Math 30-2 Polynomial Written Response Section

- For full marks, your responses must address **all** aspects of the question.
- All responses, including descriptions and/or explanations of concepts must include pertinent ideas, calculations, formulas and correct units.
- Your responses must be presented in a in a well-organized manner. For example, you may organize your responses in point form or paragraphs.

Written Response 1

Use the following information to answer the next question.

Joanna was asked to describe the graph of $f(x) = (x + 2)^2 (x - 3)$. Her response is as follows:

- The domain is $\{x \mid -2 \le x \le 3, x \in R\}$.
- The range is $\{y \mid y \in R\}$.
- There are 3 distinct x-intercepts.
- There are two turning points, at (1.33, -18.52) and (-4, 0).
- The graph extends down in quadrant 3 and up in quadrant 1.
- The degree of the polynomial is 3.
- The y-intercept is -12.
- One of the x-intercepts is 3.

- **Analyze** her response. Indicate her correct observations. Correct any errors. [3 Marks]

[*Analyze: "Make a mathematical examination of parts to determine the nature, proportion, function, interrelationships, and characteristics of the whole".*]

- **Describe** the changes that would occur if the leading coefficient was negative. [2 Marks]

[*Describe: "Give a written account of a concept".*]

Written Response 2

Use the following information to answer the next question.

The cafeteria at Bridgetown High School sells a lunch special every Friday for \$4. Last year, on average, 280 students purchased the lunch special. Needing to raise the cost for next year, a survey was conducted to estimate potential revenue for different lunch special prices. The table below summarizes the data.

Lunch Special	Potential Revenue
\$4	\$1 120
\$5	\$1 250
\$6	\$1 320
\$7	\$1 330
\$8	\$1 280

The data can be modeled by the quadratic equation,

$y = ax^2 + bx + c$, where x is the cost of the lunch special, and y is the potential revenue in dollars.

- **Determine** the value of *b* for the quadratic regression function. [1 Mark]

[*Determine: "Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations".*]

- Using the regression model, **algebraically determine** the potential revenue for a lunch special of $8.50. Round the answer to the nearest cent. [1 Mark]

[*Algebraically: "Using mathematical procedures that involve variables or symbols to represent values".*]

- **Determine** the maximum potential revenue. [2 Marks]

- A cost analysis indicates that a potential revenue below $1275 is not feasible for the cafeteria to continue to operate. **Interpret** the effect on the cost of the lunch special. [2 Marks]

[*Interpret: "Provide a meaning of something; present information in a new form that adds meaning to the original data".*]

Written Response 3

Use the following information to answer the next question.

An experiment was performed to observe the volume of air in the lungs, as a function of time. The following data was gathered and is shown in the table below:

Time (seconds)	Volume (litres)
1	0.341
2	0.796
2.5	0.989375
3	1.119
3.5	1.154125
4	
5	0.385

After plotting the points and observing the shape, it was determined that a cubic regression function would best model the data.

The regression equation is $V(t) = -0.041t^3 + 0.18t^2 + 0.202t$

- In row 6 of the table, a value is missing. **Determine** this value **algebraically**, accurate to 3 decimal places. **Explain** its meaning. [2 Marks]

[*Explain:" Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail".*]

- **Describe** the graphing characteristic that is required in order to find the length of one breath. [1 Mark]

- Assuming that negative values have no meaning in the context of this question, **determine** the domain and range, accurate to the nearest hundredth. **Justify.** [3 Marks]

[*Justify: "Indicate why a conclusion has been stated, by providing supporting reasons and/or evidence that form a mathematical argument".*]

Written Response Section Possible Solutions

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Written Response 1

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Joanna was asked to describe the graph of $f(x) = (x + 2)^2 (x - 3)$. Her response is as follows:

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- There are 3 distinct x-intercepts.
- There are two turning points, at (1.33, -18.52) and (-4, 0).
- The graph extends down in quadrant 3 and up in quadrant 1.
- The degree of the polynomial is 3.
- The y-intercept is -12.
- One of the x-intercepts is 3.

- **Analyze** her response. Indicate her correct observations. Correct any errors. [3 Marks]

Possible Solution

The domain is not correct; it should be $\{x \mid x \in R\}$.

The range is correct.

There are two distinct x –intercepts, not three. The x-intercepts are -2 and 3.

There are two turning points. The turning point at (1.33, -18.52) is correct, but the other turning point is at (-2,0).

The end behaviour described by, the graph extends down in quadrant 3 and up in quadrant 1, is correct.

The polynomial is of degree 3.

The y-intercept is -12.

One of the x-intercepts is 3.

- **Describe** the changes that would occur if the leading coefficient was negative. [2 Marks]

Possible Solution

A negative leading coefficient would change:

- The end behaviour – it now extends up in quadrant 2 and down in quadrant 4.
- The y-intercept would change from -12 to 12.
- One of the turning points would change – from (1.33, -18.52) to (1.33, 18.52).

A negative leading coefficient would result in no change to:

- Domain
- Range
- x-intercepts
- the degree

Written Response 2

Use the following information to answer the next question.

