The Factor Theorem Solutions

1. The cubic function, y = P(x) has zeros of -2, 1, and 4. If P(0) = 16, what is the value of $P\left(\frac{1}{2}\right)$, accurate to 2 decimals?

Given the zeros of the polynomial, we know the related binomials will be (x + 2) (x – 1) and (x – 4).

P(x) = a(x + 2) (x – 1) (x – 4).

P(0) = 16 means that when x = 0, y = 16. Thus, we have a point, the y-intercept, that can be used to find the value of 'a'. Once this value is determined, we can calculate $P\left(\frac{1}{2}\right)$.

16 = a(0 + 2) (0 – 1) (0 – 4)

16 = 8a

a = 2

$P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}+2\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-4\right)$

= 2(2.5) (-0.5) (-3.5)

= 8.75

2. The polynomial $P(x) = 3x^4 - 11x^3 + 3x^2 + 11x - 6$ has a linear factor of (x – 3). What is the remaining cubic factor?

Use synthetic division.

3	3	-11	3	11	-6
		9	-6	-9	6
	3	-2	-3	2	0

The remaining cubic factor is $3x^3 - 2x^2 - 3x + 2$

3. For the polynomial $P(x) = x^3 - 7x^2 - kx + 16$, one zero is -2. What is the largest zero of P(x)?

Start by finding the value of k by substituting (-2) for x. Since (-2) is zero, the value of the polynomial at (-2) is 0.

$0 = (-2)^3 - 7(-2)^2 - k(-2) + 16$

$0 = -8 - 28 + 2k + 16$

$0 = -36 + 16 + 2k$

$20 = 2k$

$k = 10$

Now use synthetic division.

-2	1	-7	-10	16
		-2	18	-16
	1	-9	8	0

Factor the resultant trinomial: $x^2 - 9x + 8 = 0$

$(x - 8)(x - 1)$. The largest zero is 8.

4. For $P(x) = x^3 - 6x^2 - 3x + 40$, the zeros can be written as $x = m$, and $x = \frac{n \pm \sqrt{p}}{2}$. What is the value of p?

The possible integral zeros are factors of the constant term 40.

Test the numbers, ±1, ±2, ±4, ±5, ±8, ±10, ±20, ±40.

Using synthetic division, we see that (x – 5) is a factor of P(x).

5	1	-6	-3	40
		5	-5	-40
	1	-1	-8	0

The quotient is $x^2 - x - 8$.

Use the quadratic formula.

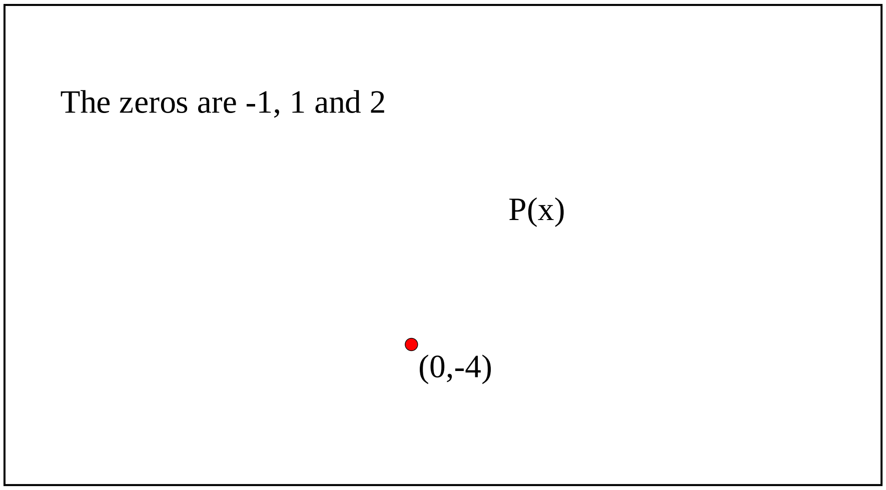
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 + 32}}{2}$$

$$x = \frac{1 \pm \sqrt{33}}{2}$$

The value of p is 33.

Use the following information to answer the next question.



