Math 30-1 Trigonometric Identities Written Response

- Write your responses as neatly as possible.
- For full marks, your responses must address **all** aspects of the question.
- All responses, including descriptions and/or explanations of concepts must include pertinent ideas, calculations, formulas and correct units.
- Your responses must be presented in a in a well-organized manner. For example, you may organize your responses in point form or paragraphs.

Use the following to answer the next question.

Example 1	$(\sin 2x)(\sec^2 x) = 2\tan x$
Example 2	$\cos 2x + 1 = 1 - \sin^2 x$

Written Response 1

- **Compare** the two examples above and **justify** which example is a trigonometric equation, and which example is a trigonometric identity. [3 Marks]

[*Compare: "Examine the character or qualities of two things by providing characteristics of both that point out their mutual similarities and differences".*]

[*Justify: "Indicate why a conclusion has been stated, by providing supporting reasons and/or evidence that form a mathematical argument".*]

- **Determine** the general solution for the trigonometric equation, in radians. [2 Marks]

[*Determine: "Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations".*]

- **Determine** the non-permissible values for the trigonometric identity, in degrees. [2 Marks]

<u>Written Response 2</u>

- **Explain** the meaning of a conjugate. **Illustrate** with an example. [2 Marks]

[*Explain: "Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail".*]

[*Illustrate: "Make clear by giving an example. The form of the example will be specified in the question: e.g., a word description, sketch, or diagram".*]

- **Prove** the identity, $\frac{\tan x}{1+\cos x}=\frac{1-\cos x}{\sin x \cos x}$, by using the concept of conjugates. [3 Marks]

[*Prove: "Establish the truth or validity of a statement by giving factual evidence or logical argument".*]

<u>Written Response 3</u>

- **Algebraically solve,** csc2x – cot2x = 1, within the domain [-π, 0]. [3 Marks]

[*Algebraically: "Using mathematical procedures that involve variables or symbols to represent values".*]

[*Solve: "Give a solution to a problem".*]

Written Response Section Possible Solutions

Use the following to answer the next question.

Example 1	$(\sin 2x)(\sec^2 x) = 2\tan x$
Example 2	$\cos 2x + 1 = 1 - \sin^2 x$

Written Response 1

- **Compare** the two examples above and **justify** which example is a trigonometric equation, and which example is a trigonometric identity. [3 Marks]

Possible Solution

A trigonometric **identity** is true for all permissible values of x. A trigonometric **equation** is true for only certain specific values of x.

When using technology, if the right side of the equal sign appears to be the exact same graph as the left side of the equal sign, it would appear that the statement is an identity. Example 1 appears to be a trigonometric identity for this reason.

When Example 2 is graphed using the left side and the right side of the equal sign, there are two separate graphs showing. Example two appears to be a trigonometric equation.

Alternatively, showing that the trigonometric equation has a specific value or values which make it true, would justify its status.

Working with example 2:

$\cos 2x + 1 = 1 - \sin^2 x$

Subtract 1 from both sides.

$\cos 2x = -\sin^2 x$

Make a substitution from the formula sheet.

$1 - 2\sin^2 x = -\sin^2 x$

Gather like terms.

$1 = \sin^2 x$

Graphing the left side as y_1 and the right side as y_2, and finding intersection points, $x = 90^0 + 180^0 n, n \in I$.

Example 2 is a trigonometric equation.

Alternatively, for the trigonometric identity, proving that the left side of the equal sign is equal to the right side of the equal sign, using algebraic techniques, would justify its status.

Working with example 1:

$(\sin 2x)(\sec^2 x) = 2\tan x$

Re-write the left side with a double angle formula sheet substitution, and re-write the reciprocal ratio with its primary ratio equivalent.

$2\sin x\cos x\left(\frac{1}{\cos^2 x}\right) = 2\tan x$

Divide out cos x.

$2\sin x\left(\frac{1}{\cos x}\right) = 2\tan x$

$\left(\frac{2\sin x}{\cos x}\right) = 2\tan x$

$2\tan x = 2\tan x$

Example 1 is a trigonometric identity.

- **Determine** the general solution for the trigonometric equation, in radians. [2 Marks]

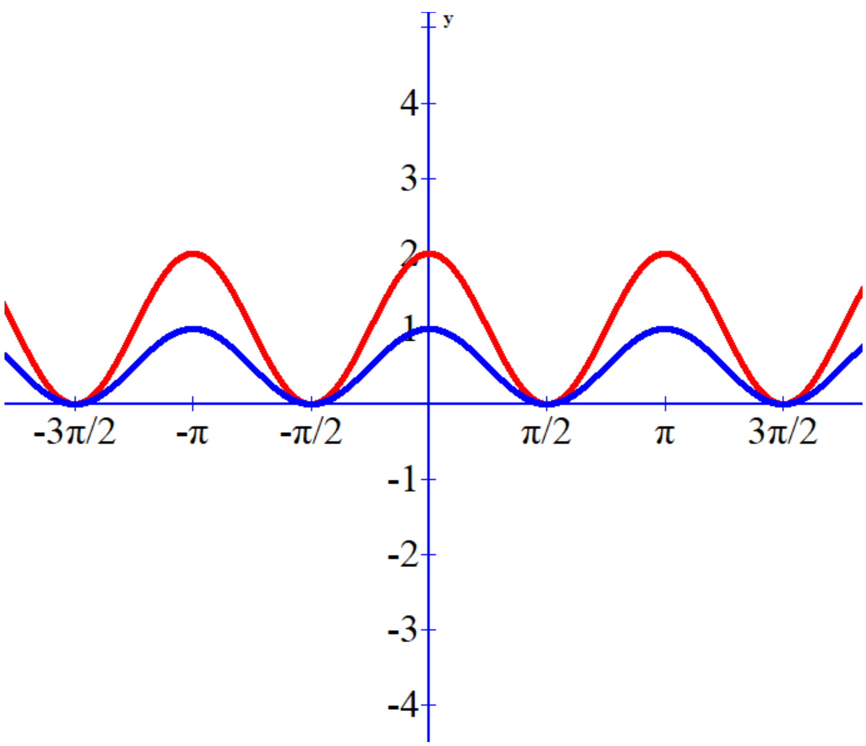
Possible Solution

As mentioned above, Example two is the trigonometric equation.

