1.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Relations and Functions	2	Procedural	Acceptable	**22.5%**

Use the following information to answer the next question.

The simplified product of $\dfrac{2n^4p}{3m} \cdot \dfrac{6m^6}{3n^2p^2}$, $m \neq 0, n \neq 0, p \neq 0,$ can be represented by

$$\dfrac{A\,n^B\,m^C}{3p}$$

where A, B, and C represent single-digit numbers.

NUMERICAL RESPONSE

In the simplified product, $\dfrac{A\,n^B\,m^C}{3p}$, the value of

A is ____ (record in the **first** column)

B is ____ (record in the **second** column)

C is ____ (record in the **third** column)

Solution

Multiply coefficients

$$\dfrac{12n^4m^6p}{9n^2mp^2}$$

Reduce the coefficients to simplest form. Divide the common bases by keeping the base and subtracting the exponents.

$$\frac{4n^2m^5}{3p}$$

A is 4

B is 2

C is 5

Possible Reasons For The Low Percentage of Correct Answers

- A limited number of examples shown in class or practiced for homework. It may have been assumed that this was a prerequisite skill worked on significantly in previous grades.
- Incorrect application of the exponent laws:
  - Multiplying the exponents even though the bases are different.
  - Adding the exponents, even though the bases are different.
  - Uncertainty of simplifying powers when dividing with the same base.
- Confusion on how to deal with coefficients.

2.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Relations and Functions	1	Procedural	Acceptable	**41.5%**

Use the following information to answer the next question.

Ken made an error in the simplification of the rational expression $\dfrac{2x+10}{2x^2-50}$, $x \neq -5, 5$. His simplification of the expression is shown below.

Step 1 $\dfrac{{}^{1}\cancel{2}(x+5)}{{}^{1}\cancel{2}(x^2-25)}$

Step 2 $\dfrac{1(x+5)}{1(x+5)(x-5)}$

Step 3 $\dfrac{1\cancel{(x+5)}}{1\cancel{(x+5)}(x-5)}$

Step 4 $(x-5)$, $x \neq -5, 5$

The step in which Ken made his error is

A) Step 1   B) Step 2   C) Step 3   D) Step 4

<u>Solution</u>

The problem is going from step 3 to step 4. Everything is good up to step 3. The final step should be $\dfrac{1}{x-5}$, not (x − 5).

The correct answer is D.

<u>Possible Reasons For The Low Percentage of Correct Answers</u>

- Thinking that there was a problem in 1 of the first 3 steps.
  - Not understanding that factoring is the first step, or not understanding the factoring process.
  - Not realizing that simplification is correctly applied in step 3.
- When simplifying, if there are no other terms in the numerator, with a denominator term such as (x – 5) in this case, there must be 1 in the numerator. The incorrect answer in step 4 indicates the binomial (x – 5) in the numerator, but in step 3 it is clearly shown in the denominator.

3.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Relations and Functions	3	Procedural	Acceptable	**28.6%**

Use the following information to answer the next question.

Over a distance of 800 km, the average speed of a small airplane is 6 times faster than the average speed of a train. This information is shown in the table below.

	Distance (km)	Speed (km/h)	Time (h)
Train	800	x	$\dfrac{800}{x}$
Airplane	800	$6x$	$\dfrac{800}{6x}$

To travel 800 km, the train requires 8 h more than the airplane. The equation shown below represents this relationship.

$$\frac{800}{x} - \frac{800}{6x} = 8$$

NUMERICAL RESPONSE

The average speed of the train, to the nearest tenth of a kilometer per hour, is _____ km/hr.

Possible Reasons For The Low Percentage of Correct Answers

- A student does not know that clearing the fraction is the first step, and/or does not know how to clear a fraction.
- A student tries to clear the fraction, but does not multiply each term by the LCD.
- A student doesn't know how to determine the LCD.
- A student can't isolate a variable.

[Note: Help with clearing fractions can be found at: https://mathpqjq.com/clearing-fractions/]

4.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Logical Reasoning	1	Problem Solving	Acceptable	**21.6%**

Use the following information to answer the next question.

Sasha has 6 white socks, 4 black socks, and 10 brown socks in the clothes dryer. Without looking, he randomly selects one sock at a time until he gets 2 socks that are the same colour.

The number of socks that Sasha must randomly select from the clothes dryer to guarantee that he gets 2 socks that are the same colour but **not** white is

    A) 9   B) 8   C) 4   D) 3

Solution

The important restriction here is that the match cannot be white; in other words, the match can only be black or brown.

