

Multiplying and Dividing Rational Expressions Practice

Use the following information to answer the first question.

A math student was given the following expression and asked to simplify:

$$\frac{2x + 14}{x^2 - x - 42} \div \frac{x + 7}{x^3 - 36x}$$

The student's work is shown below.

Step 1	$\frac{2(x + 7)}{(x + 7)(x - 6)} \div \frac{x + 7}{x(x - 6)(x + 6)}$
Step 2	$\left(\frac{2(x + 7)}{(x + 7)(x - 6)}\right) \left(\frac{x(x - 6)(x + 6)}{x + 7}\right)$
Step 3	$(2) \left(\frac{x(x + 6)}{x + 7}\right)$
Step 4	$\frac{2x(x + 6)}{x + 7}, x \neq -7$

1. Unfortunately, this student's work is not correct. The step of this student's **first** error **and** the correct answer is

- A) Step 1 and $\frac{2x(x-6)}{x+7}, x \neq -7, -6, 0, 6, 7$
- B) Step 1 and $\frac{2x(x-6)}{x-7}, x \neq -7, -6, 0, 6, 7$
- C) Step 4 and $\frac{2x(x-6)}{x+7}, x \neq -7, -6, 0, 6, 7$
- D) Step 4 and $\frac{2x(x-6)}{x-7}, x \neq -7, -6, 0, 6, 7$

2. The product of $\left(\frac{x^2+9x+20}{2x^2+6x-8}\right) \left(\frac{x^2-1}{3x+15}\right)$ is

- A) $\frac{x+1}{6}, x \neq -5, -4, 1$
- B) $\frac{x+1}{3}, x \neq -5, -4, 1$
- C) $\frac{x-1}{6}, x \neq -5, -4, 1$
- D) $\frac{x-1}{3}, x \neq -5, -4, 1$

3. The simplification of $\left(\frac{3x^2+9x}{14x}\right)\left(\frac{7x}{x+3}\right)$ can be written in the form $\frac{Mx}{K}$, $x \neq -3, 0$.
The values of M and K respectively, are ____ and ____.

4. The number of non-permissible values for the expression $\frac{2x+5}{2x^2-50} \div \frac{4x}{x^2+10x+24}$ is

A) 3

B) 4

C) 5

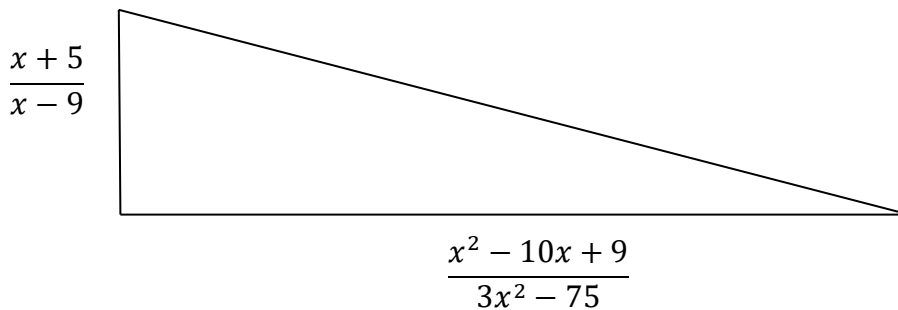
D) 6

5. Simplify and state the restrictions on the variable.

$$\frac{w+10}{10w} \div (100-w^2)$$

6. If $\left(\frac{y^2+7y+6}{2y-1}\right)\left(\frac{8y^3-2y}{12y+12}\right) = \frac{y(y+6)(2y+1)}{M}$, then the value of M is _____.

7. Find an expression for the area of the triangle.



Use the following information to answer the next question.

The simplified product of $\left(\frac{4w^3r}{5m}\right)\left(\frac{15m^7w}{8w^5mr^2}\right)$, where $m \neq 0$, $w \neq 0$, and $r \neq 0$, can be written in the form $\frac{Am^B}{2wr}$, where A and B represent single digit whole numbers.

