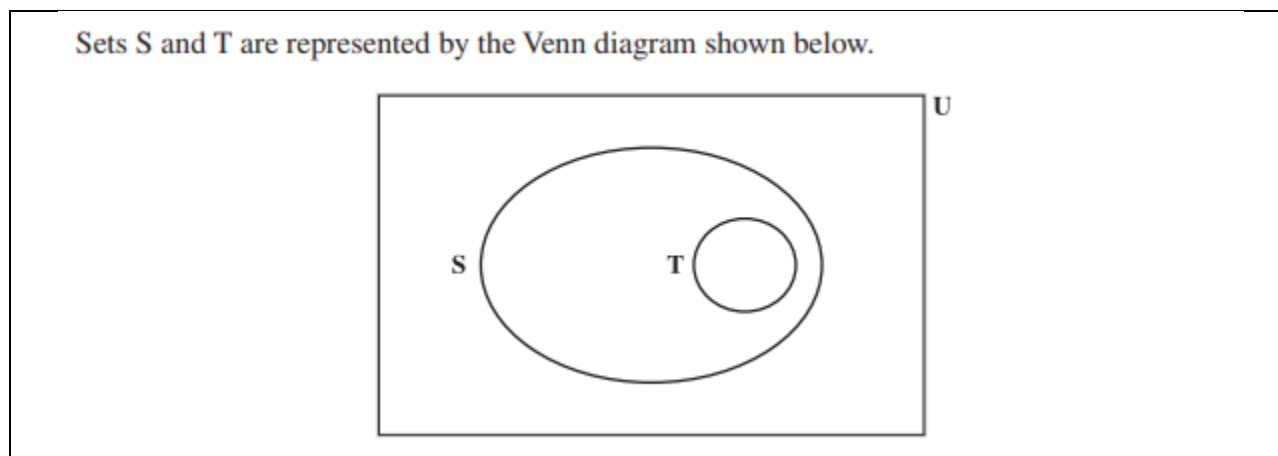


Math 30-2 Diploma Questions Below 50% (2016)

1.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Logical Reasoning	2	Conceptual	Acceptable	45.9%

Use the following information to answer the next question.



Which of the following statements is true for sets S and T?

- A) $S \subset T$
- B) $T \subset S'$
- C) $S \cap T = \emptyset$
- D) $S \cup T = S$

Solution

If $S \subset T$ is true, **all** of the elements of S must also be in T. From the diagram, we can see that some elements are, but not **all**.

Statement A is not true.

If $T \subset S'$ is true, then **all** the elements of T, must also be in **not S**. **Not S** would be everything in the rectangle (the universal set U) not in set S. From the diagram, we can see that this is false.

Statement B is not true.

If $S \cap T = \emptyset$ is true, then the intersection of sets S and T would be the empty set, or no common elements. But since one circle is imbedded within the other, there must be common elements.

Statement C is not true.

Given $S \cup T = S$, this means that the union of sets S and T is equal to set S. In other words, if all the elements of Set S and T are combined, this is the equivalent of all the elements of set S.

Statement D is true.

The correct answer is D.

Possible Reasons For The Low Percentage of Correct Answers


- With several symbols mentioned in this question, if a student does not understand the meaning of all of them, there is a strong likelihood of an incorrect answer. This question is testing conceptual understanding of set theory symbols as related to a Venn Diagram.

2.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Probability	5	Conceptual	Acceptable	31.3%

Use the following information to answer the next question.

The following symbols are selected and arranged to create various code “words.” No symbol can be repeated in a code word.



SYMBOLS

Which of the following calculations could be used to determine the number of possible code words that can be created when **at least** 8 symbols are selected and arranged?

- A) ${}_{10}P_8 + {}_{10}P_9 + {}_{10}P_{10}$
- B) ${}_{10}P_8 \cdot {}_{10}P_9 \cdot {}_{10}P_{10}$
- C) ${}_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10}$
- D) ${}_{10}C_8 \cdot {}_{10}C_9 \cdot {}_{10}C_{10}$

Solution

In this context, **at least 8** means 8 or 9 or 10 symbols. There are 3 separate cases to consider and the word ‘or’ means add.