It is possible for Sasha to pull out 8 socks without getting a match; the 8 being 6 white, 1 black and 1 brown. The next selection, i.e. the 9th sock, has to be either a black or a brown, which would then create the required match.

Sasha needs to select 9 socks before a match can be guaranteed.
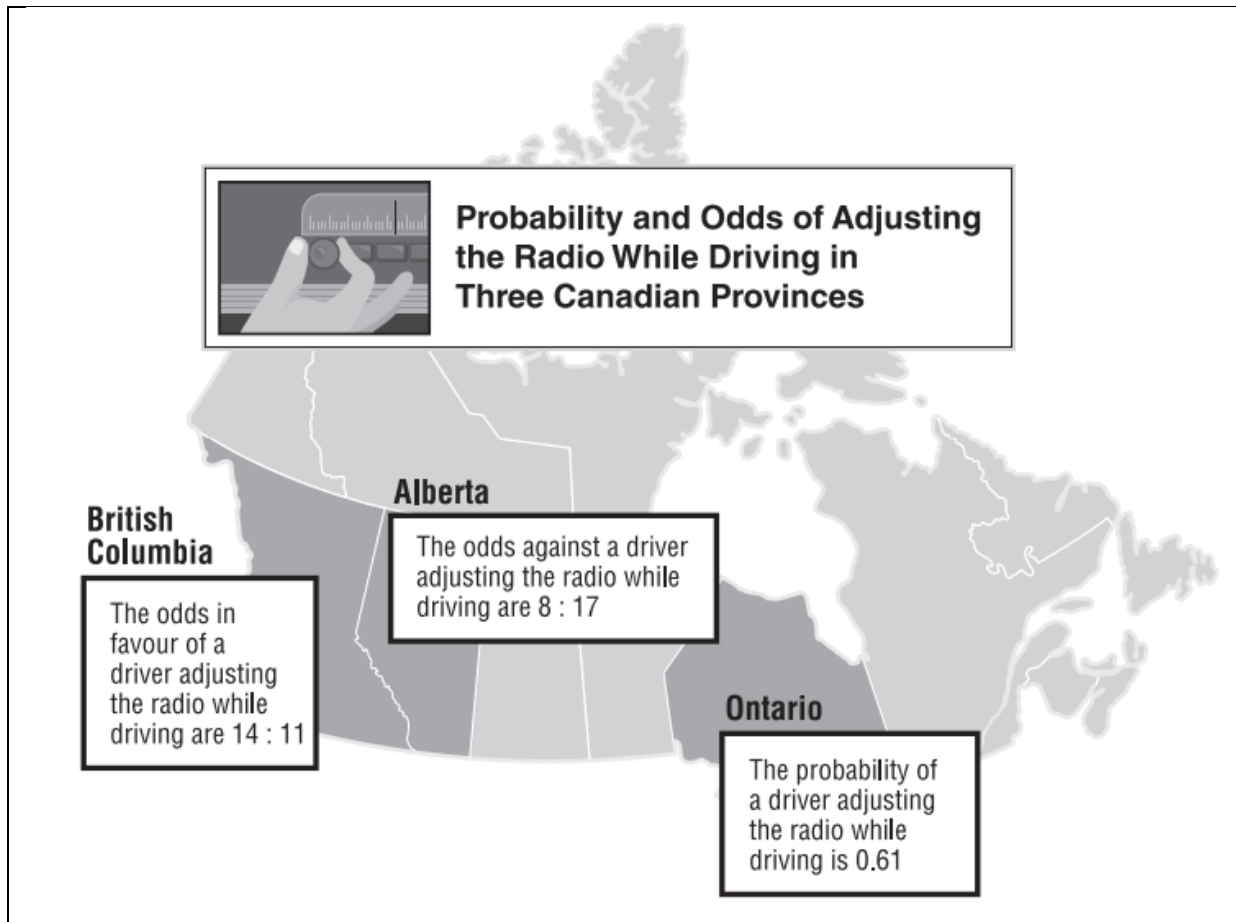
The correct answer is A.

Possible Reasons For The Low Percentage of Correct Answers

- Not being aware of the restriction and thinking that the match can be in any colour.
- Not understanding the meaning of guarantee. The thought might be that in pulling out 3 socks, for example, it could be 1 white and 2 black. This would make a match. But pulling out 3 socks could have so many more options that are **not** a match.

