Rational Equations Practice

Use the following information to answer the first question.

Jimmy and Johnny work at their father's machine shop after school. It takes Jimmy 8 minutes to sweep the floor and it takes Johnny 12 minutes to sweep the same floor, when they each work by themselves. Let t be the time it takes for both of them to sweep the floor together.

1. The **equation** used to find the time it takes for them to sweep the floor together, and the **time** it takes for them to sweep the floor together, are

   A) The **equation** is $\frac{t}{8} + \frac{t}{12} = 1$, and the **time** is 10 minutes.

   B) The **equation** is $\frac{t}{8} + \frac{t}{12} = 1$, and the **time** is 4.8 minutes.

   C) The **equation** is $\frac{t}{8} + \frac{t}{12} = 20$, and the **time** is 10 minutes.

   D) The **equation** is $\frac{t}{8} + \frac{t}{12} = 20$, and the **time** is 4.8 minutes.

2. The value for x in the rational equation, $-\frac{1}{4} + \frac{2}{x} = \frac{5}{12}$, is _____.

Use the following information to answer the next question.

While correctly solving the following rational equation $\frac{2x+9}{3} + \frac{4x}{x-5} = x$, algebraically, a student wrote an equivalent quadratic equation of the form $ax^2 + bx + c = 0$.

3. The equivalent quadratic equation could have been
   A) $2x^2 + 10x - 45 = 0$
   B) $11x^2 + 5x - 10 = 0$
   C) $x^2 - 26x + 45 = 0$
   D) $13x^2 - 5x + 10 = 0$

4. The solution to the rational equation, $\dfrac{2}{x^2-4} + \dfrac{10}{6x+12} = \dfrac{1}{x-2}$, $x \neq \pm 2$, is _____.

Use the following information to answer the next question.

The dimensions of a particular rectangle are represented by rational expressions, where x > 3, as shown in the diagram below.

$\dfrac{x+11}{x+1}$

$\dfrac{2x}{x-3}$
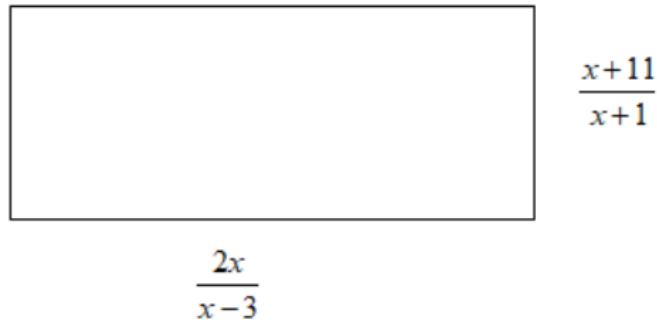
5. If the perimeter of this rectangle is 22 cm, the *numerical value* of the **width** is

   A) 2   B) 3   C) 4   D) 5

6. When solving $\dfrac{3x}{x+2} - \dfrac{5}{x-3} = \dfrac{-25}{x^2-x-6}$, the extraneous root is

   A) -2   B) 3   C) 6   D) -6

7. Solve for x in the rational equation, $\dfrac{x}{4} - \dfrac{7}{x} = 3$, $x \neq 0$ and verify the solution(s).

Use the following information to answer the next question.

When there is no wind, a runner is able to run 5000 m at an average speed of x m/s. When she runs with a tailwind, her average speed increases by 0.05 m/s and it takes her 12 seconds less to run the 5000 m.

The equation shown below represents this relationship.

$$\frac{5000}{x} - \frac{5000}{x + 0.05} = 12, x > 0$$

8. When there is no wind, the runner's average speed, to the nearest tenth of a metre per second, can be expressed in the form x = K.M , where K and M are integers. The values of K and M, respectively are, _____ and _____.

Use the following information to answer the next question.

The owner of a bookstore purchased some novels from a publisher for a total of $2016. The owner read the book and liked it so much that he took 4 books to keep and give away to relatives as gifts. He sold the remaining books for a total of $3360. He made a profit of $10 on each book.

Let x = the number of books purchased from the Publisher

9. A) Write a rational expression for the cost per book on the initial order of novels.

B) Write a rational expression for the cost per book when all but 4 of the books was sold.