Is this a combination or a permutation question? The key word used in the question is ‘arranged’. To arrange implies order, and order means we are dealing with a permutation.

The only possible answer giving a sum of permutation expressions is A.

The correct answer is A.

Possible Reasons For The Low Percentage of Correct Answers

- Given that all the possible answers are similar, an incorrect answer is likely related to one of the two different characteristics.
- First, is this a permutation or a combination question? Although groups of symbols are selected (which might lead one to choose combination), we are also told that there is an arrangement. The word arrange implies order and thus means that it is a permutation question.
- Second, is the operator between terms add or multiply? As soon as there are multiple cases, as indicated by the phrase “at least 8”, we know that the answer involves case 1 or case 2 or case 3, etc. The word “or” means add.
- Errors could be attributed to a lack of understanding of the phrase “at least”, being unable to distinguish between a combination or a permutation, or not knowing how to manage individual cases.

3.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Probability	6	Conceptual	Excellence	45.9%

Use the following information to answer the next question.

Stéphane is going to randomly select 10 songs from the list shown below to create a playlist.

Category	Number of Songs
Rock 'n' Roll	6
Classical	5
Jazz	8
Country	10

Which of the following calculations could be used to determine the number of 10-song playlists that contain 2 or 3 country songs?

- A) ${}_{10}C_2 \cdot {}_{10}C_3$
- B) ${}_{10}C_2 + {}_{10}C_3$
- C) ${}_{10}C_2 \cdot {}_{19}C_8 \cdot {}_{10}C_3 \cdot {}_{19}C_7$
- D) ${}_{10}C_2 \cdot {}_{19}C_8 + {}_{10}C_3 \cdot {}_{19}C_7$

Solution

Recall that “or” means add; “and” means multiply.

$$\begin{array}{lcl} \text{Two Songs} & \text{or} & \text{Three Songs} \\ 2 \text{ Country Songs and } 8 \text{ Others} & + & 3 \text{ Country Songs and } 7 \text{ Others} \\ 2 \text{ From } 10 \text{ Country and } 8 \text{ From } 19 & + & 3 \text{ From } 10 \text{ Country and } 7 \text{ From } 19 \\ {}_{10}C_2 \cdot {}_{19}C_8 & + & {}_{10}C_3 \cdot {}_{19}C_7 \end{array}$$

The correct answer is D.

Possible Reasons For The Low Percentage of Correct Answers

- The first two possible options, A and B, do not consider that the combinations of 2 or 3 country songs, have to be combined with the remaining number of songs that are not country (19) to form a 10 song playlist.
- The difference between options C and D is that one has a multiplication sign between the 2nd and 3rd terms, and the other has an addition sign. If a student incorrectly picked option C (the multiplication sign), he/she probably didn't understand that there are two separate cases that need to be added.

4.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Probability	3	Problem Solving	Acceptable	27.3%

Use the following information to answer the next question.

A box contains 6 red balls, 2 green balls and 5 blue balls. Renee randomly selects 2 balls, one after the other, with replacement.

NUMERICAL RESPONSE

To the nearest hundredth, the probability that both balls that Renee selects from the box are red is _____.

Solution

Since the first ball is replaced following the first selection, these events are independent. The outcome of the second selection is not affected in any way by the outcome of the first selection.

The formula for independent probability is:

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(\text{Red and Red}) = \left(\frac{6}{13}\right) \left(\frac{6}{13}\right)$$

$$P(\text{Red and Red}) = \left(\frac{36}{169}\right)$$

$$P(\text{Red and Red}) = 0.213017\dots$$

To the nearest hundredth, the probability that both balls that Renee selects from the box are red is 0.21.

Possible Reasons For The Low Percentage of Correct Answers

- A student might have exchanged the meaning of replacement with non-replacement and thought $P(\text{Red and Red}) = \left(\frac{6}{13}\right) \left(\frac{5}{12}\right)$.
- A student may have considered just the red balls (and not green and blue) and thought $P(\text{Red and Red}) = \left(\frac{2}{6}\right) \left(\frac{2}{6}\right)$.
- Or, confused the above scenario with replacement and thought $P(\text{Red and Red}) = \left(\frac{2}{6}\right) \left(\frac{1}{5}\right)$.