5.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Probability	1	Problem Solving	Acceptable	**37.1%**

Use the following information to answer the next question.

Probability and Odds of Adjusting the Radio While Driving in Three Canadian Provinces

British Columbia
The odds in favour of a driver adjusting the radio while driving are 14 : 11

Alberta
The odds against a driver adjusting the radio while driving are 8 : 17

Ontario
The probability of a driver adjusting the radio while driving is 0.61

If the Provinces listed above are arranged in order from **lowest** probability of a driver adjusting the radio to the **highest** probability of a driver adjusting the radio, the order will be

A) Alberta, British Columbia, Ontario
B) Alberta, Ontario, British Columbia
C) British Columbia, Ontario, Alberta
D) British Columbia, Alberta, Ontario

<u>Solution</u>

Convert all odds to probability equivalents.

British Columbia

Odds in Favor are 14:11

Probability $\frac{14}{25} = 56\%$

Alberta

Odds Against are 8:17

Odds in Favor are 17:8

Probability $\frac{17}{25} = 68\%$

Ontario

The Probability is 61%.

The order of lowest to highest probability is British Columbia, Ontario, Alberta.

The correct answer is C.

<u>Possible Reasons For The Low Percentage of Correct Answers</u>

- An inability to convert odds to its probability equivalent.
- Not reading and interpreting the fact that the two odds given in British Columbia and Alberta are different in that one is "in favour" and the other is "against". For the "against" odds in Alberta, the two given numbers must be reversed before getting the correct probability.
- The answer would be incorrect if stating the Provinces from highest to lowest instead of lowest to highest.

6.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Probability	2	Conceptual	Acceptable	**47.1%**

Use the following information to answer the next question.

Three Experiments and Related Events

I   Experiment I involves rolling a fair die once. Event X is rolling a 2. Event Y is rolling a 6.

II   Experiment II involves randomly selecting one number from 1 to 10 inclusive. Event X is selecting an odd number. Event Y is selecting a number that is prime.

III   Experiment III involves randomly choosing one marble from a bag. Event X is choosing a red marble. Event Y is choosing a green marble.

The mutually exclusive events are described in experiments

A) I and II only　　　B) I and III only　　　C) II and III only　　　D) I, II, and III

Solution

Mutually exclusive events cannot happen at the same time.

Experiment 1

> The events of rolling a 2 and rolling a 6 cannot happen at the same time. These events **are** mutually exclusive.

Experiment II

> The events of selecting an odd number and selecting a prime number can happen at the same time. For example, 3 is both an odd number and a prime number. These events **are not** mutually exclusive.

Experiment III

> The events of choosing one red marble and choosing one green marble cannot happen at the same time. These events **are** mutually exclusive.

Possible Reasons For The Low Percentage of Correct Answers

- The meaning of mutually exclusive events is not understood.
- Not knowing the meaning of a prime number.

7.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Probability	3	Problem Solving	Acceptable	**31.8%**

Use the following information to answer the next question.

An automobile association collected data on the probability of an accident occurring at different time periods over the course of a day. It also collected separate data on the probability of an accident occurring in each month of the year. These data are shown in the two tables below.

Accident Occurrence by Time of Day

24-h Clock Time Period	Probability of An Accident Occurring
23:00 – 02:59	0.074
03:00 – 06:59	0.061
07:00 – 10:59	0.180
11:00 – 14:59	0.241
15:00 – 18:59	0.286
19:00 – 22:59	0.158
Total	**1.000**

Accident Occurrence by Month

Month	Probability of An Accident Occurring
January	0.105
February	0.094
March	0.072
April	0.075
May	0.066
June	0.073
July	0.073
August	0.068
September	0.072
October	0.079
November	0.089
December	0.134
Total	**1.000**

NUMERICAL RESPONSE

Assuming that these are independent events, the probability to the nearest hundredth, that a randomly selected accident will occur in the time period 15:00 – 18:59 in December is _____.

Solution

The probability of two independent events is the product of the probability of each individual event.

Let P(A) = Probability of an accident occurring in the time period 15:00 – 18:59

Let P(B) = Probability of an accident occurring in December.

P(A and B) = P(A) x P(B)

P(A and B) = (0.286) (0.134) [these numbers come from the table]

P(A and B) = 0.038324

Rounded to the nearest hundredth, P(A and B) = 0.04

Assuming that these are independent events, the probability to the nearest hundredth, that a randomly selected accident will occur in the time period 15:00 – 18:59 in December is 0.04.

