Math 30-2 Rational Epressions Written Response Section

- For full marks, your responses must address **all** aspects of the question.
- All responses, including descriptions and/or explanations of concepts must include pertinent ideas, calculations, formulas and correct units.
- Your responses must be presented in a in a well-organized manner. For example, you may organize your responses in point form or paragraphs.

Written Response 1

Use the following information to answer the next question.

Two different students were asked to simplify $\frac{5x}{5x^2+x}$ and their work is shown below.

	Student A	Student B
Step 1	$\frac{5x}{x(5x+1)}$	$\frac{5x}{x(5x+1)}$
Step 2	$\frac{5}{5x+1}$	$\frac{5}{5x+1}$
Final Answer	$\frac{1}{x+1}, x \neq -1, 0, -\frac{1}{5}$	$\frac{5}{5x+1}, x \neq -\frac{1}{5}$

- **Analyze** and **compare** the work shown by Student A and Student B. [3 Marks]

[*Analyze: "Make a mathematical examination of parts to determine the nature, proportion, function, interrelationships, and characteristics of the whole".*]

[*Compare: "Examine the character or qualities of two things by providing characteristics of both that point out their mutual similarities and differences".*]

- Select one of the correct non-permissible values for either Student A or Student B. **Explain** why the value you have chosen is non-permissible. [2 Marks]

[*Explain: "Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail".*]

Written Response 2

- Given the expression $\frac{x+3}{7x^2-7} \div \frac{Kx+12}{7x^3(x+1)}$, where the non-permissible values are -3, -1, 0, 1, **determine** the value of K. [2 Marks]

[*Determine: "Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations".*]

- **Simplify** the expression. [2 Marks]

Written Response 3

Use the following information to answer the next question.

Consider the following rational equation.

$$\frac{1}{x-5}-\frac{x}{2-x}=\frac{3}{x^2-7x+10}$$

- **Determine** the least common denominator. **Explain** why it is necessary to have the LCD. [2 Marks]

- An equivalent equation can be written in the form $ax^2 - bx - c = 0$. **Determine** the value of b. [1 Mark]

- Does this equation have an *extraneous* root? **Explain.** [2 Marks]

- **Determine** the solution to the equation. **Verify** the solution. [2 Marks]

[*Verify: "Establish, by substitution for a particular case or by geometric comparison, the truth of a statement".*]

Written Response Section Possible Solutions

- For full marks, your responses must address **all** aspects of the question.
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Written Response 1

Use the following information to answer the next question.

Two different students were asked to simplify $\frac{5x}{5x^2+x}$ and their work is shown below.

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Step 1	$\frac{5x}{x(5x+1)}$	$\frac{5x}{x(5x+1)}$
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Final Answer	$\frac{1}{x+1}, x \neq -1, 0, -\frac{1}{5}$	$\frac{5}{5x+1}, x \neq -\frac{1}{5}$

- **Analyze** and **compare** the work shown by Student A and Student B. [3 Marks]

Possible Solution

For step 1 and step 2, each student has shown the correct work.

The final answer for Student A is incorrect. A 5 was divided out of two terms in the expression. It is not possible to do this because if a common factor is divided out of a rational expression, it must be divided out of all of the terms. There are three terms in this expression, and the 5 was not divided out of all three terms.

Student A had two of the three correct non-permissible values. Based on the correct simplification at step one, the non-permissible values are 0 and $-\frac{1}{5}$. Since the simplification is not correct, the stated non-permissible value of -1 is also not correct.

Student B has shown the correct simplification. However, the non-permissible values are not correct. This student has shown one correct non-permissible value, but has missed the fact that 0 is also a non-permissible value. The concept error made here is that non-permissible values must be stated prior to the simplification process. Student B stated the non-permissible values only at the final simplification step.

- Select one of the correct non-permissible values for either Student A or Student B. **Explain** why the value you have chosen is non-permissible. [2 Marks]

Possible Solution

Student A correctly selected 0 as a non-permissible value. Non-permissible values are determined when all factoring has taken place and before any simplification has occurred. At the step showing $\frac{5x}{x(5x+1)}$

the non-permissible values should be stated. At this step, if x = 0, the denominator would be equal to zero. If the denominator of a rational expression is equal to zero, the expression would be considered undefined. Hence, zero is a non-permissible value.

