Laws of Logarithms Solutions

1. Express as a single log.

   a) $4\log x - \dfrac{\log y}{3} + 5\log z$

Remember that $\dfrac{\log y}{3}$ is the same as $\left(\dfrac{1}{3}\right)\log y$.

If there is a number in front of the log, move it to the exponential position [the power law allows for this to happen]

$$\log x^4 - \log y^{\frac{1}{3}} + \log z^5$$

Since each log has the same base (i.e 10), look to apply the product and/or quotient laws to combine into a single logarithmic expression.

A subtraction sign between logarithmic expressions means divide the values of the powers, and an addition sign means to multiply the values of the powers.

Apply the quotient law first.

$$\log\left(\frac{x^4}{y^{\frac{1}{3}}}\right) + \log z^5$$

Now apply the product law.

$$\log\left(\frac{x^4 z^5}{y^{\frac{1}{3}}}\right)$$

   b) $6\log_c k - \log_c k^2 - \log_c \sqrt{k}$

When there are two subtraction signs, it is convenient to re-write the expression in its equivalent as: $6\log_c k - (\log_c k^2 + \log_c \sqrt{k})$

Move any numbers in front of the logs to the exponential position. Convert radicals to fractional exponent equivalents.

$$\log_c k^6 - (\log_c k^2 + \log_c k^{\frac{1}{2}})$$

$$\log_c k^6 - \log_c k^{2.5}$$

$$= \quad \log_c\left(\frac{k^6}{k^{2.5}}\right)$$

$$= \quad \log_c k^{3.5}$$

c) $m\log_2 5 + \log_2 5 - \log_2 5^{m+3}$

$\log_2 5^m + \log_2 5 - \log_2 5^{m+3}$

$$= \quad \log_2\left(\frac{5^{m+1}}{5^{m+3}}\right)$$

$= \quad \log_2 5^{(m+1)-(m+3)}$

$= \quad \log_2 5^{-2}$

d) $2 + \log_2 3x - 2\log_2 x$

In this situation, where there are two logarithmic expressions with the same base, and an integer, look to express the integer as an equivalent with $\log_2$.

The equivalent of 2 is $\log_2 4$, because $2^2 = 4$. Substitute this in for 2 in the expression.

$\log_2 4 + \log_2 3x - 2\log_2 x$

Use the power law to move any numbers from in front of the log to the exponential position.

$\log_2 4 + \log_2 3x - \log_2 x^2$

Use the product and the quotient laws to combine, and then simplify.

$$\log_2\left(\frac{(4)(3x)}{x^2}\right)$$

$$= \quad \log_2\left(\frac{12}{x}\right)$$

2. If $\log_6 4 = m$, and $\log_6 5 = n$, then $\log_6 80$ expressed in terms of m and n can be written as

   a) $m + n$ b) $2m + 2n$ c) $2m + n$ d) $m + 2n$

Since the base of all the logarithmic expressions is the same (i.e. 6), look for combinations of 4 and 5 that can be multiplied together to equal 80. The operation here is multiplication because all of the answers listed are summed; which means the application of the product law.

4 X 4 X 5 = 80.

$\log_6 \mathbf{4} + \log_6 \mathbf{4} + \log_6 \mathbf{5} = \log_6 \mathbf{80}$

Therefore, $m + m + n = \log_6 \mathbf{80}$

$\log_6 \mathbf{80}$expressed in terms of m and n is **2m + n.**

3. The simplification of $\left(4^{\log m}\right)\left(4^{\log m}\right)$ is

   a) $4^{\log m^2}$ b) $4^{\log 2m}$ c) $16^{\log m^2}$ d) $16^{\log 2m}$

The exponents in this question are logarithmic expressions. We are multiplying two powers of 4. The exponent laws state that we keep the base and add the exponents.

$$4^{\log m+\log m}$$

We are now adding logarithmic expressions, which means the product law is applied and we have to multiply the values of the powers, i.e. (m)(m).

$$= \quad 4^{\log m^2}$$

4. Evaluate

   a) $\left(6^{\log 20}\right)\left(6^{\log 5}\right)$

When multiplying two powers with the same base, keep the base and add the exponents.

$$6^{\log 20+\log 5}$$

Combine the two logarithmic expressions into one, using the product law.

$$6^{\log 100}$$

Since log 100 is equal to 2, the expression is equal to 6^2 or 36.

   b) $-\log_3 9-3\log_3\left(\frac{1}{3}\right)$

Use the power law to move numbers in front of the logs to the exponential position.

$$\log_3 9^{-1}-\log_3\left(\frac{1}{3}\right)^3$$

= $\log_3\left(\frac{1}{9}\right) - \log_3\left(\frac{1}{27}\right)$

= $\log_3\left(\frac{\frac{1}{9}}{\frac{1}{27}}\right) = \log_3 3$

= 1

b) $\frac{1}{2}\log_4 729 - \left(\log_4 2 + \frac{3\log_4 81}{4}\right)$

Use the power law to move numbers from in front of the log to the exponential position.

