

Math 20-1 Sequence and Series Written Response

- Write your responses as neatly as possible.
- For full marks, your responses must address **all** aspects of the question.
- All responses, including descriptions and/or explanations of concepts must include pertinent ideas, calculations, formulas and correct units.
- Your responses must be presented in a well-organized manner. For example, you may organize your responses in point form or paragraphs.

WRITTEN RESPONSE 1

Tom was asked to find the general term for the geometric sequence, $8r^3, 16r^4, 32r^5, \dots$

His work is shown below.

Step 1	$\frac{16r^4}{8r^3} = 2r$
Step 2	$t_n = 8r^3(2r)^{n-1}$
Step 3	$t_n = (2r)^3(2r)^{n-1}$
Step 4	$t_n = (2r)^{3n-3}$

- **Analyze** his work and **describe** the error made by Tom. [2 Marks]

Analyze – “Make a mathematical examination of parts to determine the nature, proportion, function, interrelationships, and characteristics of the whole”.

Describe – “Give a written account of a concept”.

- Make the correction. **Explain.** [1 Mark]

Explain – “*Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail*”.

- **Determine** the coefficient of t_8 . [1 Mark]

Determine – “*Find a solution, to a specified degree of accuracy to a problem by showing appropriate formulas, procedures and/or calculations*”.

WRITTEN RESPONSE 2

Domestic bees make their honeycomb by starting with a single hexagonal cell, then forming ring after ring of hexagonal cells around the initial cell. The number of cells in successive rings form an arithmetic sequence.

- Write a rule for the number of cells in the n th ring. **Justify.** [2 Marks]

Justify – “*Indicate why a conclusion has been stated, by providing supporting reasons and/or evidence that form a mathematical argument*”.

- **Algebraically determine** the total number of cells in the honeycomb after the 11th ring has formed. (do not forget to count the initial cell) [1 Mark]

Algebraically – “Using mathematical procedures that involve variables or symbols to represent values”.

- If the total number of cells is 816, **determine** the number of rings that have been formed. **Sketch** a graph and **explain** how it pertains to the answer. [2 Marks]

Sketch: “Provide a drawing that represents the key features or characteristics of an object or graph”.

Written Response Possible Solutions

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- **Analyze** his work and **describe** the error made by Tom. [2 Marks]

Focus on the directing words and recall their meaning.

Analyze – “Make a mathematical examination of parts to determine the nature, proportion, function, interrelationships, and characteristics of the whole”.

Describe – “Give a written account of a concept”.

Solution

In step 1, Tom began by determining the common ratio, r . He knew that the common ratio can be determined by dividing any term, other than the first term, by the previous term. He decided to use $r = \frac{t_2}{t_1} = \frac{16r^4}{8r^3} = 2r$. This is correct.

In step 2, he utilized the general form of a geometric sequence, $t_n = (t_1)(r)^{n-1}$, by substituting the expressions for t_1 and r . This step is also correct.

In step 3, he rewrote $8r^3$ in an equivalent form of $(2r)^3$. He did this because he can now combine 2 powers into a single power. This step is correct.

His error occurred in step 4.

- Make the correction. **Explain.** [1 Mark]

Focus on the directing word and recall its meaning.

Explain – “Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail”.

Solution

Although Tom knew that two powers can be combined into one, he didn't completely remember how to do this. He did keep the base $(2r)$ the same, but he multiplied the exponents. He should have added the exponents.

Step 4 should be:

$$t_n = (2r)^{n+2}$$

- **Determine** the coefficient of t_8 . [1 Mark]

Focus on the directing word and recall its meaning.

Determine – “Find a solution, to a specified degree of accuracy to a problem by showing appropriate formulas, procedures and/or calculations”.

Solution

We are asked to find a specific term, the eighth term (t_8); and then state its coefficient.

Use the general form of a geometric sequence. $t_n = (t_1) (r)^{n-1}$

We know that $t_1 = 8r^3$ and that $r = 2r$.

$$t_8 = (8r^3)(2r)^{n-1}$$

From bullet two above, we will use the simplified version of the general term.

$$t_n = (2r)^{n+2}$$

$$t_8 = (2r)^{8+2}$$

$$t_8 = (2r)^{10}$$

$$t_8 = 2^{10}r^{10}$$

$$t_8 = 1024r^{10}$$

The coefficient of t_8 is 1024.

WRITTEN RESPONSE 2

Domestic bees make their honeycomb by starting with a single hexagonal cell, then forming ring after ring of hexagonal cells around the initial cell. The number of cells in successive rings from an arithmetic sequence.

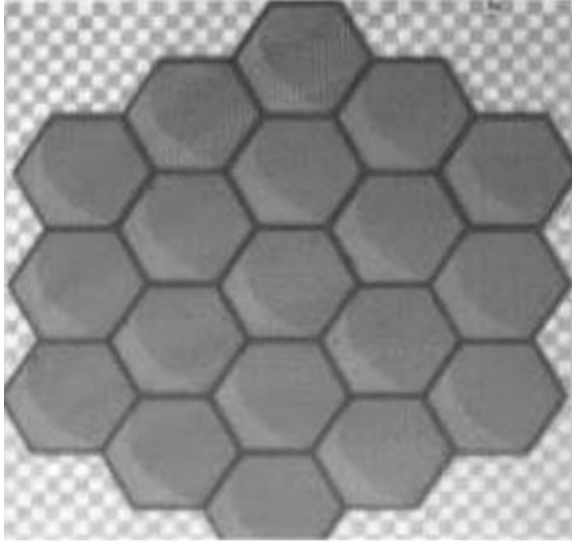
- Write a rule for the number of cells in the n th ring. **Justify.** [2 Marks]

Focus on the directing word and recall its meaning.

