<u>Adding and Subtracting Rational Expressions Practice</u>

Use the following information to answer the first question.

Consider the following four statements.

Statement 1	$\frac{x+9}{x(x+1)} + \frac{2x}{5}$
Statement 2	$\frac{-6y}{x^2} - \frac{y-1}{3x}$
Statement 3	$\frac{(x+2)(x-1)}{x+1} + \frac{5}{x+5}$
Statement 4	$\frac{9x+1}{x^2-1} - \frac{3x^2}{4x+4}$

The following is a list of potential Lowest Common Denominators.

A	5(x+1)	E	$3x^2$
B	(x+1)(x+5)	F	3x
C	(x+1)(x+5)(x+2)(x-1)	G	5x(x+1)
D	$4x^2+1$	H	4(x+1)(x-1)

1. Match the statement with the correct LCD, using the letters A-H.
   Statement 1 ___
   Statement 2 ___
   Statement 3 ___
   Statement 4 ___

2. Simplify $\frac{2x^2-x}{(x-3)(x-1)} - \frac{3+6x}{(x-3)(x-1)} + \frac{8}{(x-3)(x-1)}$, where $x \neq 1, 3$.
   A) $\frac{2x^2-7x+5}{(x-3)(x-1)}$
   B) $\frac{2x-5}{x-3}$
   C) $\frac{2x^2+7x-5}{(x-3)(x-1)}$
   D) $\frac{2x+5}{x-3}$

Use the following information to answer the next question.

A math student was asked to simplify

$$\frac{5x+3}{x^2-2x}-\frac{8-x}{x-2}, x \neq 0, 2$$

Analyze the student's work below.

Step 1	$\frac{5x+3}{x(x-2)}-\frac{8-x}{x-2}$
Step 2	$\frac{5x+3}{x(x-2)}-\frac{x(8-x)}{x(x-2)}$
Step 3	$\frac{5x+3-8x+x^2}{x(x-2)}$
Step 4	$\frac{x^2-3x+3}{x(x-2)}$
Step 5	$\frac{(x-1)(x-2)}{x(x-2)}$
Step 6	$\frac{x-1}{x}$

3. The student made an error in step

   A) 3　　B) 4　　C) 5　　D) 6

4. The simplification of $\frac{2x+1}{8x^2}+\frac{4x-4}{3x}-\frac{8-x}{6}, x \neq 0$, can be simplified in the form $\frac{Ax^3-Bx+3}{24x^2}$, where A and B are integers. The **sum** of A and B is ____.

5. Simplify $\frac{y^2-\frac{1}{4}}{y-\frac{1}{2}}$

   A) $\frac{y+1}{2}$　　B) $\frac{y+1}{4}$　　C) $\frac{2y+1}{4}$　　D) $\frac{2y+1}{2}$

6. Subtract, simplify and state the non-permissible values for

$$\frac{6x}{x-5} - \frac{240}{x^2 - 2x - 15}$$

Use the following information to answer the next question.

The image found by a convex lens is described by the equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where *f* is the focal length (distance from the lens to the focus), *u* is the distance from the object to the lens, and *v* is the distance from the image to the lens. All distances are measured in centimetres. A math student was asked to do two things:

- Isolate *f*
- Determine the value of *f* when *u* = 60 and *v* = 12.

7. The correct isolation of *f* and the correct value of *f* when *u* = 60 and *v* = 12 are

A) $f = \frac{uv}{u+v}$, **and** *f* = 20

B) $f = \frac{uv}{u+v}$, **and** *f* = 10

C) $f = \frac{u+v}{uv}$, **and** *f* = 20

D) $f = \frac{u+v}{uv}$, **and** *f* = 10

Use the following information to answer the next question.

Given

$$\frac{x+1}{x+6} - \frac{x^2-4}{x^2+2x} \div \frac{2x^2+7x+3}{2x^2+x}$$ [***NOTE: Division is done before subtraction***]

The following statements are made.

Statement 1	Following the division simplification, the LCD used in the subtraction of the remaining two terms is (x+6)(x+2)
Statement 2	The non-permissible values are x = -6, -3, -2, -½ , 0.
Statement 3	The final simplification is $\frac{15}{(x+6)(x+3)}$.
Statement 4	The final simplification is $\frac{9}{(x+6)(x+2)}$.