Possible Reasons For The Low Percentage of Correct Answers

- Difficulty reading the table.
- Not realizing the individual probabilities needed to be multiplied. In other words, not sure how to deal with independent events.

8.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Probability	6	Procedural	Acceptable	**43.2%**

Use the following information to answer the next question.

A local restaurant offers milkshakes in 20 different flavours. The owner has decided to offer customers the choice of blending two different flavours together to create more flavour options.

The total number of different possible two-flavour blended milkshakes is

A) 40   B) 190   C) 380   D) 400

Solution

Blending two flavours together means that there is a combination situation. The order of the blend does not matter and is therefore not a permutation.

In the formula, $_nC_r$, n represents the total number of objects (or flavours in this case) and r represents how many of the total we wish to combine.

The answer is the determination of $_{20}C_2$.

$_{20}C_2 = 190$.

The correct answer is B.

Possible Reasons For The Low Percentage of Correct Answers

- Not realizing that the problem is an application of solving a combination scenario.
- Confusing a combination with a permutation.

9.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Probability	6	Conceptual	Acceptable	**37.3%**

Use the following information to answer the next question.

In the Braille system, each Braille character consists of 6 dot positions. Dots can be raised to represent letters, digits, symbols, or common letter combinations.

The Braille character for the letter "v" is shown below. Dots that are raised are represented by solid dots.

v
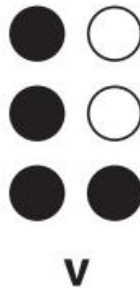
NUMERICAL RESPONSE

The number of different Braille characters that have 4 of the 6 dots raised is _____.

Solution

There are 6 available dots, with each one either raised or not.

We would like 4 of these 6 to be raised. It is a combination scenario where the answer is determined by $_6C_4$.

$_6C_4 = 15$

The number of different Braille characters that have 4 of the 6 dots raised is 15.

Possible Reasons For The Low Percentage of Correct Answers

- Thinking that it is permutation rather than a combination.

10.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Relations and Functions	7	Conceptual	Acceptable	**39.6%**

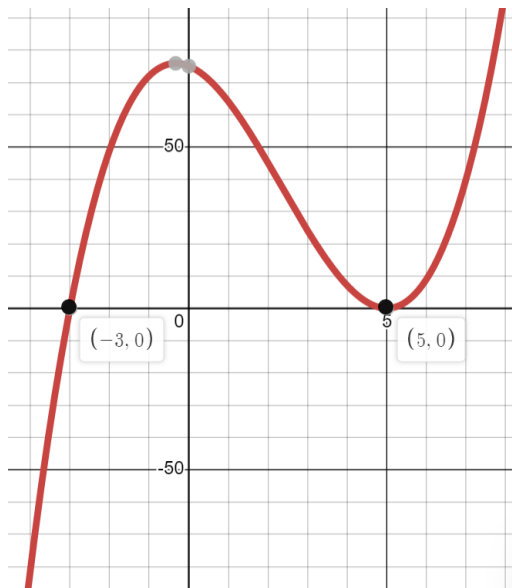Use the following information to answer the next question.

The following statements were made about the polynomial function $y = (x-5)^2(x+3)$.

I   The x-intercepts are -3 and 5.

II   The function has a degree of 3.

III   The domain is $-3 \leq x \leq 5$, $x \in R$.

IV   The range is $y \in R$.

The statements that are true are

A) I and II only
B) III and IV only
C) I, II and IV
D) I, III and IV

Solution

- The x-intercepts are -3 and 5. Statement I is true.
- The function has a degree of 3. Statement II is true.
- The function has a domain of x Є R. Statement III is false.
- The function has a range of y Є R. Statement IV is true.

The correct answer is C.

Possible Reasons For The Low Percentage of Correct Answers

- Difficulty relating x-intercepts to the factored form of a polynomial equation.
- Observing an exponent of 2 on one binomial and thinking that it is a quadratic instead of a cubic function.
- Typically the domain and range for a cubic function are the elements of the real number system. Therefore, there could have been an error in reading the domain and incorrectly thinking the domain was only values between the x-intercepts.