$\log_4 729^{\frac{1}{2}} - \left(\log_4 2 + \log_4 81^{\frac{3}{4}}\right)$

= $\log_4 \sqrt{729} - \left(\log_4 2 + \log_4 \sqrt[4]{81^3}\right)$

= $\log_4 27 - (\log_4 2 + \log_4 27)$

= $\log_4 27 - (\log_4 54)$

= $\log_4\left(\frac{27}{54}\right)$

= $\log_4\left(\frac{1}{2}\right)$

= -0.5

e) $$\frac{\left(\sqrt{2}^{\log_6 27}\right)\left(\sqrt{2}^{\log_6 16}\right)}{\sqrt{2}^{\log_6 12}}$$

On the numerator, multiplying 2 powers with the same base means to keep the base and add the exponents.

$$\frac{\sqrt{2}^{\log_6 27+\log_6 16}}{\sqrt{2}^{\log_6 12}}$$

Use the product law of logarithms to combine the 2 logarithm expressions into 1.

$$\frac{\sqrt{2}^{\log_6 432}}{\sqrt{2}^{\log_6 12}}$$

Dividing powers with the same base means keep the base and subtract the exponents.

$$\sqrt{2}^{\log_6 432-\log_6 12}$$

Use the quotient law of logarithms to combine the 2 logarithm expressions into 1.

$$\sqrt{2}^{\log_6\left(\frac{432}{12}\right)}$$

$= \sqrt{2}^{\log_6 36}$

$= \left(\sqrt{2}\right)^2$

= 2

f) $\log_n n + \log_m m^4 - \log_c 1 - 2\log_{0.5}32$

Recall that $\log_b b^k = k$;   and $\log_b 1 = 0$

$\log_n n = 1$

$\log_m m^4 = 4$

$\log_c 1 = 0$

$2\log_{0.5}32 = (2)\left(\frac{\log 32}{\log 0.5}\right) = -10$

The answer is 1 + 4 – 0 – (-10), which is equal to 15.

Use the following information to answer the next question.

A Math 30-1 student was asked to horizontally stretch $y = \log_2 x$ by a factor of $\frac{1}{24}$ about the y-axis and then state an equivalent transformation. Analyze the following steps.

Step 1 $y = \log_2 24x$

Step 2 $y = \log_2 8 + \log_2 3x$

Step 3 $y = 3 + \log_2 3x$

Step 4 Compared to $y = \log_2 x$, $y = \log_2 24x$ has been been horizontally stretched by a factor of $\frac{1}{3}$ about the y-axis and translated 3 units up.

5. Did the Math 30-1 student make an error in any step? If so, identify the step, the error and make any corrections.

There is no error.

6. If $\log_5 x$ is horizontally stretched by a factor of 125 about the y-axis, this would be the equivalent of $\log_5 x$ vertically translated _3__ units _down____(up or down).

If $\log_5 x$ is horizontally stretched by a factor of 125 about the y-axis, this means that x has been replaced with $\left(\frac{1}{125}\right)x$.

Splitting the 1 logarithmic expression into 2 expressions using the product law,

$$\log_5\left(\frac{1}{125}\right)x = \log_5\left(\frac{1}{125}\right) + \log_5 x$$

$$= -3 + \log_5 x$$

$$= \log_5 x - 3$$

7. The sound of a car horn is about 110 dB. If another sound, which is just above normal conversation, is about 39 811 times less intense as the car horn, what is the decibel level of the other sound?

Each bel is a 10 fold increase or decrease in intensity on the sound scale. Convert 110 dB to bels. It is the equivalent of 11 bels.

$$\frac{10^{11}}{10^x} = 39811$$

$10^{11-x} = 39811$

Input $y_1 = 10^{11-x}$, and $y_2 = 39811$, into the graphing calculator and determine the x-coordinate of the intersection point. The value is 6.4 bels or 64 dB.

The decibel level of the other sound is 64 dB.

8. The logarithmic scale to express the pH of a solution is pH = -log[H+], where [H+] is the hydrogen ion concentration, in moles per litre (mol/L). Lactic acidosis is a medical condition characterized by elevated lactates and a blood pH of less than 7.35. A patient is severely ill when his or her blood pH is 7.0. Find the hydrogen ion concentration in a patient with a blood pH of 7.0.

$7.0 = -\log_{10}[H+]$

$-7.0 = \log_{10}[H+]$

$10^{-7} = [H+]$

The hydrogen ion concentration in a patient with a blood pH of 7.0 is 10^{-7} mol/l.

9. Evaluate $\log_2\left(\sin\frac{\pi}{4}\right)+\log_2\left(\cos\frac{\pi}{4}\right)$

The sin and cos of $\frac{\pi}{4}$ are both $\frac{\sqrt{2}}{2}$.

$\log_2\left(\frac{\sqrt{2}}{2}\right) + \log_2\left(\frac{\sqrt{2}}{2}\right) = \log_2\left(\frac{2}{4}\right)$

$= \log_2\left(\frac{1}{2}\right)$

$= \frac{\log\frac{1}{2}}{\log 2}$

$= -1$