The cafeteria at Bridgetown High School sells a lunch special every Friday for $4. Last year, on average, 280 students purchased the lunch special. Needing to raise the cost for next year, a survey was conducted to estimate potential revenue for different lunch special prices. The table below summarizes the data.

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The data can be modeled by the quadratic equation,

$y = ax^2 + bx + c$, where x is the cost of the lunch special, and y is the potential revenue in dollars.

- **Determine** the value of *b* for the quadratic regression function. [1 Mark]

Solution

The regression equation is $y = -30x^2 + 400x$. The value of *b* is 400.

- Using the regression model, **algebraically determine**, the potential revenue for a lunch special of $8.50. Round the answer to the nearest cent. [1 Mark]

Solution

$y = -30(8.5)^2 + 400(8.5)$

$y = -2167.50 + 3400$

$y = 1232.50$

The potential revenue for a lunch special of $8.50 is $1232.50.

- **Determine** the maximum potential revenue. [2 Marks]

Solution

Use the graphing calculator to determine the maximum value. The maximum potential revenue is $1333.33.

- A cost analysis indicates that a potential revenue below $1275 is not feasible for the cafeteria to continue to operate. **Interpret** the effect on the cost of the lunch special. [2 Marks]

Possible Solution

If the cost of the lunch special is below, or above, a particular number, the cafeteria will cease operations. Providing a price that is too low will not cover costs, and providing a price too high, will likely mean that fewer students will buy lunch; therefore, once again, not being able to cover costs.

In the calculator, input $y_1 = -30x^2 + 400x$, and $y_2 = 1275$. There are two points of intersection. The first point is (5.27, 1275) and the second point is (8.06, 1275).

The cost for the lunch special should be between $5.27 and $8.06.

Written Response 3

Use the following information to answer the next question.

An experiment was performed to observe the volume of air in the lungs, as a function of time. The following data was gathered and is shown in the table below:

Time (seconds)	Volume (litres)
1	0.341
2	0.796
2.5	0.989375
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5	0.385

After plotting the points and observing the shape, it was determined that a cubic regression function would best model the data.

The regression equation is $V(t) = -0.041t^3 + 0.18t^2 + 0.202t$

- In row 6 of the table, a value is missing. **Determine** this value **algebraically**, accurate to 3 decimal places. **Explain** its meaning. [2 Marks]

Solution

Substitute (4) into the regression equation to find the missing volume.

$V(4) = -0.041(4)^3 + 0.18(4)^2 + 0.202(4)$

$= \quad -2.624 \quad + \quad 2.88 \quad + \quad 0.808$

$= \quad 1.064$

After 4 seconds of taking a breath, the volume of air in the lungs is 1.064 litres.

- **Describe** the graphing characteristic that is required in order to find the length of one breath. [1 Mark]

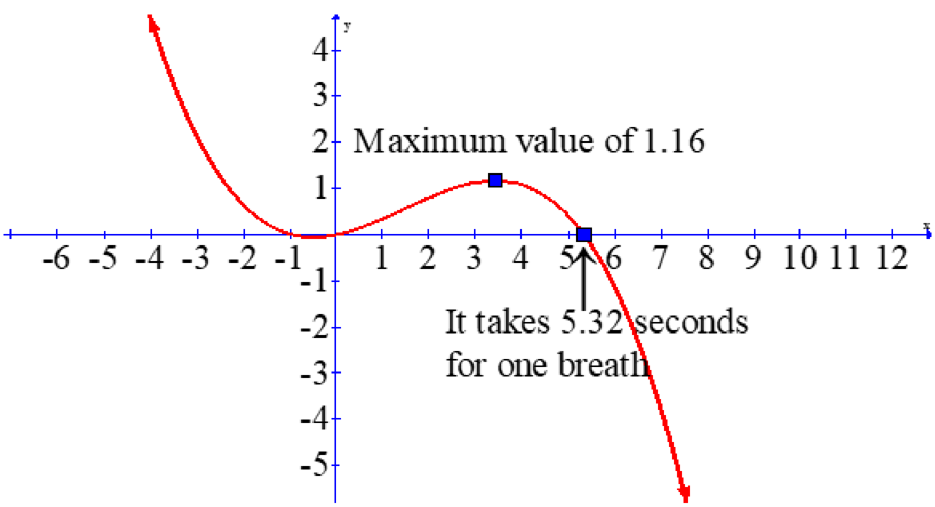
Possible Solution

The independent variable is time. At the end of one breath, the air in the lungs is zero. When V = 0, the graph has touched the x-axis.

The graphing characteristic needed is the x-intercept.

- Assuming that negative values have no meaning in the context of this question, **determine** the domain and range, accurate to the nearest hundredth. **Justify**. [3 Marks]

Possible Solution



Although the graph is a cubic function that extends up in quadrant 2 and down in quadrant 4, since we are not considering any negative values, the domain and range are determined by the values in quadrant 1.

Using technology, determine the maximum value and the positive x-intercept. These numbers will establish the necessary limits. In both cases, 0 is the lowest value.

The domain is $\{x \mid 0 \leq x \leq 5.32, x \in R\}$.

The range is $\{y \mid 0 \leq y \leq 1.16, y \in R\}$.