5.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Relations and Functions	6	Conceptual	Acceptable	45%

Use the following information to answer the next question.

Amin is determining how many ancestors he had from the past 10 generations. He notes that 1 generation ago, he had 2 ancestors, 2 generations ago he had 4 ancestors, 3 generations ago he had 8 ancestors, and so on.

These data could **most appropriately** be modelled using

- A) Linear regression
- B) Quadratic regression
- C) Sinusoidal regression
- D) Exponential regression

Solution

Extending the pattern:

Generations Ago	Ancestors
1	2
2	4
3	8
4	16
5	32
6	64
7	128

We can see that for every extra generation, the number of ancestors doubles. The growth of this graph rises quickly in an exponential fashion. If points were plotted on a graph, the shape would be distinctly exponential.

From a graphical shape perspective, the linear (straight line), quadratic (parabola) and sinusoidal (periodic wave) options would be dismissed as incorrect.

If these points were put into the lists in the calculator and the various types of regressions were accessed, the exponential graph would be a precise $y = 2^x$. This

would indicate a high correlation, especially since the others are either repeating decimals, or there is an error indication for the sinusoidal regression.

The correct answer is D.

Possible Reasons For The Low Percentage of Correct Answers

- Not thinking about generating more ordered pairs to get a sense of the shape of the graph.
- Not have a solid grasp of the shapes of all the potential options given here.
- Not thinking about finding a regression equation using the calculator.
- Using the regression function, but not being able to interpret how the equation could lead to the correct answer.

6.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Relations and Functions	6	Procedural	Acceptable	48.8%

Use the following information to answer the next question.

Once growing conditions are ideal, the population of a certain bacteria doubles every 20 minutes. Starting from a single bacterium, the number of bacteria, E , present after m minutes can be modelled by the formula

$$E = 2^{\frac{m}{20}}.$$

NUMERICAL RESPONSE

To the nearest minute, the time it will take for there to be **at least** 6800 bacteria is _____ min.

Solution

1. Algebraically using logarithms.

$$6800 = 2^{\frac{m}{20}}$$

$$\log 6800 = \log 2^{\frac{m}{20}}$$

$$\log 6800 = \left(\frac{m}{20}\right) \log 2$$

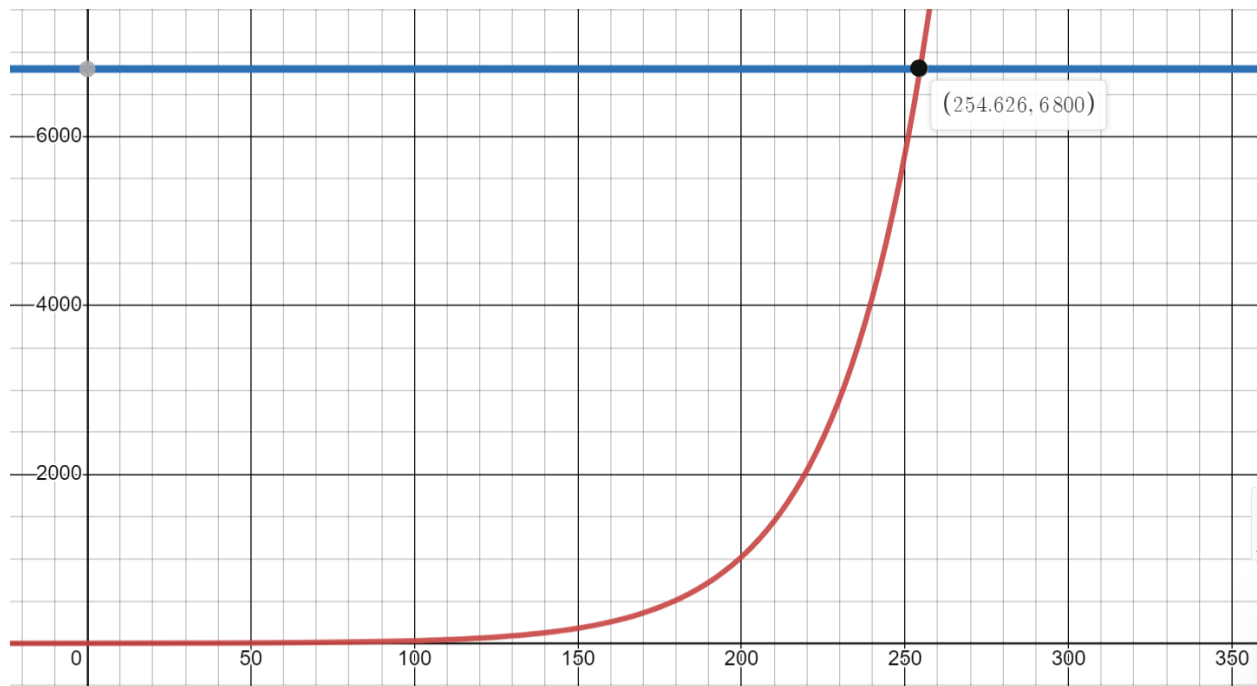
$$\frac{\log 6800}{\log 2} = \left(\frac{m}{20}\right)$$