8. The two true statements are

   A) 1 and 4 B) 2 and 3 C) 2 and 4 D) 1 and 3

9. Add the following and state the restrictions.

$$\frac{6p}{p^2-9} + \frac{5}{3-p}$$

Adding and Subtracting Rational Expressions PracticeSolutions

Use the following information to answer the first question.

Consider the following four statements.

Statement 1	$\frac{x+9}{x(x+1)}+\frac{2x}{5}$
Statement 2	$\frac{-6y}{x^2}-\frac{y-1}{3x}$
Statement 3	$\frac{(x+2)(x-1)}{x+1}+\frac{5}{x+5}$
Statement 4	$\frac{9x+1}{x^2-1}-\frac{3x^2}{4x+4}$

The following is a list of potential Lowest Common Denominators.

A	5(x+1)	E	$3x^2$
B	(x+1)(x+5)	F	3x
C	(x+1)(x+5)(x+2)(x-1)	G	5x(x+1)
D	$4x^2+1$	H	4(x+1)(x-1)

1. Match the statement with the correct LCD, using the letters A-H.
   Statement 1 _G_
   Statement 2 _E_
   Statement 3 _B_
   Statement 4 _H_

Solution

The lowest common denominator (LCD) is the smallest number, or smallest expression in this question, that all factors divide evenly into. Typically, it will be the product of all separate factors.

Statement 1

The individual factors in both denominators are (x)(5)(x+1). This expression is the LCD.

Statement 1 is matched with letter G.

Statement 2

The individual factors are (x) and (3). If a factor occurs twice in any denominator, include it twice when determining the LCD. The LCD is $3x^2$.

Statement 2 is matched with letter E.

Statement 3

The first term has a denominator of (x + 1) and the second term has a denominator of

(x + 5). The LCD is the product of these two factors.

Statement 3 is matched with letter B.

Statement 4

Before determining the LCD, both denominators need to be factored**.**

$$\frac{9x+1}{x^2-1} - \frac{3x^2}{4x+4}$$

$$= \frac{9x+1}{(x+1)(x-1)} - \frac{3x^2}{4(x+1)}$$

The separate factors from both denominators are (x + 1)(x – 1) (4). The LCD is the product of these factors.

Statement 4 is matched with letter H.

2. Simplify $\frac{2x^2-x}{(x-3)(x-1)} - \frac{3+6x}{(x-3)(x-1)} + \frac{8}{(x-3)(x-1)}$, where $x \neq 1, 3$.

   A) $\frac{2x^2-7x+5}{(x-3)(x-1)}$

   B) $\frac{2x-5}{x-3}$

   C) $\frac{2x^2+7x-5}{(x-3)(x-1)}$

   D) $\frac{2x+5}{x-3}$

Solution

There are 3 terms, each having the same denominator. Perform the operations on the numerators.

$$\frac{2x^2-x}{(x-3)(x-1)} - \frac{3+6x}{(x-3)(x-1)} + \frac{8}{(x-3)(x-1)},$$

$$\frac{2x^2 - x - 3 - 6x + 8}{(x-3)(x-1)}$$

$$\frac{2x^2 - 7x + 5}{(x-3)(x-1)}$$

Factor the numerator.

$$\frac{(2x-5)(x-1)}{(x-3)(x-1)}$$

Simplify by dividing out the common binomial factor (x – 1).

$$\frac{2x-5}{x-3}$$

The correct answer is B.

Use the following information to answer the next question.

A math student was asked to simplify

$$\frac{5x+3}{x^2-2x} - \frac{8-x}{x-2}, x \neq 0, 2$$

Analyze the student's work below.