C) A rational equation to model this situation would be:

Cost per book, selling   -   Cost per book, buying   = 10

   ‾‾‾‾‾   ‾‾‾‾‾

Fill in the blanks above.

D) Determine the number of books purchased from the Publisher.

Use the following information to answer the next question.

A math student was asked to solve the following rational equation.

$$\frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2 - 9y + 18}, y \neq 3,6$$

Their work is shown below.

Step 1	$\dfrac{9}{y-3} - \dfrac{4}{y-6} = \dfrac{18}{(y-3)(y-6)}$
Step 2	$(y-3)(y-6)\left[\dfrac{9}{y-3} - \dfrac{4}{y-6} = \dfrac{18}{(y-3)(y-6)}\right]$
Step 3	$9y - 54 - 4y - 12 = 18$
Step 4	$5y = 84$
Step 5	$y = 16.8$

10. Unfortunately, this student's work is not correct. Find the error and make the correction to determine the correct answer.

<u>Rational Equations Practice</u><u>Solutions</u>

Use the following information to answer the first question.

Jimmy and Johnny work at their father's machine shop after school. It takes Jimmy 8 minutes to sweep the floor and it takes Johnny 12 minutes to sweep the same floor, when they each work by themselves. Let t be the time it takes for both of them to sweep the floor together.

1. The **equation** used to find the time it takes for them to sweep the floor together, and the **time** it takes for them to sweep the floor together, are

    A) The **equation** is $\frac{t}{8} + \frac{t}{12} = 1$, and the **time** is 10 minutes.

    B) The **equation** is $\frac{t}{8} + \frac{t}{12} = 1$, and the **time** is 4.8 minutes.

    C) The **equation** is $\frac{t}{8} + \frac{t}{12} = 20$, and the **time** is 10 minutes.

    D) The **equation** is $\frac{t}{8} + \frac{t}{12} = 20$, and the **time** is 4.8 minutes.

Solution

Make a table to organize the information.

	Time to sweep floor in minutes.	Fraction of word done in 1 min	Fraction of work done in t minutes
Jimmy	8	$\frac{1}{8}$	$\left(\frac{1}{8}\right)(t)$ *or* $\frac{t}{8}$
Johnny	12	$\frac{1}{12}$	$\left(\frac{1}{12}\right)(t)$ *or* $\frac{t}{12}$
Together	t	$\frac{1}{t}$	$\frac{t}{t}$ or 1

The **equation** is $\frac{t}{8} + \frac{t}{12} = 1$.

The LCD is 24. Multiply each term by 24 to clear the fraction.

$$24\left(\frac{t}{8} + \frac{t}{12} = 1\right)$$

3t + 2t = 24

5t = 24

$$t = \frac{24}{5}$$

t = 4.8 minutes.

When Jimmy and Johnny work together, it will take them 4.8 minutes to sweep the floor.

The correct answer is B.

2. The value for x in the rational equation, $-\frac{1}{4} + \frac{2}{x} = \frac{5}{12}$, is __3__.

Solution

The LCD is 12x because this is the smallest expression that each denominator (4, x, and 12) will divide evenly into. Multiply each of the three terms in this equation by 12x to clear the fraction.

$$12x\left(-\frac{1}{4} + \frac{2}{x} = \frac{5}{12}\right)$$

-3x + 24 = 5x

24 = 8x

x = 3

<u>Verify</u>

$$-\frac{1}{4} + \frac{2}{x} = \frac{5}{12}$$

$$-\frac{1}{4} + \frac{2}{(3)} = \frac{5}{12}$$

$$-\frac{3}{12} + \frac{8}{12} = \frac{5}{12}$$

$$\frac{5}{12} = \frac{5}{12}$$

The value for x in this equation is 3.

Use the following information to answer the next question.

While correctly solving the following rational equation $\frac{2x+9}{3} + \frac{4x}{x-5} = x$, algebraically, a

student wrote an equivalent quadratic equation of the form $ax^2 + bx + c = 0$.

3. The equivalent quadratic equation could have been
   A) $2x^2 + 10x - 45 = 0$
   B) $11x^2 + 5x - 10 = 0$
   C) $x^2 - 26x + 45 = 0$
   D) $13x^2 - 5x + 10 = 0$

Solution

The LCD is $(3)(x - 5)$, which is the product of all separate factors in the denominator.