8. The value of A is ____ and the value of B is ____.

9. An expression for the volume of a rectangular prism is $\frac{2x+8}{x^2}$. If an expression for the width is $\frac{2x+2}{x-3}$, and an expression for the length is $\frac{5x-15}{x^2+x}$, determine an expression for the height.

10. Simplify $\left(\frac{c^2-9}{d^3-d}\right)\left(\frac{d^2-d^4}{c+3}\right) \div (c-3)$, and state all non-permissible values.

Multiplying and Dividing Rational Expressions Practice **Solutions**

Use the following information to answer the first question.

A math student was given the following expression and asked to simplify:

$$\frac{2x + 14}{x^2 - x - 42} \div \frac{x + 7}{x^3 - 36x}$$

The student's work is shown below.

Step 1	$\frac{2(x + 7)}{(x + 7)(x - 6)} \div \frac{x + 7}{x(x - 6)(x + 6)}$
Step 2	$\left(\frac{2(x + 7)}{(x + 7)(x - 6)}\right) \left(\frac{x(x - 6)(x + 6)}{x + 7}\right)$
Step 3	$(2) \left(\frac{x(x + 6)}{x + 7}\right)$
Step 4	$\frac{2x(x + 6)}{x + 7}, x \neq -7$

1. Unfortunately, this student's work is not correct. The step of this student's **first** error **and** the correct answer is

- A) Step 1 **and** $\frac{2x(x-6)}{x+7}, x \neq -7, -6, 0, 6, 7$
- B) Step 1 **and** $\frac{2x(x-6)}{x-7}, x \neq -7, -6, 0, 6, 7$
- C) Step 4 **and** $\frac{2x(x-6)}{x+7}, x \neq -7, -6, 0, 6, 7$
- D) Step 4 **and** $\frac{2x(x-6)}{x-7}, x \neq -7, -6, 0, 6, 7$

Solution

Step one is incorrect. The factoring on the denominator of the first expression should be $(x - 7)(x + 6)$, **not** $((x + 7)(x - 6))$.

$$\frac{2(x + 7)}{(x - 7)(x + 6)} \div \frac{x + 7}{x(x - 6)(x + 6)}$$

$$\left(\frac{2(x + 7)}{(x - 7)(x + 6)}\right) \left(\frac{x(x - 6)(x + 6)}{x + 7}\right)$$

Divide out the two common binomial factors, $(x + 7)$ and $(x + 6)$

$$\left(\frac{2}{(x - 7)}\right) \left(\frac{x(x - 6)}{1}\right)$$

$$= \frac{2x(x-6)}{x-7}, x \neq -7, -6, 0, 6, 7$$

The correct answer is B.

2. The product of $\left(\frac{x^2+9x+20}{2x^2+6x-8}\right)\left(\frac{x^2-1}{3x+15}\right)$ is

A) $\frac{x+1}{6}, x \neq -5, -4, 1$

B) $\frac{x+1}{3}, x \neq -5, -4, 1$

C) $\frac{x-1}{6}, x \neq -5, -4, 1$

D) $\frac{x-1}{3}, x \neq -5, -4, 1$

Solution

Begin by factoring.

$$\left(\frac{(x+4)(x+5)}{2(x+4)(x-1)}\right)\left(\frac{(x+1)(x-1)}{3(x+5)}\right)$$

Determine the non-permissible values. By looking at the denominators, determine the values for the variables that would make the denominators equal to zero.

If $x = -5, -4,$ or $1,$ then we would have a denominator equal to zero, which would make the rational expression undefined.

There are three common binomials in the numerators and the denominators; $(x + 4), (x + 5)$ and $(x - 1).$ When these common factors are divided out, the simplification is

$$\frac{x+1}{6}, x \neq -5, -4, 1$$

The correct answer is A.

3. The simplification of $\left(\frac{3x^2+9x}{14x}\right)\left(\frac{7x}{x+3}\right)$ can be written in the form $\frac{Mx}{K}, x \neq -3, 0.$

The values of M and K respectively, are 3 and 2.