5. The polynomial P(x) can be written in the form $y = a(x + b)^2 (x - c) (x - d)$. What is the value of a?

The exponent of 2 on $(x + b)^2$, indicates that zero having a multiplicity of 2. The value of the zero is negative, and there is only 1 zero to the left of the origin. Thus, b = 1.

For the two positive zeros, 1 and 2, the corresponding ‘c’ and ‘d’ values are 1 and 2.

Substitute these values and the given point to find ‘a’.

$-4 = a((0) + 1)^2 ((0) - 1) ((0) - 2)$

$-4 = a (1) (-1) (-2)$

$-4 = 2a$

a = -2

Use the following information to answer the next question.

The graph of P(x) below has a zero of 1 with a multiplicity of 2.

6. When P(x) is written in the form, $y = 0.2(x + b)(x - c)^2$, where $b, c \in N$, if the y-intercept is 0.8, what is the value of the other zero?

The value of c is 1 because the positive zero as indicated on the graph has a multiplicity of 2 (hence the exponent of 2 on $(x - c)^2$). Substitute the y-intercept of (0, 0.8) to find the value of 'b'.

$0.8 = 0.2(x + b)(x - 1)^2$

$0.8 = 0.2((0) + b)((0) - 1)^2$

$0.8 = 0.2(b)(-1)^2$

$0.8 = 0.2b$

$b = 4$

The value of the other zero is -4.

7. Given the polynomial function, $P(x) = 3x^4 - 4x^3 - 11x^2 + 16x - 4$, which of the following statements is incorrect?

a) $P(1) = 0$ Correct

$P(1) = 3(1)^4 - 4(1)^3 - 11(1)^2 + 16(1) - 4$

$= 3 - 4 - 11 + 16 - 4$

$= 19 - 19$

$= 0$

b) The potential zeroes are ±1, ±2, ±4. Correct

The factors of the constant, -4, are potential zeros.

c) $P(x) \div (3x^2 + 7x + 2) = (x - 1)(x + 2)$ Incorrect

If the sign in front of the 7x was negative, it would be true.

d) $(3x - 1)$ is a factor of $P(x)$. Correct

$P(\frac{1}{3}) = 3(\frac{1}{3})^4 - 4(\frac{1}{3})^3 - 11(\frac{1}{3})^2 + 16(\frac{1}{3}) - 4$

$= 3(\frac{1}{81}) - 4(\frac{1}{27}) - 11(\frac{1}{9}) + \frac{16}{3} - \frac{12}{3}$

$= \frac{1}{27} - \frac{4}{27} - \frac{33}{27} + \frac{144}{27} - \frac{108}{27}$

$= 0$

8. If $(x + 4)$ is a factor of $x^3 + 2x^2 - kx + 4$, determine the value of k.

a) -7   b) 25   c) -25   d) 7

Substitute -4 into the polynomial for x. Since it is a factor, the remainder is 0.

$(-4)^3 + 2(-4)^2 - (-4)(k) + 4 = 0$

$-64 + 32 + 4k + 4 = 0$

$-32 + 4 + 4k = 0$

$-28 + 4k = 0$

$4k = 28$

k = 7

9. The polynomial function $P(x) = x^3 + bx^2 - 7x + 2b$, where $b \in N$, has a factor of $(x - 1)$. When written as $P(x) = (x + k)(x - 1)^2$, find the value of k.

Since $(x - 1)$ is a factor, substituting 1 for x means that the remainder is 0. Substitute 1 for x in order to find the value of b.

$(1)^3 + b(1)^2 - 7(1) + 2b = 0$

$1 + b - 7 + 2b = 0$

$-6 + 3b = 0$

$3b = 6$

$b = 2$

Now use synthetic division.

1	1	2	-7	4
		1	3	-4
	1	3	-4	0

The remaining binomial is $x^2 + 3x - 4$.

Factoring this binomial, we get $(x+4)(x-1)$.

The complete factorization is: $(x + 4)(x - 1)^2$

The value of k is 4.

10. When $x^3 - x^2 - 16x - 4m$ is divided by $(x - m)$, the remainder is 0. Find m.

$m^3 - m^2 - 16m - 4m = 0.$

$m(m^2 - m - 20) = 0$

$m(m - 5)(m + 4) = 0$

$m = 5$ or -4