Multiply the LCD by each of the three terms in the rational equation.

$$(3)(x - 5)\left[\frac{2x+9}{3} + \frac{4x}{x-5} = x\right]$$

$[(x - 5)(2x + 9)] + [(3)(4x)] = (x)(3)(x - 5)$

$[2x^2 - x - 45] + [12x] = 3x^2 - 15x$

$2x^2 + 11x - 45 = 3x^2 - 15x$

$0 = x^2 - 26x + 45$

The correct answer is C.

4. The solution to the rational equation, $\frac{2}{x^2-4} + \frac{10}{6x+12} = \frac{1}{x-2}$, x ≠ ± 2, is __5__.

Solution

Begin by factoring.

$$\frac{2}{(x - 2)(x + 2)} + \frac{10}{6(x + 2)} = \frac{1}{x - 2}$$

The LCD is $(x - 2)(x +2)(6)$ which is the smallest expression that each of the denominators will divide evenly into.

$$[6(x-2)(x+2)]\left[\frac{2}{(x-2)(x+2)}+\frac{10}{6(x+2)}=\frac{1}{x-2}\right]$$
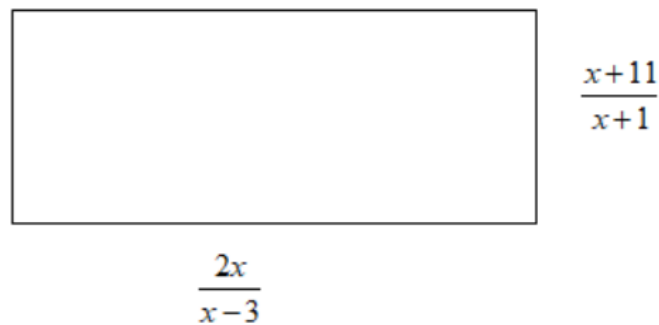
$(12) + 10(x-2) = 6(x+2)$

$12 + 10x - 20 = 6x + 12$

$10x - 8 = 6x + 12$

$4x = 20$

$x = 5$

Use the following information to answer the next question.

The dimensions of a particular rectangle are represented by rational expressions, where $x > 3$, as shown in the diagram below.

5. If the perimeter of this rectangle is 22 cm, the *numerical value* of the **width** is

A) 2   B) 3   C) 4   D) 5

Solution

In order to find the *numerical value* of the width, first solve for the value of the variable *x*.

To find the perimeter, add the 4 sides together.

Perimeter = 2(length) + 2(width)

We now have an equation. Substitute the values or expressions we know.

$$22 = 2\left(\frac{2x}{x-3}\right) + 2\left(\frac{x+11}{x+1}\right)$$

$$22 = \left(\frac{4x}{x-3}\right) + \left(\frac{2x+22}{x+1}\right)$$

The LCD is (x − 3)(x + 1). Multiply each of the 3 terms by this LCD.

$$(x-3)(x+1)\left[22 = \frac{4x}{x-3} + \frac{2x+22}{x+1}\right]$$

22(x − 3)(x + 1) = 4x(x + 1) + (2x + 22)(x − 3)

22(x² - 2x - 3) = 4x² + 4x + 2x² + 16x − 66

22x² - 44x − 66 = 6x² + 20x − 66

16x² − 64x = 0

16x(x − 4) = 0

x = 4

Since the expression for the width is $\frac{x+11}{x+1}$, substitute x = 4 into the expression.

$$\frac{(4)+11}{(4)+1} = \frac{15}{5} = 3$$

The *numerical value* of the **width** is 3.

The correct answer is B.

6. When solving $\frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2-x-6}$, the extraneous root is

    A) -2   B) 3   C) 6   D) -6

Solution

Begin by factoring.

$$\frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{(x+2)(x-3)}$$

The non-permissible values are x = -2 and x = 3

The LCD is the product of all separate factors, which in this case is (x + 2)(x − 3). Multiply each of the 3 terms by the LCD.

$$(x + 2)(x - 3)\left[\frac{3x}{x + 2} - \frac{5}{x - 3} = \frac{-25}{(x + 2)(x - 3)}\right]$$

3x(x − 3) − 5(x + 2) = -25

3x² − 9x − 5x − 10 = -25

3x² − 14x + 15 = 0

(3x − 5)(x − 3) = 0

x = $\frac{5}{3}$ or x = 3

Since 3 is a non-permissible value, x = 3 is an extraneous root.