Justify – “Indicate why a conclusion has been stated, by providing supporting reasons and/or evidence that form a mathematical argument”.

Solution

A hexagon has 6 sides. When a new ring is added, there are 6 more cells compared to the previous ring. The common difference, d , is 6. The first term, t_1 , is 6.



Ring 1 has 6 cells.

Ring 2 has 12 cells.

Ring 3 has 18 cells.

The sequence is: 6, 12, 18, ...

The general term for an arithmetic sequence is $t_n = t_1 + (n - 1)d$, where t_1 is the first term, n is the number of terms, and d is the common difference. Substitute the known values into the general term.

$$t_n = 6 + (n - 1)6$$

$$t_n = 6 + 6n - 6$$

$$t_n = 6n$$

NOTE – The term number corresponds to the ring number.

- **Algebraically determine** the total number of cells in the honeycomb after the 11th ring has formed. (do not forget to count the initial cell) [1 Mark]

Focus on the directing word and recall its meaning.

Algebraically – “Using mathematical procedures that involve variables or symbols to represent values”.

Solution

When determining the total number of cells in the honeycomb, we need to use a sum formula.

However, we first need to find the number of cells in the 11th ring.

Using the general term for the arithmetic sequence, $t_n = 6n$,

$$t_{11} = 6(11)$$

$$t_{11} = 66$$

Since we know t_1 (6), the last term t_n (66), and the number of terms, n , (11), use the formula

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_{11} = \frac{11}{2}(6 + 66)$$

$$S_{11} = 396$$

Remember to include the first cell.

The total number of cells after the 11th ring is 397.

- If the total number of cells is 816, **determine** the number of rings that have been formed. **Sketch** a graph and **explain** how it pertains to the answer. [2 Marks]

Focus on the directing words and recall their meaning.

Sketch: "Provide a drawing that represents the key features or characteristics of an object or graph".

Solution

We can't use the same formula from the bullet above,

$$S_n = \frac{n}{2}(t_1 + t_n)$$

because we do not know the last term, t_n

Instead, we will use the formula, $S_n = \frac{n}{2} [2t_1 + (n - 1)d]$

$$S_n = 816$$

$$t_1 = 6$$

$$d = 6$$

$$816 = \frac{n}{2} [2(6) + (n - 1)6]$$

Multiply both sides of the equation to clear the fraction.

$$1632 = n(12 + 6n - 6)$$

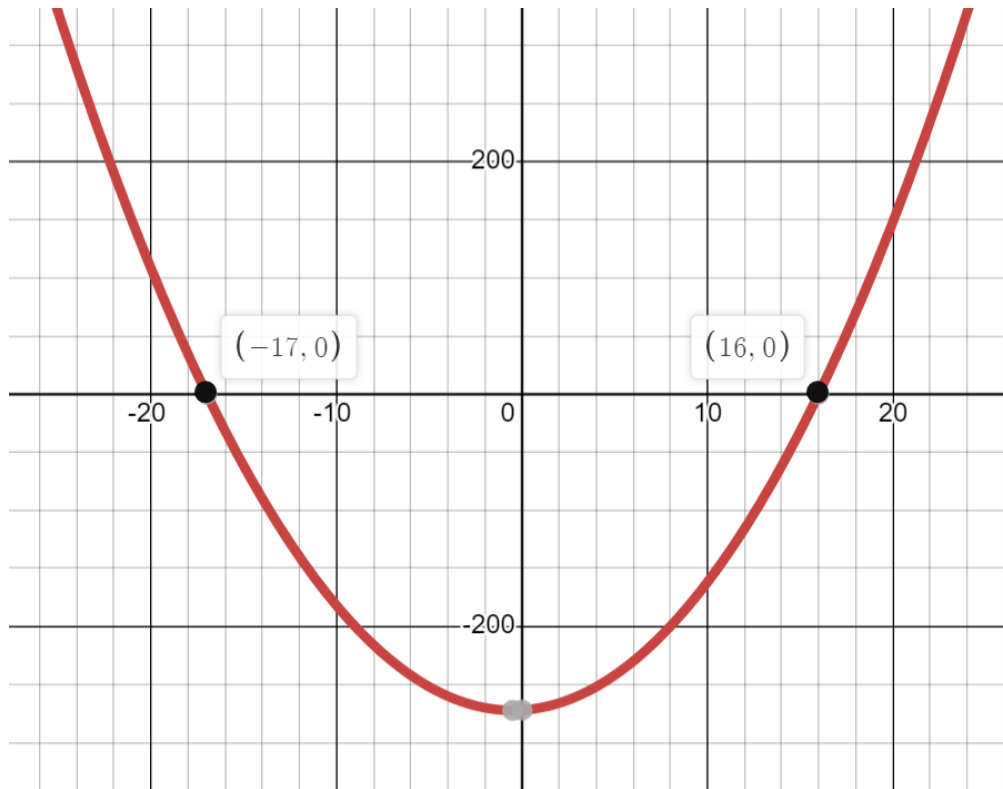
$$1632 = n(6n + 6)$$

$$1632 = 6n^2 + 6n$$

$$0 = 6n^2 + 6n - 1632$$

$$0 = 6(n^2 + n - 272)$$

Graph the equation $y = x^2 + x - 272$.



The solutions to this quadratic equation are determined by the x-intercepts. There are two x-intercepts, -17 and 16. Since the negative value doesn't make sense in this context, we will reject it. Our answer is 16.

When the total number of cells is 816, the number of rings that have been formed is 16.