$$\frac{(20)(\log 6800)}{\log 2} = m$$

$$255 \text{ min} = m$$

2. Graphically

Graph $y_1 = 6800$ and $y_2 = 2^{m/20}$



The x-coordinate of the intersection point is 254.626....

To the nearest minute, the time it will take for there to be at least 6800 bacteria is 255 minutes.

Possible Reasons For The Low Percentage of Correct Answers

- If the mindset of the student was to graph to determine the solution, they could have had problems with the window adjustment. Or, they might not have realized that the solution was the x-coordinate of the intersection point.
- If the mindset was to solve algebraically, they may have only thought about equating the bases, which cannot be done in this case (6800 cannot be expressed as a base of 2). Or, if using logs, a breakdown could have occurred at any of the multiple stages of the process.
- The notion of a fractional exponent may have caused an uncertainty about how to proceed.

7.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Relations and Functions	4	Conceptual	Acceptable	30.4%

Given $\log_{bc} = a$, $b \neq 1$, which of the following statements must be true?

- A) $b > 0$ and $a > 0$
- B) $b > 0$ and $c > 0$
- C) $b > c$ and $a > 0$
- D) $a > b$ and $c > 0$

Solution

If we can show any parts of these statements to be false, then we eliminate them as potential correct answers.

By definition of a logarithmic function, b must be greater than 0.

Statement A

The first part is true. But a doesn't have to be greater than zero. For example, it is possible to have the correct logarithmic equation, $\log_2\left(\frac{1}{2}\right) = -1$. Therefore, c doesn't have to be greater than zero. Statement A cannot be the correct answer.

Statement C

The first part of statement C is $b > c$. However, given the logarithmic equation, $\log_2 8 = 2$, b is not greater than c (2 is not greater than 8). Statement C cannot be correct.

Statement D

The first part of statement D is $a > b$. However, given the logarithmic equation, $\log_2 2 = 1$, a is not greater than b (1 is not greater than 2). Statement D cannot be correct.

Statement B

For the equation, $\log_b c = a$, given any value of b which must be positive by definition, raised to any exponent a , whether it be negative, positive will always result in a positive value for c .

For example, $\log_4 c = -\frac{1}{2}$, means $c = 4^{-0.5}$, which is equal to $\frac{1}{2}$.

Or, for example, $\log_5 c = 2$, means $c = 5^2$, or 25.

Thus, the value of c must be greater than zero.

The two statements that must be true are $b > 0$ and $c > 0$.

The correct answer is B.

