Math 20-1 Trigonometry Written Response

- Write your responses as neatly as possible.
- For full marks, your responses must address **all** aspects of the question.
- All responses, including descriptions and/or explanations of concepts must include pertinent ideas, calculations, formulas, and correct units.
- Your responses must be presented in a in a well-organized manner. For example, you may organize your responses in point form or paragraphs.

WRITTEN RESPONSE 1

Consider the trigonometric equation, $cos\theta = \frac{\sqrt{2}}{2}$, where $0^0 \leq \theta < 360^0$.

- **Determine** the reference angle? [1 Mark]

[Determine: *Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations*]

- In which quadrant or quadrants lies the terminal arm? **Explain.** [1 Mark]

[*Explain: Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail*]

- **Sketch** the largest possible value of θ in standard position. **Explain.** [1 Mark]

[Sketch: *Provide a drawing that represents the key features or characteristics of an object or graph*]

- **Solve** the equation. [1 Mark]

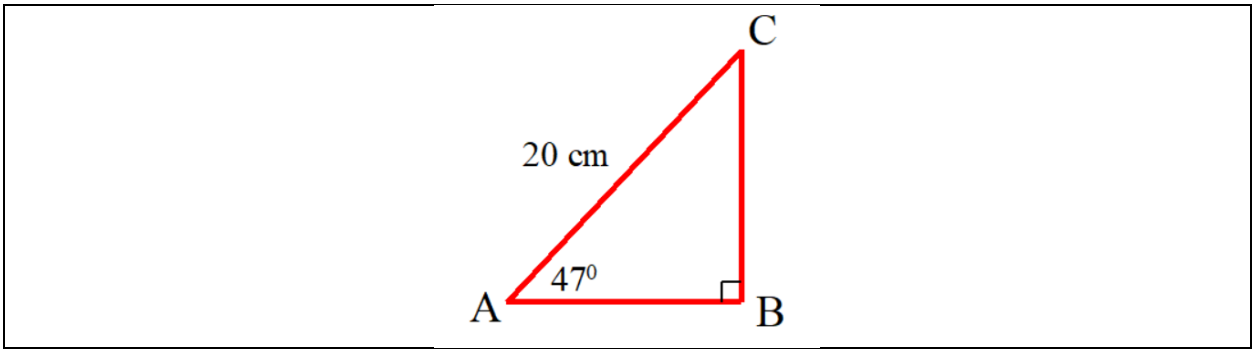
[Solve: *Give a solution to a problem*]

- **Verify** your solution. [1 Mark]

[Verify: *Establish, by substitution for a particular case or by geometric comparison, the truth of a statement*]

WRITTEN RESPONSE 2

Use the diagram below to answer the next question.



- **Determine** the length of BC, accurate to one decimal. [1 Mark]

[Determine: *Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations*]

- In general, suppose the side opposite ∠ A is labelled H. **Interpret** the meaning of the following if:
  - H < BC
  - H > BC, but less than 20 cm. [1 Mark]

[Interpret: *Provide a meaning of something; present information in a new form that adds meaning to the original data*]

- Suppose H = 16 cm. Tom **concluded** that the largest possible measure of ∠ B is 114^0. Is he correct? **Justify.** [2 Marks]

[Conclude: *Make a logical statement based on reasoning and/or evidence*]

[Justify*: Indicate why a conclusion has been stated, by providing supporting reasons and/or evidence that form a mathematical argument*]

Written Response Possible Solutions

- Write your responses as neatly as possible.
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WRITTEN RESPONSE 1

Consider the trigonometric equation, $cos\theta = \frac{\sqrt{2}}{2}$, where $0^0 \leq \theta < 360^0$.

- **Determine** the reference angle? [1 Mark]

[Determine: *Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations*]

Solution

Determine the reference angle using the calculator. Use second function cosine and when given a ratio, we can find the reference angle.

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^0$.

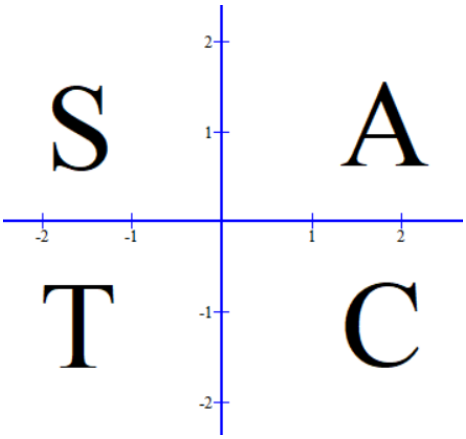
The reference angle is 45^0.

- In which quadrant or quadrants lies the terminal arm? **Explain.** [1 Mark]

[*Explain: Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail*]

Solution

The CAST rule helps us to remember that the cosine ratio is positive in quadrants 1 and 4.