Solution

Factor.

$$\left(\frac{3x(x+3)}{14x}\right)\left(\frac{7x}{x+3}\right)$$

Divide out the common binomial $(x+3)$.

$$\left(\frac{3x}{14x}\right)\left(\frac{7x}{1}\right)$$

$$= \frac{21x^2}{14x}$$

$$= \frac{3x}{2}$$

The values of M and K respectively, are 3 and 2.

4. The number of non-permissible values for the expression $\frac{2x+5}{2x^2-50} \div \frac{4x}{x^2+10x+24}$ is

A) 3

B) 4

C) 5

D) 6

Solution

Begin by factoring.

$$\frac{2x+5}{2(x+5)(x-5)} \div \frac{4x}{(x+4)(x+6)}$$

We have to account for values of the variable that would make both denominators, and the numerator of the divisor, equal to zero.

$x = -6, -5, -4, 0$ and 5 .

The correct answer is C.

5. Simplify and state the restrictions on the variable.

$$\frac{w+10}{10w} \div (100-w^2)$$

Solution

Division is the same as multiplication by the reciprocal of the divisor.

$$\left(\frac{w+10}{10w}\right)\left(\frac{1}{100-w^2}\right)$$

For the denominator in the second bracket, it will be advantageous to divide out (-1) prior to factoring by difference of squares. This will allow us to create a common binomial factor.

$$\left(\frac{w+10}{10w}\right)\left(\frac{1}{-1(w+10)(w-10)}\right)$$

The non-permissible values are -10, 0, 10.

Simplify by moving the -1 to the numerator and dividing out the common binomial (w + 10).

$$\left(\frac{1}{10w}\right)\left(\frac{-1}{(w-10)}\right)$$

The final simplification is

$$\left(\frac{-1}{10w(w-10)}\right), w \neq -10, 0, 10$$

6. If $\left(\frac{y^2+7y+6}{2y-1}\right)\left(\frac{8y^3-2y}{12y+12}\right) = \frac{y(y+6)(2y+1)}{M}$, then the value of M is 6.

Solution

Factor.

$$\left(\frac{(y+6)(y+1)}{2y-1}\right)\left(\frac{2y(2y+1)(2y-1)}{12(y+1)}\right)$$

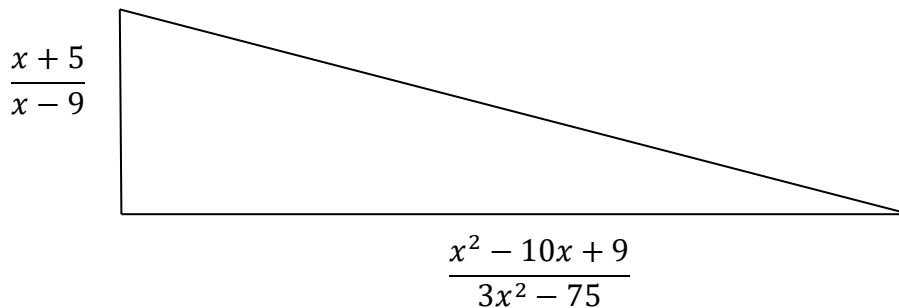
$$\left(\frac{y+6}{1}\right)\left(\frac{2y(2y+1)}{12}\right)$$

$$\left(\frac{2y(2y+1)(y+6)}{12}\right)$$

$$\left(\frac{y(y+6)(2y+1)}{6}\right)$$

The value of M is 6.

7. Find an expression for the area of the triangle.



Solution

The formula for the area for a triangle is $A = \frac{(base)(height)}{2}$.

An expression for the base of the triangle is $\frac{x^2-10x+9}{3x^2-75}$.