11.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Relations and Functions	7	Problem Solving	Acceptable	**35.4%**

A rain gutter is made from sheets of aluminium that are 28 cm wide. The first step in forming the rain gutter is to turn the edges up to form right angles, as shown in the diagram below.
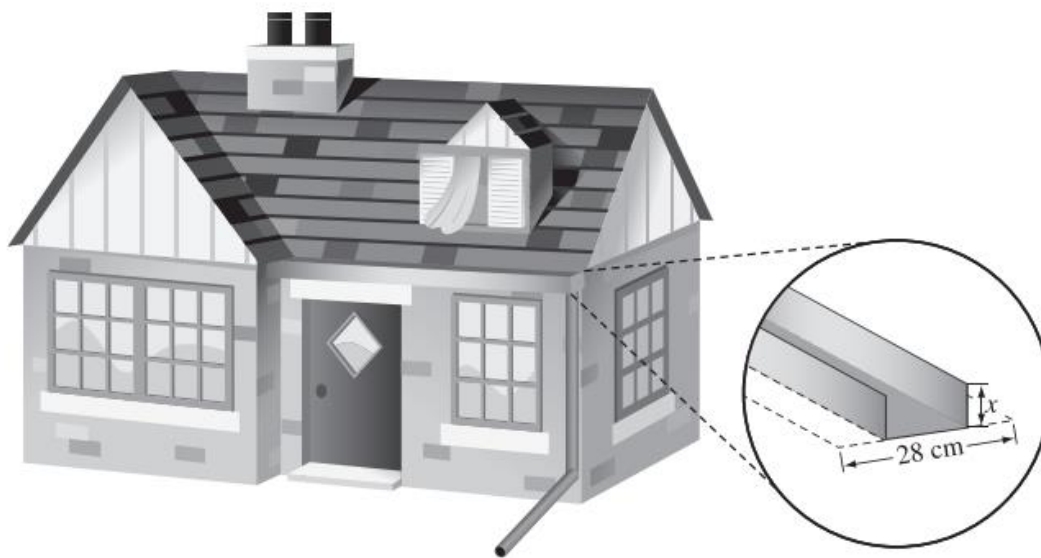
The cross-sectional area formed by the turned-up edges affects the water flow. This cross-sectional area, A, can be modelled by the function
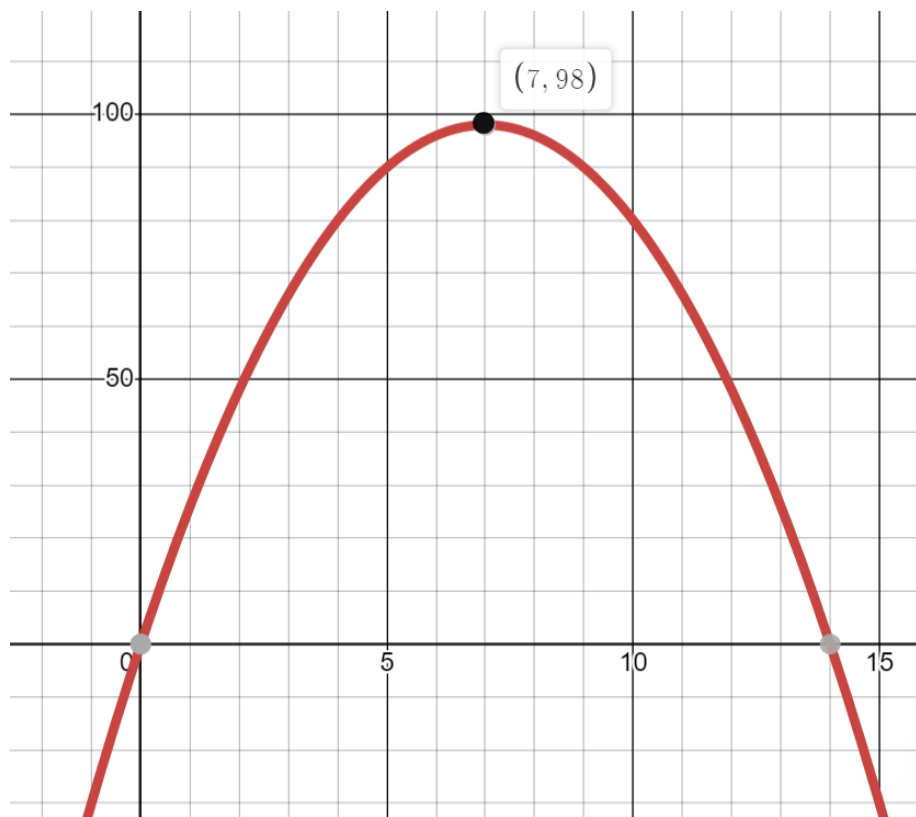
$$A = x\,(28 - 2x)$$

where x is the height of the turned-up edges.