The correct answer is B.

7. Solve for x in the rational equation, $\frac{x}{4} - \frac{7}{x} = 3, x \neq 0$, and verify the solution(s).

Solution

The LCD is 4x. Multiply each of the 3 terms by 4x to clear the fraction.

$$4x\left[\frac{x}{4} - \frac{7}{x} = 3\right]$$

x² − 28 = 12x

x² − 12x − 28 = 0

(x − 14) (x + 2) = 0

x = 14 or x = -2

Verify x = 14

$$\frac{(14)}{4} - \frac{7}{(14)} = 3$$

$$\frac{14}{4} - \frac{1}{2} = 3$$

$$\frac{7}{2} - \frac{1}{2} = 3$$

$$\frac{6}{2} = 3$$

$$3 = 3$$

<u>Verify x = -2</u>

$$\frac{(-2)}{4} - \frac{7}{(-2)} = 3$$

$$\frac{-1}{2} + \frac{7}{2} = 3$$

$$\frac{6}{2} = 3$$

$$3 = 3$$

Use the following information to answer the next question.

When there is no wind, a runner is able to run 5000 m at an average speed of x m/s. When she runs with a tailwind, her average speed increases by 0.05 m/s and it takes her 12 seconds less to run the 5000 m.

The equation shown below represents this relationship.

$$\frac{5000}{x} - \frac{5000}{x + 0.05} = 12, x > 0$$

8. When there is no wind, the runner's average speed, to the nearest tenth of a metre per second, can be expressed in the form x = K.M , where K and M are integers. The values of K and M, respectively are, __4__ and __5__.

Solution

To clear the fractions, multiply each of the 3 terms by the LCD, which is (x)(x + 0.05).

$$(x)(x + 0.05)\left[\frac{5000}{x} - \frac{5000}{x + 0.05} = 12\right]$$

5000(x + 0.05) − 5000x = 12(x)(x + 0.05)

5000x + 250 − 5000x = 12x² + 0.6x

0 = 12x² + 0.6x − 250

Use the quadratic formula, where a = 12, b = 0.6, and c = -250.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.6) \pm \sqrt{(0.6)^2 - 4(12)(-250)}}{2(12)}$$

$$x = \frac{-(0.6) \pm \sqrt{12000.36}}{24}$$

$$x = \frac{-(0.6) \pm 109.5461547}{24}$$

$$x = \frac{-0.6 + 109.5461547}{24} \quad or \quad \frac{-0.6 - 109.5461547}{24}$$

x = 4.539…   or   -4.589…

The negative root does not make sense in this context. The value of x, rounded to one decimal, is 4.5.

The values of K and M, respectively, are 4 and 5.

Use the following information to answer the next question.

The owner of a bookstore purchased some novels from a publisher for a total of $2016. The owner read the book and liked it so much that he took 4 books to keep and give away to relatives as gifts. He sold the remaining books for a total of $3360. He made a profit of $10 on each book.

Let x = the number of books purchased from the Publisher

9. A) Write a rational expression for the cost per book on the initial order of novels.

Solution

Divide the total cost of the order by the number of books, represented by the variable x.

The rational expression is $\dfrac{2016}{x}$**.**

B) Write a rational expression for the cost per book when all but 4 of the books was sold.

Solution

Divide the total money made on the sale of the books, by the number of books purchased, less the 4 books.

The rational expression is $\dfrac{3360}{x-4}$**.**

C) A rational equation to model this situation would be:

Cost per book, selling   -   Cost per book, buying   = 10

$\dfrac{3360}{x-4}$ $\dfrac{2016}{x}$

Fill in the blanks above.

D) Determine the number of books purchased from the Publisher.