Step 1	$\frac{5x+3}{x(x-2)} - \frac{8-x}{x-2}$
Step 2	$\frac{5x+3}{x(x-2)} - \frac{x(8-x)}{x(x-2)}$
Step 3	$\frac{5x+3-8x+x^2}{x(x-2)}$
Step 4	$\frac{x^2-3x+3}{x(x-2)}$
Step 5	$\frac{(x-1)(x-2)}{x(x-2)}$
Step 6	$\frac{x-1}{x}$

3. The student made an error in step
   A) 3 B) 4 C) 5 D) 6

Solution

Look at step 5.

$$\frac{(x-1)(x-2)}{x(x-2)}$$

The factoring of the numerator is incorrect. It cannot be factored, since there are not two numbers that multiply to 3 and add to -3.

The correct answer is C.

4. The simplification of $\frac{2x+1}{8x^2} + \frac{4x-4}{3x} - \frac{8-x}{6}, x \neq 0$, can be simplified in the form $\frac{Ax^3-Bx+3}{24x^2}$, where A and B are integers. The **sum** of A and B is _30__.

Solution

The LCD is $24x^2$.

Rewrite all terms with an equivalent expression having a common denominator.

$$\frac{(3)(2x+1)}{(3)8x^2} + \frac{(8x)(4x-4)}{(8x)3x} - \frac{(4x^2)(8-x)}{(4x^2)6}$$

$$\frac{6x+3}{24x^2} + \frac{32x^2-32x}{24x^2} - \frac{(32x^2-4x^3)}{24x^2}$$

$$\frac{6x+3+32x^2-32x-32x^2+4x^3}{24x^2}$$

$$\frac{4x^3-26x+3}{24x^2}$$

The value of A is 4 and the value of B is 26.

The value of A and B is 30.

5. Simplify $\frac{y^2-\frac{1}{4}}{y-\frac{1}{2}}$

$A)\ \frac{y+1}{2}$ B) $\frac{y+1}{4}$ C) $\frac{2y+1}{4}$ D) $\frac{2y+1}{2}$

Solution

Given this complex fraction, the initial strategy is to rewrite the numerator and the denominator with a single term.

$$\frac{y^2-\frac{1}{4}}{y-\frac{1}{2}}=\frac{\frac{4y^2}{4}-\frac{1}{4}}{\frac{2y}{2}-\frac{1}{2}}=\frac{\frac{4y^2-1}{4}}{\frac{2y-1}{2}}$$

Rewrite the division statement as multiplication by the reciprocal of the divisor.

$$\left(\frac{4y^2-1}{4}\right)\left(\frac{2}{2y-1}\right)$$

Factor.

$$\left(\frac{(2y+1)(2y-1)}{4}\right)\left(\frac{2}{2y-1}\right)$$

Simplify.

$$\frac{2y+1}{2}$$

The correct answer is D.

6. Subtract, simplify and state the non-permissible values for

$$\frac{6x}{x-5} - \frac{240}{x^2-2x-15}$$

Solution

Begin by factoring.

$$\frac{6x}{x-5} - \frac{240}{(x-5)(x+3)}$$

The non-permissible values are x = -3 and x = 5.

The LCD is the product of all separate factors. The LCD is (x – 5)(x + 3). Rewrite the first term as an equivalent expression having a common denominator.

$$\frac{(x+3)6x}{(x+3)(x-5)} - \frac{240}{(x-5)(x+3)}$$

$$\frac{6x^2+18x}{(x+3)(x-5)} - \frac{240}{(x-5)(x+3)}$$

$$\frac{6x^2+18x-240}{(x+3)(x-5)}$$

$$= \frac{6(x^2+3x-40)}{(x+3)(x-5)}$$

$$= \frac{6(x+8)(x-5)}{(x+3)(x-5)}$$

$$= \boldsymbol{\frac{6(x+8)}{(x+3)}, x \neq -3, 5}$$

Use the following information to answer the next question.

The image found by a convex lens is described by the equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where *f* is the focal length (distance from the lens to the focus), *u* is the distance from the object to the lens, and *v* is the distance from the image to the lens. All distances are measured in centimetres. A math student was asked to do two things:

- Isolate *f*
- Determine the value of *f* when *u* = 60 and *v* = 12.