$\cos 2x + 1 = 1 - \sin^2 x$

Method 1 – Graphically

The intersection points of the two graphs represent the solutions. As shown below, the graphs intersect at $\frac{\pi}{2}$ and at every interval of π going either left or right. The general solution is $x = \frac{\pi}{2} + \pi n$, $n \in I$.



Method 2 – Algebraically

From the formula sheet, substitute an equivalent expression for the cosine double-angle identity.

$\cos 2x + 1 = 1 - \sin^2 x$

$(2\cos^2 x - 1) + 1 = 1 - \sin^2 x$

The formula sheet identity, $\sin^2 x + \cos^2 x = 1$, has two equivalent versions:

$\cos^2 x = 1 - \sin^2 x$ and $\sin^2 x = 1 - \cos^2 x$

Now substitute on the right side of the equation.

$(2\cos^2 x - 1) + 1 = \cos^2 x$

Set the equation equal to zero and simplify.

$2\cos^2 x - \cos^2 x - 1 + 1 = 0$

$\cos^2 x = 0$

Take the square root of both sides.

$\cos x = 0$

Using the calculator, $\cos^{-1}(0) = \frac{\pi}{2}$. The next values will occur at every interval of π.

The general solution is $x = \frac{\pi}{2} + \pi n,\ n \in I$.

- **Determine** the non-permissible values for the trigonometric identity, in degrees. [2 Marks]

Possible Solution

As mentioned above, Example 1 is the trigonometric identity.

A non-permissible value may occur when a variable is present in the denominator of a rational expression. Division by zero is not allowed and any values of the variable which makes the denominator equal to zero are non-permissible values.

For trigonometric identities, check all denominators, as well as any ratios that can be written in an equivalent form with a denominator.

Given Example 1, $(\sin 2x)(\sec^2 x) = 2\tan x$, there is no denominator shown; but, $\sec^2 x$ can be written as $\frac{1}{\cos^2 x}$, and tanx can be written as $\frac{\sin x}{\cos x}$. Non-permissible values will occur for values of x which make cosx = 0.

$\cos^{-1}(0) = 90^0$

If either the left side of the equal sign, or the right side of the equal sign is graphed, the x-intercepts of the equivalent tangent graph, will occur at 90^0 and then at every interval of 180^0.

The non-permissible values are $x \neq 90^0 + 180^0 n,\ n \in I$.

Written Response 2

- **Explain** the meaning of a conjugate. **Illustrate** with an example. [2 Marks]

Possible Solution

A math conjugate is formed by changing the signs between two terms in a binomial. For example, given the binomial (2x + 5), its conjugate is (2x – 5).

- **Prove** the identity, $\frac{\tan x}{1+\cos x} = \frac{1-\cos x}{\sin x \cos x}$, by using the concept of conjugates. [3 Marks]

Possible Solution

Work the left side of the equal sign, and begin by multiplying the numerator and the denominator by the conjugate of the denominator.

$$\frac{\tan x}{1+\cos x} X \frac{1-\cos x}{1-\cos x}$$

$$= \frac{\tan x - \tan x \cos x}{1-\cos^2 x}$$

$$= \frac{\frac{\sin x}{\cos x} - \left(\frac{\sin x}{\cos x}\right)\cos x}{1-\cos^2 x}$$

Re-write the denominator as $\sin^2 x$.

$$= \frac{\frac{\sin x}{\cos x} - \frac{\sin x \cos x}{\cos x}}{\sin^2 x}$$

Multiply by the reciprocal of the denominator, and factor the numerator.

$$= \frac{\sin x(1-\cos x)}{\cos x} X \frac{1}{\sin^2 x}$$

Divide out a sinx on the numerator and the denominator.

$$= \frac{1-\cos x}{\cos x \sin x}$$

The left side of the equal sign is now equal to the right side. Thus, the identity has been proved using conjugates.

Written Response 3

- **Algebraically solve,** csc2x – cot2x = 1, within the domain [-π, 0]. [3 Marks]

$$\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x} = 1$$

$$\frac{1-\cos 2x}{\sin 2x} = 1$$

Substitute (1 – $2\sin^2 x$) for (cos2x) and (2sinxcosx) for (sin2x) [from the formula sheet]

$$\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}=1$$

Simplify the numerator.

$$= \quad \frac{2\sin x\sin x}{2\sin x\cos x}=1$$

Divide out the common 2sinx in the numerator and the denominator.

$$= \quad \frac{\sin x}{\cos x} \textit{ or } \tan x$$

= tanx = 1

In the calculator, type $\tan^{-1}(1)$ and determine that the reference angle in radians is $\frac{\pi}{4}$. The tangent ratio is positive in quadrants one and three.

Within the given domain, starting at zero degrees, rotate clockwise to $-\frac{3\pi}{4}$

$x = -\frac{3\pi}{4}$