Written Response 2

- Given the expression $\frac{x+3}{7x^2-7} \div \frac{Kx+12}{7x^3(x+1)}$, where the non-permissible values are -3, -1, 0, 1, **determine** the value of K. [2 Marks]

Possible Solution

Factor the expression.

$$\frac{x+3}{7x^2-7} \div \frac{Kx+12}{7x^3(x+1)} = \frac{x+3}{7(x+1)(x-1)} \div \frac{Kx+12}{7x^3(x+1)}$$

When analyzing the *denominators* of the two terms, the non-permissible values are x = -1, 0 and 1. We know that the numerator of the second term will move to the denominator when the 'multiply by the reciprocal' is applied. Therefore, Kx + 12 cannot be equal to zero.

We are told in the question that x = -3 is a non-permissible value.

Solve K(-3) + 12 = 0

-3K = -12

K = 4

The value of k is 4.

- **Simplify** the expression. [2 Marks]

Possible Solution

$$\frac{x+3}{7(x+1)(x-1)} \div \frac{4x+12}{7x^3(x+1)}$$

$$= \frac{x+3}{7(x+1)(x-1)} \div \frac{4(x+3)}{7x^3(x+1)}$$

= $\dfrac{x+3}{7(x+1)(x-1)} \bullet \dfrac{7x^3(x+1)}{4(x+3)}$

Divide out the common terms in the numerator and the denominator.

= $\dfrac{x^3}{4(x-1)}$, $x \neq -3, -1, 0, 1$

The simplified expression is $\dfrac{x^3}{4(x-1)}$, $x \neq -3, -1, 0, 1$.

Written Response 3

Use the following information to answer the next question.

Consider the following rational equation.

$$\frac{1}{x-5} - \frac{x}{2-x} = \frac{3}{x^2-7x+10}$$

- **Determine** the least common denominator. **Explain** why it is necessary to have the LCD. [2 Marks]

Possible Solution

Factor the expression.

$$\frac{1}{x-5} - \frac{x}{2-x} = \frac{3}{(x-2)(x-5)}$$

The denominator of the middle term can be written in an equivalent form by dividing out -1. In doing so, it will then be in the same form as a binomial in the last term.

$$\frac{1}{x-5}-\frac{x}{-(x-2)}=\frac{3}{(x-2)(x-5)}$$

The subtraction sign can now be written as an addition sign.

$$\frac{1}{x-5}+\frac{x}{(x-2)}=\frac{3}{(x-2)(x-5)}$$

The least common denominator is the product of all common and unique factors of the denominators. The LCD is (x – 2) (x – 5).

It is necessary to determine the LCD because the fractions can then be cleared, allowing us to solve the equation with algebraic techniques.

- An equivalent equation can be written in the form $ax^2 - bx - c = 0$. **Determine** the value of *b*. [1 Mark]

Possible Solution

Multiply each term by the LCD.

$$[(x-2)(x-5)]\frac{1}{x-5}+[(x-2)(x-5)]\frac{x}{(x-2)}=[(x-2)(x-5)]\frac{3}{(x-2)(x-5)}$$

= $(x - 2) + x(x - 5) = 3$

= $x - 2 + x^2 - 5x = 3$

= $x^2 - 4x - 5 = 0$

The value of b is 4.

- Does this equation have an *extraneous* root? **Explain.** [2 Marks]

Possible Solution

Factor $x^2 - 4x - 5 = 0$ and apply the zero product principle.

(x – 5) (x + 1) = 0

It appears that the roots are 5 and -1. However, by looking at the original factored expression, we can see that 5 is a non-permissible value.

Therefore, x = 5 is an extraneous root.

- **Determine** the solution to the equation. **Verify** the solution. [2 Marks]

Possible solution

The quadratic function, $x^2 - 4x - 5 = 0$, is a simplified expression when the original rational expression is re-written by clearing the fractions.

Applying the zero product principle, (x – 5) = 0, or (x + 1) = 0. The roots are 5 and -1. As stated in the bullet above, 5 is an extraneous root.

The solution is x = -1, x ≠ 2, 5.

To verify, substitute the value of x = -1 into the original equation.

$$\frac{1}{(-1)-5} - \frac{(-1)}{2-(-1)} = \frac{3}{(-1)^2 - 7(-1) + 10}$$

$$\frac{-1}{6} + \frac{1}{3} = \frac{3}{1+7+10}$$

$$\frac{-1}{6} + \frac{2}{6} = \frac{3}{18}$$

$$\frac{1}{6} = \frac{1}{6}$$

Since the left side of the equation is equal to the right side of the equation, the solution is verified.