7. The correct isolation of *f* and the correct value of *f* when *u* = 60 and *v* = 12 are

   A) $f = \frac{uv}{u+v}$, **and** *f* = 20

   B) $f = \frac{uv}{u+v}$, **and** *f* = 10

   C) $f = \frac{u+v}{uv}$, **and** *f* = 20

   D) $f = \frac{u+v}{uv}$, **and** *f* = 10

Solution

Given the original equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, combine the two terms on the right side of the equal sign into one term. The LCD is uv.

$\frac{1}{f} = \frac{(v)1}{(v)u} + \frac{(u)1}{(u)v}$,

$$\frac{1}{f} = \frac{v+u}{uv}$$

Cross multiply.

(1)(uv) = (*f*) (v + u)

$\frac{uv}{v+u} = f$

Substitute u = 60 and v = 12.

$$\frac{(60)(12)}{(12)+(60)} = f$$

$$\frac{720}{72} = f$$

f= 10

The correct answer is B.

Use the following information to answer the next question.

Given

$$\frac{x+1}{x+6} - \frac{x^2-4}{x^2+2x} \div \frac{2x^2+7x+3}{2x^2+x}$$ [***NOTE: Division is done before subtraction***]

The following statements are made.

Statement 1	Following the division simplification, the LCD used in the subtraction of the remaining two terms is (x+6)(x+2)
Statement 2	The non-permissible values are x = -6, -3, -2, -½ , 0.
Statement 3	The final simplification is $\frac{15}{(x+6)(x+3)}$.
Statement 4	The final simplification is $\frac{9}{(x+6)(x+2)}$.

8. The two true statements are
   A) 1 and 4 B) 2 and 3 C) 2 and 4 D) 1 and 3

Solution

Perform the division first. Factor all expressions.

$$\frac{x+1}{x+6} - \frac{(x+2)(x-2)}{x(x+2)} \div \frac{(2x+1)(x+3)}{x(2x+1)}$$

Change the division statement to the equivalent of multiplication by the reciprocal of the divisor.

$$\frac{x+1}{x+6} - \frac{(x+2)(x-2)}{x(x+2)} X \frac{x(2x+1)}{(2x+1)(x+3)}$$

Divide out common factors.

$$\frac{x+1}{x+6} - \frac{x-2}{x+3}$$

Following the division simplification, the LCD is (x + 6) (x + 3).

Statement 1 is false.

The non-permissible values are determined after factoring and prior to simplification. We have to remember that when dividing, non-permissible values can also be found on the numerator of the divisor.

The non-permissible values are x = -6, -3,-2, -½ , 0.

Statement 2 is true.

For the final simplification, subtract

$$\frac{x+1}{x+6} - \frac{x-2}{x+3}$$

The LCD is (x + 6)(x + 3).

$$\frac{(x+1)(x+3)}{(x+6)(x+3)} - \frac{(x-2)(x+6)}{(x+3)(x+6)}$$

$$\frac{x^2+4x+3}{(x+6)(x+3)} - \frac{(x^2+4x-12)}{(x+3)(x+6)}$$

$$\frac{x^2+4x+3-x^2-4x+12}{(x+6)(x+3)}$$

$$\frac{15}{(x+6)(x+3)}$$

Statement 3 is true and Statement 4 is false.

The correct answer is B.

9. Add the following and state the restrictions.

$$\frac{6p}{p^2-9}+\frac{5}{3-p}$$

Solution

Begin by factoring.

$$\frac{6p}{(p-3)(p+3)}+\frac{5}{3-p}$$

Notice that one factor in the first denominator (p – 3) is similar to the second denominator (3 – p). Since each term has opposite signs, we divide (-1) out of either term to make an equivalent binomial.

$$\frac{6p}{(p-3)(p+3)}+\frac{5}{-1(p-3)}$$

An equivalent form with a positive denominator is:

$$\frac{6p}{(p-3)(p+3)}+\frac{-5}{(p-3)}$$

The LCD is (p – 3)(p + 3). Set each term with a common denominator.

$$\frac{6p}{(p-3)(p+3)}+\frac{-5(p+3)}{(p-3)(p+3)}$$

$$\frac{6p-5p-15}{(p-3)(p+3)}$$

$$=\frac{p-15}{(p-3)(p+3)}, p \neq \pm 3.$$