Possible Reasons For The Low Percentage of Correct Answers

- The explanations given above are reasons why $b > 0$ and $c > 0$, either by definition or by logical reasoning. A student could easily get bogged down with either understanding what the letters represent, or determining the possible values the letters could be.
- The primary resource for this course states, "A logarithmic function has the form $f(x) = a \log_b x$, where $b > 0$, $b \neq 1$, and $a \neq 0$, and a and b are real numbers. The domain is $x > 0$." The choice of letters for this question is unfortunate since in this

definition from the text, a represents a vertical stretch, but in the question, a represents the log or the exponent. A student who didn't or couldn't reason out the answer and got it incorrect, likely did not recall this information from their notes or text.

8.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Relations and Functions	4	Procedural	Acceptable	35.9%

NUMERICAL RESPONSE

When $\log_6 40 - 3\log_6 2$ is simplified and written in the form $\log_6 a$, the value of a is _____.

Solution

Use the power law to move the coefficient of 3 on the second term to the exponential position.

$$\log_6 40 - \log_6 2^3$$

Use the quotient law to re-write as a single logarithm.

$$\log_6 \left(\frac{40}{8} \right)$$

$$\log_6 5$$

The value of a is 5.

Possible Reasons For The Low Percentage of Correct Answers

- The student did not know how to apply the power law, the quotient law or both of these logarithmic laws.

9.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Relations and Functions	2	Procedural	Acceptable	37.9%

Use the following information to answer the next question.

The simplified sum of $\frac{x+4}{3x} + \frac{1}{2}$, $x \neq 0$, can be written in the form

$$\frac{\boxed{A}x + \boxed{B}}{\boxed{C}x}$$

where A , B , and C represent single-digit numbers.

NUMERICAL RESPONSE

In the simplified sum $\frac{\boxed{A}x + \boxed{B}}{\boxed{C}x}$, the value of

A is ____

B is ____

C is ____

Solution

Express each of the two terms with a common denominator of $(3x)(2)$, or $6x$

$$\frac{(2)(x+4)}{(2)(3x)} + \frac{(3x)(1)}{(3x)(2)}$$

$$\frac{2x+8}{6x} + \frac{3x}{6x}$$

$$\frac{5x+8}{6x}$$

A is 5

B is 8

C is 6

Possible Reasons For The Low Percentage of Correct Answers

- Not knowing that adding rational expressions requires a common denominator.
- Not knowing how to determine the LCD.
- Not knowing how to write equivalent rational expressions.

10.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Relations and Functions	7	Problem Solving	Acceptable	14.9%

Use the following information to answer the next question.