NUMERICAL RESPONSE

To the nearest centimetre, the height of the turned up edge, x, that will maximize the cross-sectional area is _____ cm.

Solution

In the graphing calculator, input x(28 – 2x) into y_1.

Use the maximum function to determine the vertex of (7, 98). This means that the maximum cross-sectional area is 98 cm², for a turned up edge value, x, or 7 cm.

To the nearest centimetre, the height of the turned up edge, x, that will maximize the cross-sectional area is 7 cm.

Possible Reasons For The Low Percentage of Correct Answers

- For any word problem, understanding the meaning of the words can be barriers to determining a solution. It may have been difficult for students to understand

and visualize the cross-sectional area, even though the given diagram was meant to highlight it.

- A student may have thought to solve for x algebraically, and then couldn't remember how to execute these skills.
- The thought of graphing may not have come to mind.
- If they did graph, a lack of understanding of the meaning of the dependent and independent variables (maximum area and height of the turned up edge), may not have lead to determining the vertex.

12.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Relations and Functions	8	Conceptual	Acceptable	**45.8%**

Use the following information to answer the next question.

A family is going to take a ride on the London Eye, which is a Ferris wheel that constantly rotates. Once the family is in a capsule on the ride, the height of the capsule above the ground, y, in metres, can be modelled by the sinusoidal function

$$y = a \cdot \sin(bx + c) + d$$

where x is the time, in seconds, after the family gets in the capsule.

In this context, the y-intercept represents

A) length of time it takes for the family to reach the median height of the Ferris Wheel.
B) length of time it takes for the Ferris Wheel to complete one revolution.
C) height of the family at the highest point above the ground.
D) height of the family the moment they get in the capsule.
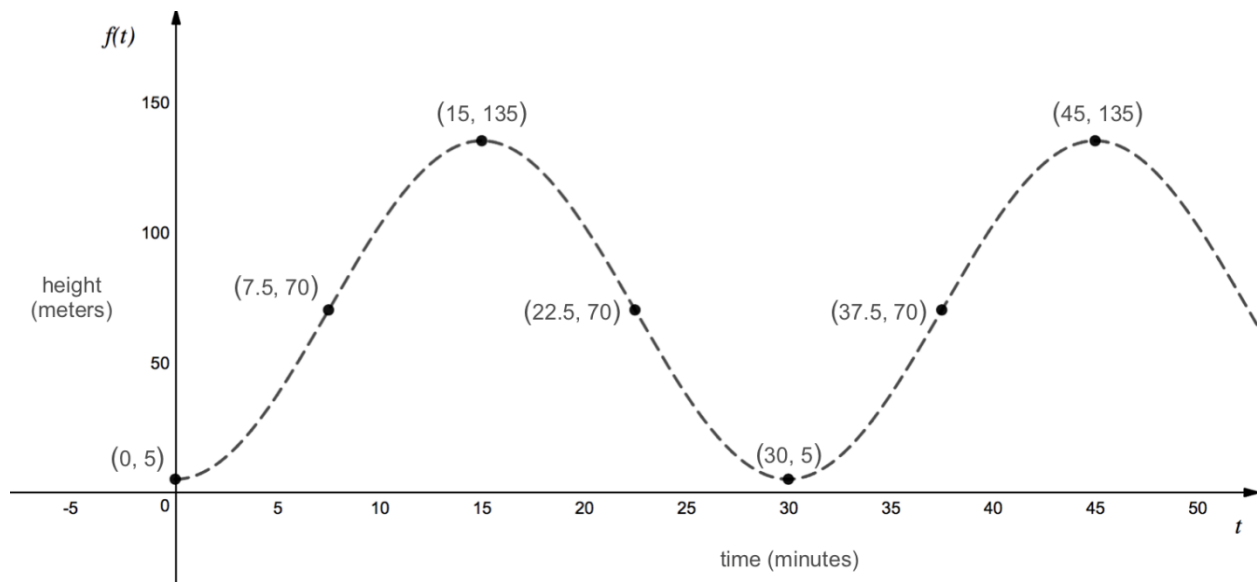
The graph above is an example of a sinusoidal graph that could represent a Ferris Wheel, such as The London Eye. The graph shows the height above the ground as a function of time. In other words, the height above the ground depends on the time elapsed.

