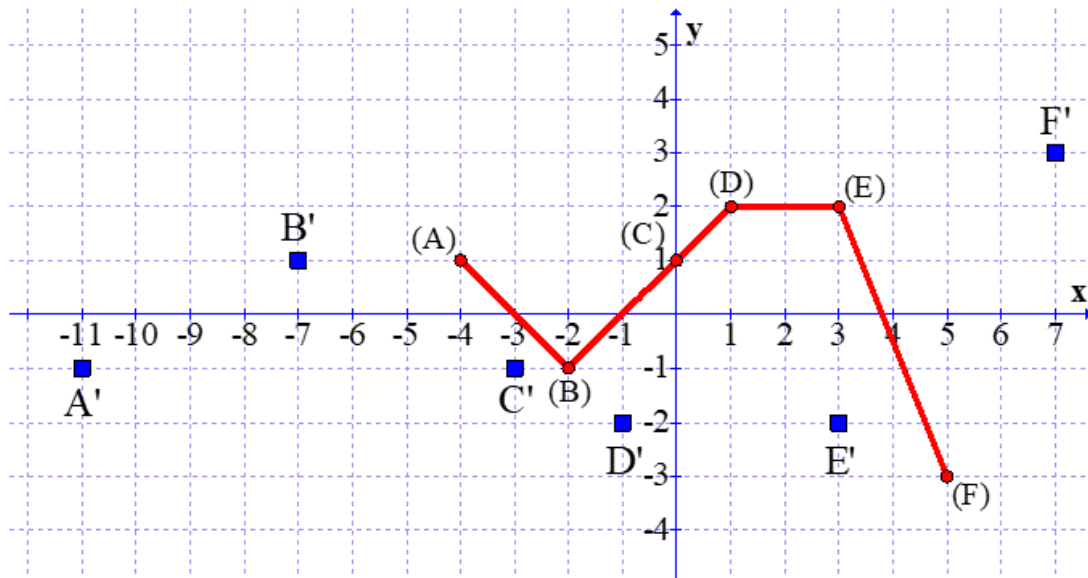
Thus $c = 2$ and $d = 4$.

The blue line in quadrant 1 represents a general visual of how a function might have the new domain and range.

Use the following information to answer the next question.

The graph of $y = f(x)$ is shown below. The following transformations are applied to $y = f(x)$:

1. A reflection in the x-axis.
2. A horizontal stretch by a factor of 2 about the y-axis.
3. A horizontal translation 3 units left.



18. a) After the transformations are applied, how many of the points (A,B,C,D,E,F) will be positioned in quadrant three? Place answer in first box below.
- b) After the transformations are applied, what are the coordinates of point F? Place answer in the second and third box below.

3	7	3	
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19. The point $K(3,27)$ is on the exponential function $y = 3^x$. When this function undergoes the transformations described by, $y - m = 3^{nx}$, the point K now becomes $(1.5, 31)$. The values of m and n respectively are 4 and 2.

For the x-coordinate:

$$3 \times \left(\frac{1}{n}\right) = 1.5$$

$$\left(\frac{3}{n}\right) = 1.5$$

$$\left(\frac{3}{1.5}\right) = n \quad n = 2$$

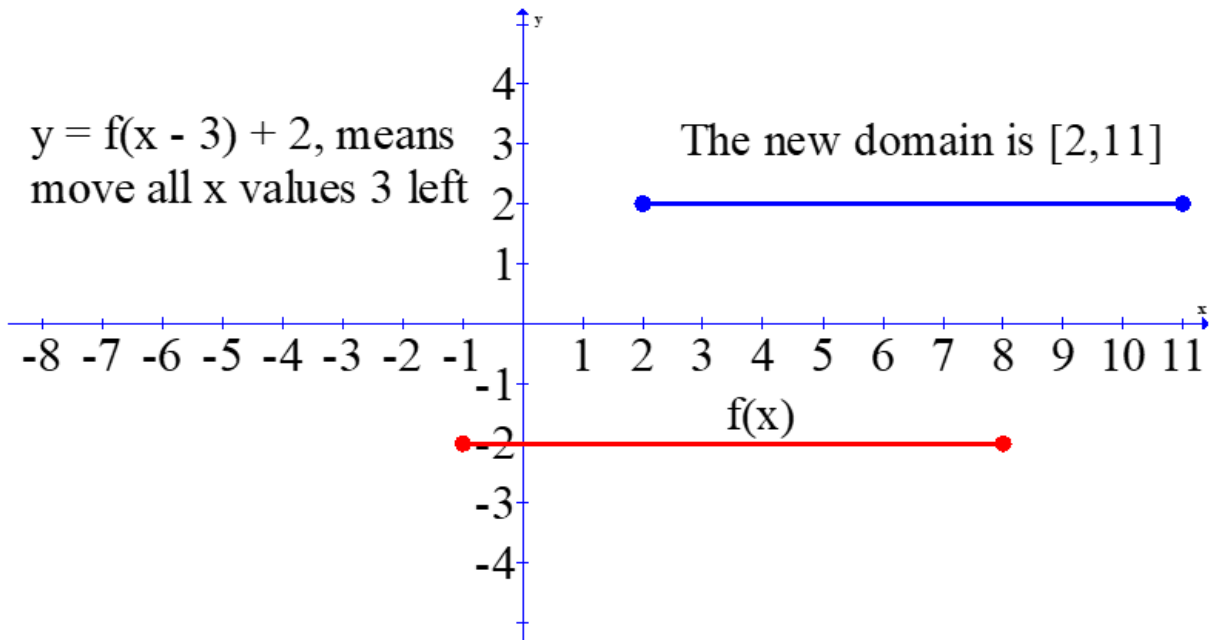
For the y-coordinate:

$$27 + m = 31$$

$$m = 4$$

Part C Show all work and provide all explanations to receive full marks in this section.

20. a) The domain of $y = f(x)$ is $\{x \mid -1 \leq x \leq 8, x \in \mathbb{R}\}$. What is the domain of $y = f(x - 3) + 2$? [Provide a picture to go with your explanation and answer]



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b) As a result of the transformations on the function, $y = f(x)$, will the range be any different? Explain.

— There is a vertical translation 2 units up. If the original range was, $y \in \mathbb{R}$, the
1 range will not change. If the original range was not $y \in \mathbb{R}$, the range will change.

21. The point $M(-3,5)$ is on the graph of $y = f(x)$. As a result of the transformations described by $y = \frac{-1}{5}f((bx) + 1) + 7$, point M is now located at $(-2,6)$.

a) What is the value of b ?

— $-3\left(\frac{1}{b}\right) - 1 = -2$

2 $\left(\frac{-3}{b}\right) = -1$ $b = 3$

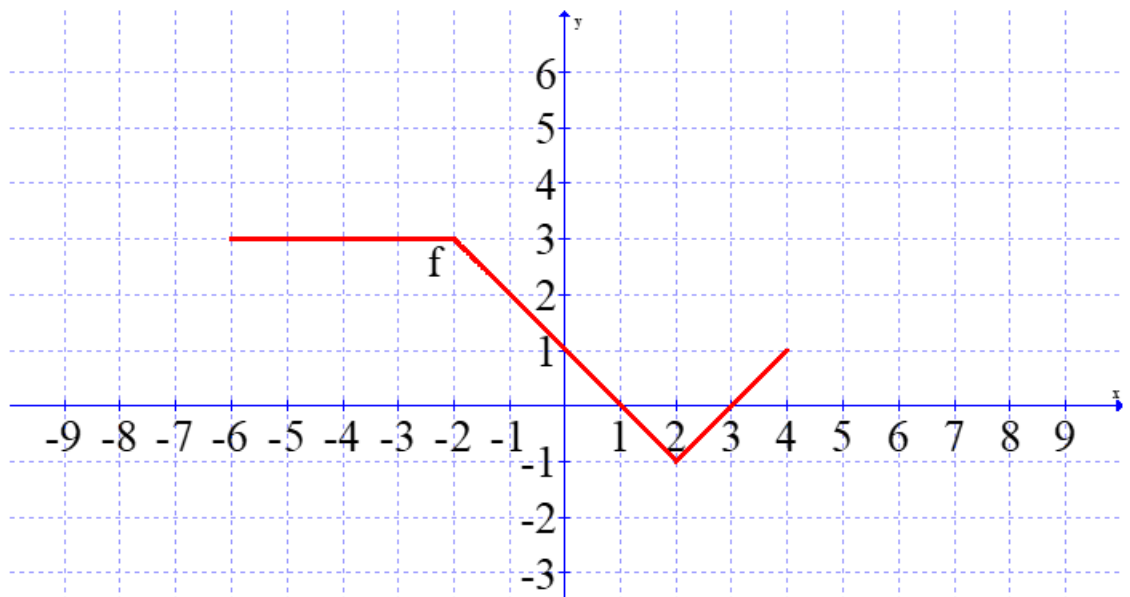
b) Describe how $\frac{-1}{5}$ and 7 affect the original function, $y = f(x)$.