Solution

To clear the fractions, multiply each of the 3 terms by the LCD, which is (x)(x − 4).

$$(x)(x-4)\left[\frac{3360}{x-4}-\frac{2016}{x}=10\right]$$
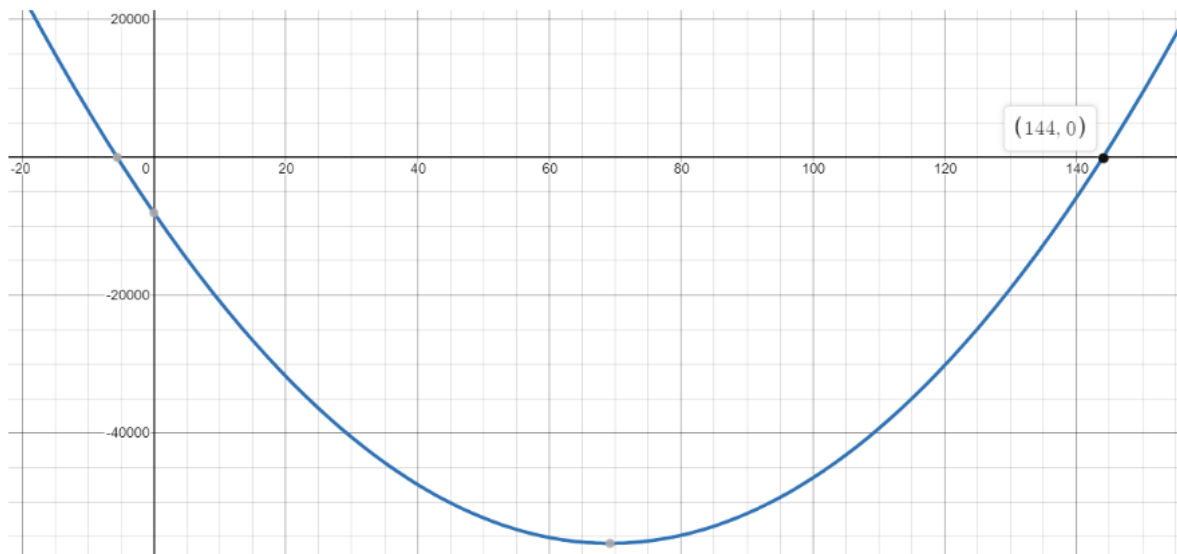
3360x − 2016(x − 4) = 10x(x − 4)

3360x − 2016x + 8064 = 10x^2 − 40x

0 = 10x^2 − 1384x − 8064

Solving Method 1

Graph the equation and determine the positive x-intercept.

A possible window setting is [-20, 200, 20, -60000, 30000, 300]

The positive x-intercept is 144.

There were 144 books purchased from the Publisher.

<u>Solving Method 2</u>

Use the quadratic formula.

$0 = 10x^2 - 1384x - 8064$

$a = 10$, $b = -1384$, $c = -8064$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1384) \pm \sqrt{(-1384)^2 - 4(10)(-8064)}}{2(10)}$$

$$x = \frac{1384 \pm \sqrt{2\,238\,016}}{20}$$

$$x = \frac{1384 + 1496}{20} \quad or \quad \frac{1384 - 1496}{20}$$

$x = 144$ or -5.6

Reject the negative root as it does not make sense in this context.

There were 144 books purchased from the Publisher.

Use the following information to answer the next question.

A math student was asked to solve the following rational equation.

$$\frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2 - 9y + 18}, y \neq 3,6$$

Their work is shown below.

Step 1	$\dfrac{9}{y-3} - \dfrac{4}{y-6} = \dfrac{18}{(y-3)(y-6)}$
Step 2	$(y-3)(y-6)\left[\dfrac{9}{y-3} - \dfrac{4}{y-6} = \dfrac{18}{(y-3)(y-6)}\right]$
Step 3	9y − 54 − 4y − 12 = 18
Step 4	5y = 84
Step 5	y = 16.8

10. Unfortunately, this student's work is not correct. Find the error and make the correction to determine the correct answer.

Solution

Steps 1 and 2 are good.

Coming out of step 2, the initial result would be:

9(y − 6) − 4(y − 3) = 18

Upon clearing the brackets, the result would be:

9y − 54 − 4y + 12 = 18.

The error is in step 3, as the 4th term shows (-12), but it should be (+12).

The correction is 9y − 54 − 4y + 12 = 18.

5y − 42 = 18

5y = 60

y = 12