For example, the graph above has a y-intercept of (0,5). This means that at a time of 0 seconds, the height above the ground is 5 m. The ride has not yet begun and the height is 5 m.

In this context, the y-intercept represents the height of the capsule the moment they get into the capsule.

The correct answer is D.
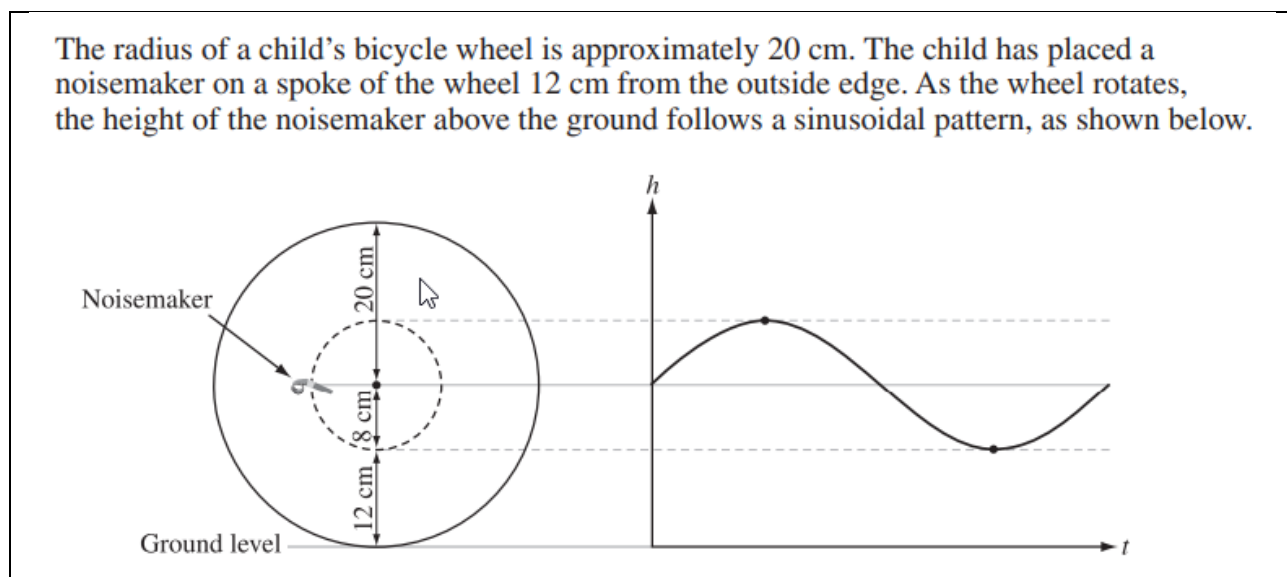
Possible Reasons For The Low Percentage of Correct Answers

- The reason or reasons are fairly complex. In sinusoidal graphs, each of the possible 4 parameters (a, b, c, and d) in y = a sin(bx + c) + d has a specific meaning. Without an over-all understanding of these parameters, it would be easy to select one of the incorrect answers.
- A student would have to recall that for any y-intercept, the value of x, or the independent variable, is 0. Then, there would have to be an understanding of the context; i.e. that the graph illustrates height above the ground as a function of time. When time = 0, the y-coordinate represents the height above the ground

prior to beginning, or stated another way, the moment someone enters the capsule.

13.

Year	Topic	Outcome	Cognitive Level	Standard	Difficulty
2013	Relations and Functions	8	Conceptual	Acceptable	**44.8%**

Use the following information to answer the next question.

The radius of a child's bicycle wheel is approximately 20 cm. The child has placed a noisemaker on a spoke of the wheel 12 cm from the outside edge. As the wheel rotates, the height of the noisemaker above the ground follows a sinusoidal pattern, as shown below.

If ground level is the reference point, the amplitude and median of the sinusoidal function are, respectively,

    A) 20 and 20
    B) 20 and 8
    C) 8 and 20
    D) 8 and 8

The median, or middle of the graph, slices the graph symmetrically horizontally. Vertically, this line is 20 m from the t axis, or the ground.