An expression for the height of the triangle is $\frac{x+5}{x-9}$.

$$A = \left(\frac{x^2 - 10x + 9}{3x^2 - 75}\right)\left(\frac{x+5}{x-9}\right) \div 2$$

Factor.

$$A = \left(\frac{(x-9)(x-1)}{3(x+5)(x-5)} \right) \left(\frac{x+5}{x-9} \right) \div 2$$

Divide out the common binomials, $(x-9)$ and $(x+5)$.

$$A = \left(\frac{(x-1)}{3(x-5)} \right) \left(\frac{1}{1} \right) \div 2$$

Which is the same as:

$$A = \left(\frac{(x-1)}{3(x-5)} \right) \left(\frac{1}{2} \right)$$

Which is equal to:

$$A = \frac{x-1}{6(x-5)}$$

An expression for the area of the triangle is $\frac{x-1}{6(x-5)}$.

Use the following information to answer the next question.

The simplified product of $\left(\frac{4w^3r}{5m} \right) \left(\frac{15m^7w}{8w^5mr^2} \right)$, where $m \neq 0$, $w \neq 0$, and $r \neq 0$, can be written in the form $\frac{Am^B}{2wr}$, where A and B represent single digit whole numbers.

8. The value of A is 3 and the value of B is 5.

Solution

Multiply the coefficients and the powers with the same base. Recall, that when multiplying powers with the same base, keep the base and **add** the exponents. Recall, that when dividing powers with the same base, keep the base and **subtract** the exponents.

$$\left(\frac{4w^3r}{5m} \right) \left(\frac{15m^7w}{8w^5mr^2} \right) = \frac{60w^4m^7r}{40w^5m^2r^2}$$

Which is equal to:

$$\frac{3m^5}{2wr}$$

The value of A is 3 and the value of B is 5.

9. An expression for the volume of a rectangular prism is $\frac{2x+8}{x^2}$. If an expression for the width is $\frac{2x+2}{x-3}$, and an expression for the length is $\frac{5x-15}{x^2+x}$, determine an expression for the height.

Solution

The formula for the volume of a rectangular prism is $V = (\text{Length})(\text{Width})(\text{Height})$

We know that an expression for the volume is $\frac{2x+8}{x^2}$.

We know that an expression for the length is $\frac{5x-15}{x^2+x}$.

We know that an expression for the width is $\frac{2x+2}{x-3}$.

Substitute these expressions into the formula.

$$\frac{2x+8}{x^2} = \left(\frac{5x-15}{x^2+x}\right) \left(\frac{2x+2}{x-3}\right) (\text{Height})$$

To determine the height, multiply the length by the width, and then divide this product into the volume.

Factor.

$$\frac{2(x+4)}{x^2} = \left(\frac{5(x-3)}{x(x+1)}\right) \left(\frac{2(x+1)}{x-3}\right) (\text{Height})$$

Divide out common factors.

$$\frac{2(x+4)}{x^2} = \left(\frac{5}{x}\right) \left(\frac{2}{1}\right) (\text{Height})$$

$$\frac{2(x+4)}{x^2} = \left(\frac{10}{x}\right) (\text{Height})$$

$$\left(\frac{2(x+4)}{x^2}\right)\left(\frac{x}{10}\right) = (\text{Height})$$

$$\frac{x+4}{5x} = \text{Height}$$

An expression for the height is $\frac{x+4}{5x}$.

10. Simplify $\left(\frac{c^2-9}{d^3-d}\right)\left(\frac{d^2-d^4}{c+3}\right) \div (c-3)$, and state all non-permissible values.

Solution

Factor.

$$\left(\frac{(c+3)(c-3)}{d(d+1)(d-1)}\right)\left(\frac{-d^2(d^2-1)}{c+3}\right) \div (c-3)$$

=

$$\left(\frac{(c+3)(c-3)}{d(d+1)(d-1)}\right)\left(\frac{-d^2(d+1)(d-1)}{c+3}\right)\left(\frac{1}{c-3}\right)$$

The non-permissible values are $d = -1, 0, 1$ and $c = -3, 3$

Divide out the common binomial factors in the numerator and the denominator.

$$\left(\frac{1}{d}\right)\left(\frac{-d^2}{1}\right)\left(\frac{1}{1}\right)$$

= -d