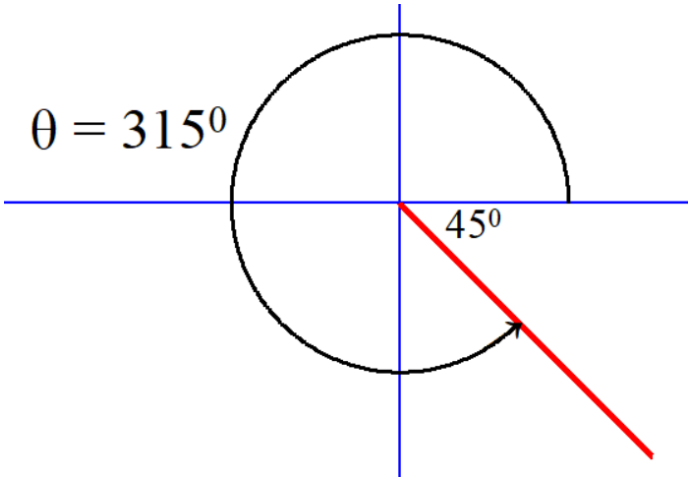


The letters indicate which quadrants have positive ratios. Cosine is positive in quadrants 1 and 4 because the adjacent side in the triangles we will construct are made up of positive x-values on the x-axis. With the hypotenuse of these triangles always positive, any adjacent/hypotenuse (cosine) ratios must be positive.

- **Sketch** the largest possible value of θ in standard position. **Explain.** [1 Mark]

[Sketch: *Provide a drawing that represents the key features or characteristics of an object or graph*]

Solution



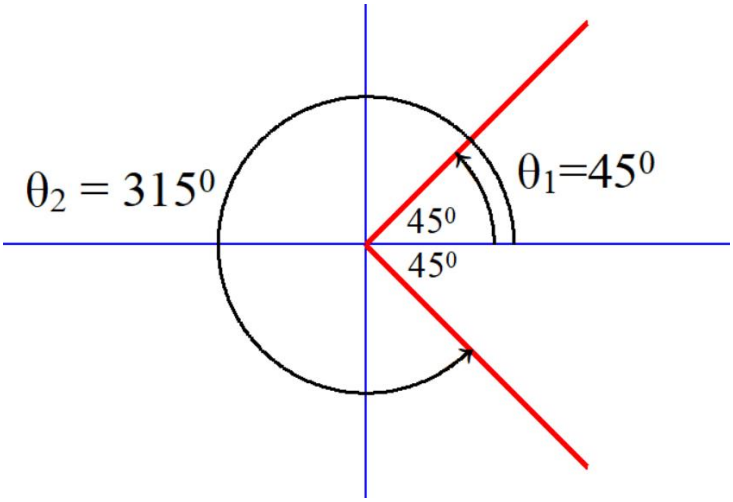
The largest value of θ is in quadrant 4 which is 315^0. This number is found by taking a complete revolution of 360^0 and subtracting the reference angle, or 45^0, from it.

- **Solve** the equation. [1 Mark]

[Solve: *Give a solution to a problem*]

Solution

As shown above, after determining the reference angle of 45^0 with the calculator and using the CAST rule to state the quadrants having positive cosine ratios, a diagram helps to complete the solution.



Within the given domain of $0^0 \leq \theta < 360^0$, the solutions are 45^0 and 315^0.

- **Verify** your solution. [1 Mark]

[Verify: *Establish, by substitution for a particular case or by geometric comparison, the truth of a statement*]

Solution

Substitute each solution into the equation for θ and show that the left side of the equal sign is equal to the right side. Use the calculator to show the decimal equivalents.

$$\cos 45 = \frac{\sqrt{2}}{2}$$

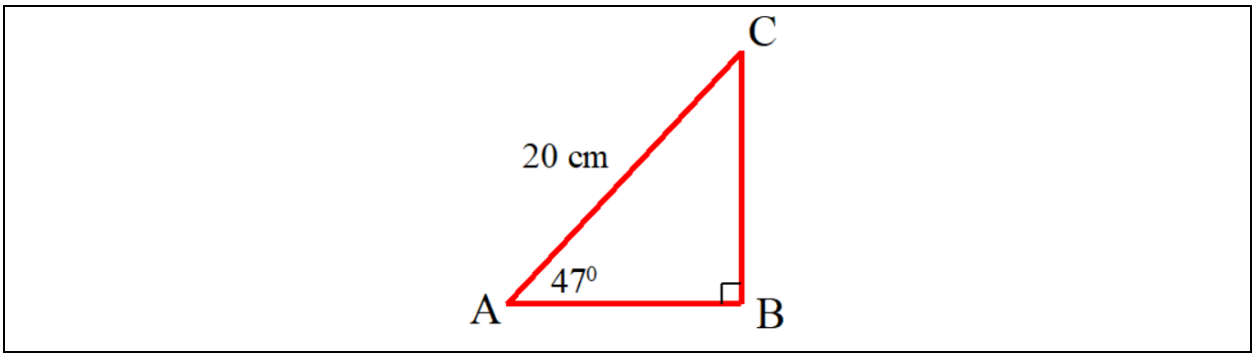
$$0.7071\ldots = 0.7071\ldots$$

$$\cos 315 = \frac{\sqrt{2}}{2}$$

$$0.7071\ldots = 0.7071\ldots$$

WRITTEN RESPONSE 2

Use the diagram below to answer the next question.