The amplitude is from the median to the highest point, or the median to the lowest point. Reading again from the original graph, this distance is 8 m.
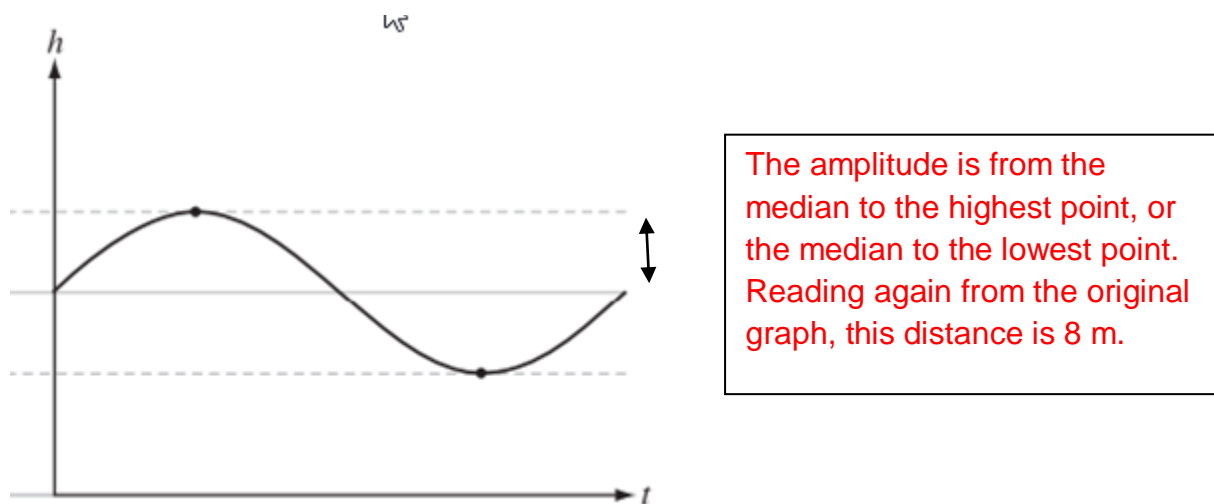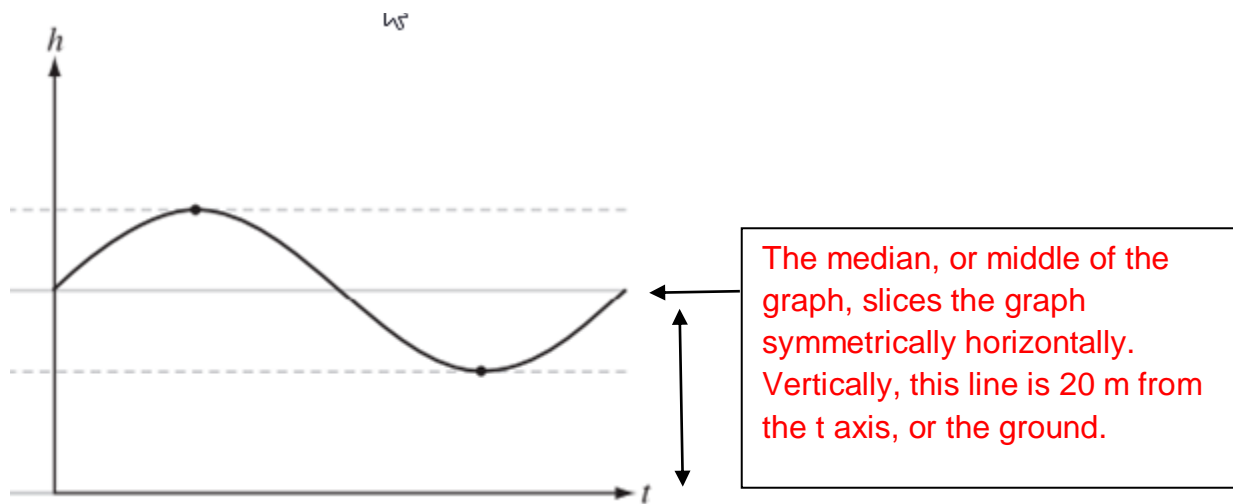
The amplitude is 8 m and the median is 20 m.

The correct answer is C.

Possible Reasons For The Low Percentage of Correct Answers

- Difficulty in reading the graph; seeing the relationship between the circle and the sine graph. Could be especially challenging if this type of picture had never been seen before.
- Not understanding how to find the amplitude or median given a sine graph.
- Not understanding the meaning of the amplitude or median.