— Vertical stretch by a factor of $1/5$ about the x -axis.

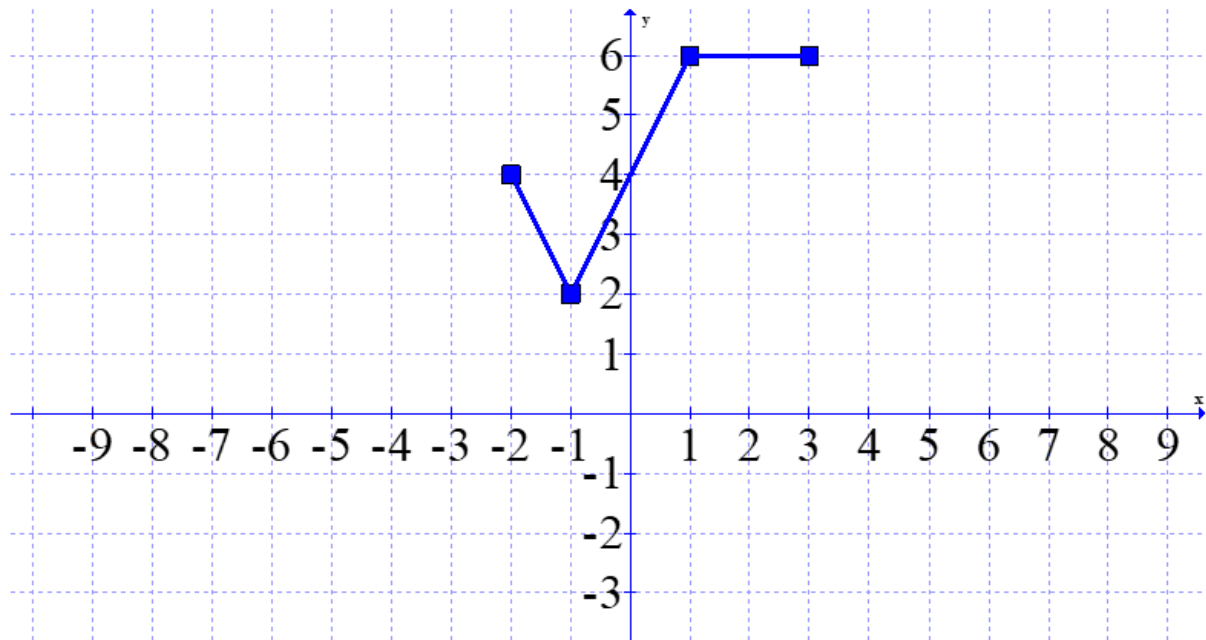
1 Reflection in the x -axis.

Vertical translation 7 units up.

22. The graph of $y = f(x)$ is shown below.



The graph of $y = f(x)$ is reflected in the line $x = 0$, horizontally stretched by a factor of $\frac{1}{2}$ about the y -axis and translated 3 units up. Sketch the graph below.



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