- **Determine** the length of BC, accurate to one decimal. [1 Mark]

[Determine: *Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations*]

Solution

The side opposite the given angle is BC. Since we know the hypotenuse and we are dealing with a right-angled triangle, set up the sine ratio:

$$sin47 = \frac{BC}{20}$$

BC = (sin47)(20)

BC = 14.627…

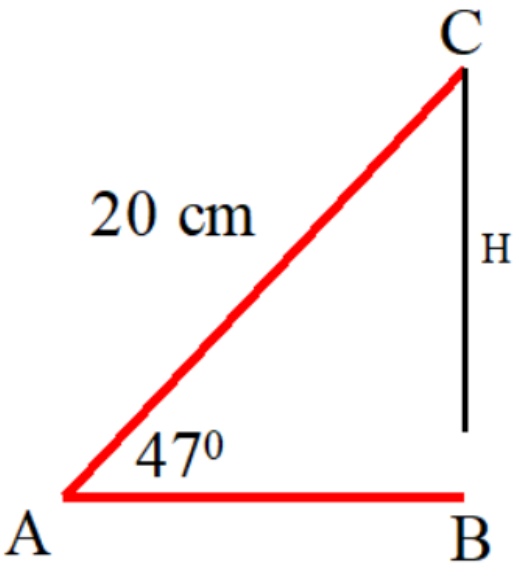
Accurate to one decimal, the length of BC is 14.6 cm.

- In general, suppose the side opposite ∠ A is labelled H. **Interpret** the meaning of the following if:
  - H < BC
  - H > BC, but less than 20 cm. [1 Mark]

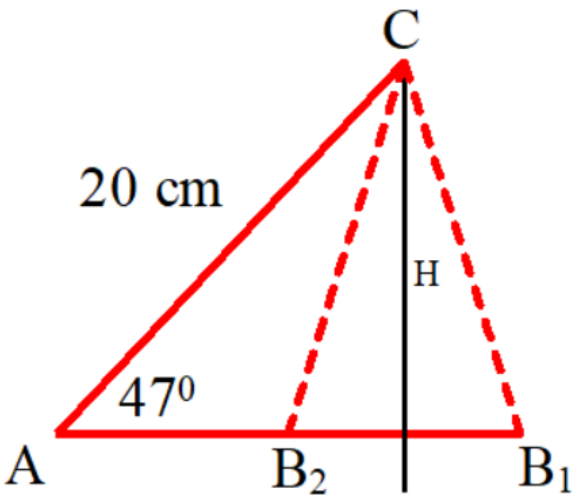
[Interpret: *Provide a meaning of something; present information in a new form that adds meaning to the original data*]

Solution

If H < BC, the creation of a triangle with the given 47^0 angle and side length of 20 cm is not possible. The side opposite the 47^0 angle (H) is not long enough to connect to side AB to form a triangle.



If H > BC, and less than the given side of 20 cm, it is now possible to draw 2 distinct triangles. With this information, an example of the ambiguous case of the sine law is presented.

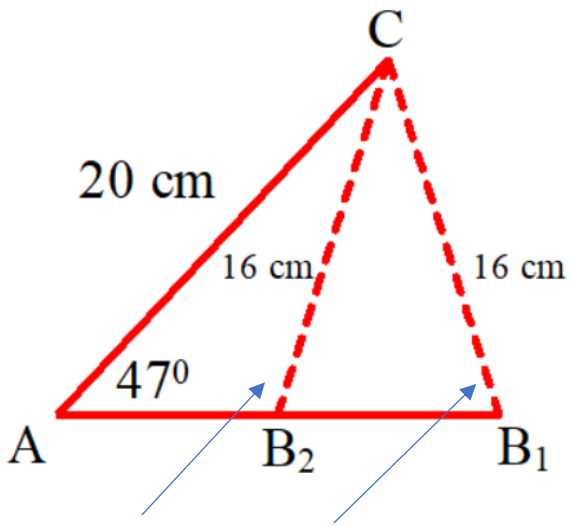


If H stays connected to point C at the top and then swings left or right, it will intersect with line AB, creating two triangles, Δ AB_2C and Δ AB_1C.

- Suppose H = 16 cm. Tom **concluded** that the largest possible measure of ∠ B is 114^0. Is he correct? **Justify.** [2 Marks]

[Conclude: *Make a logical statement based on reasoning and/or evidence*]

[Justify*: Indicate why a conclusion has been stated, by providing supporting reasons and/or evidence that form a mathematical argument*]



Tom calculated two possible angles for B.

$$\frac{\sin 47}{16} = \frac{\sin B_1}{20}$$

$$sinB_1 = \frac{(\sin 47)(20)}{16}$$

$$sinB_1 = 0.9141\ldots$$

$\sin^{-1}(0.9141\ldots) = 66.091\ldots$

To the nearest degree, $B_1 = 66^0$.

Tom knows that B_1 and B_2 are supplementary.

Therefore, $B_2 = 180^0 - 66^0$

$B_2 = 114^0$

Tom is correct that the largest possible angle for B is 114^0.