The volume, V , in cubic inches, of a fish tank can be modelled by the function

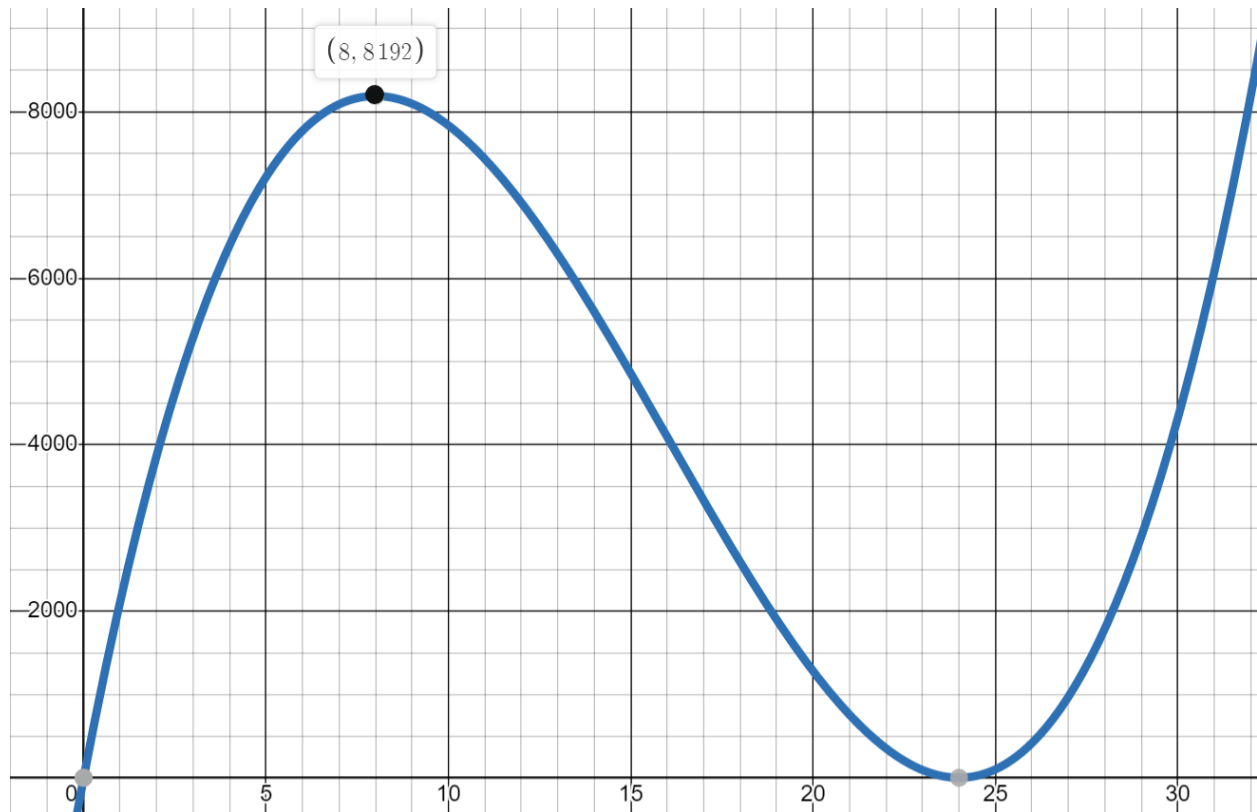
$$V = h(48 - 2h)^2$$

where h is the height of the fish tank in inches and $0 < h < 24$.

NUMERICAL RESPONSE

To the nearest cubic inch, the maximum volume of this fish tank is _____ in³.

Solution



Graph $y = x(48 - 2x)^2$

Create appropriate window settings.

On the x-axis, from 0 to 24, the highest point, or maximum value, read off of the y-axis is 8192.

To the nearest cubic inch, the maximum volume of this fish tank is 8192 in³.

Possible Reasons For The Low Percentage of Correct Answers

- A student didn't realize that graphing this function was the first step.
- A student may have graphed the function, but struggled to get the proper window settings.

- A student may have thought that this cubic function, without a restricted domain, would have no maximum value. Therefore, they may not have understood the importance of the domain.
- They couldn't remember how to access and/or use the maximum function on the graphing calculator.
- They may have found the point (8, 8192) but did not correctly interpret the meaning of this point, i.e. the importance of the y-coordinate.

11.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2016	Relations and Functions	8	Problem Solving	Acceptable	43.7%

Use the following information to answer the next question.

The sunrise time for a particular Alberta city can be modelled by the sinusoidal regression function

$$S = 1.51 \sin(0.0172d + 1.51) + 7.00$$

where S is the sunrise time in hours after midnight, and d is the number of days since the beginning of the year (January 1 = 1, January 2 = 2, etc.).

NUMERICAL RESPONSE

According to the sinusoidal regression function, the sunrise time on August 15 (day 227) to the nearest hundredth of an hour, is _____ h after midnight.

Solution

Make sure the calculator is in radian mode.

Substitute $d = 227$ into the regression function.

$$S = 1.51 \sin(0.0172(227) + 1.51) + 7.00$$

$$S = 1.51 \sin(5.4144) + 7.00$$

$$S = 1.51 (-0.7635...) + 7.00$$

$$S = -1.1529\dots + 7.00$$

$$S = 5.8470\dots$$

According to the sinusoidal regression function, the sunrise time on August 15 (day 227) to the nearest hundredth of an hour, is 5.85 h after midnight.

Possible Reasons For The Low Percentage of Correct Answers

- They didn't realize that for all practical problems involving sinusoidal functions, the mode needs to be radians.
- The thought of substituting $d = 227$ did not cross their mind.
- The order of operations was not followed correctly.
- Incorrect rounding.