Rational Expressions – Simplifying and NPV Practice

Use the following information to answer the first question.

Consider the following rational expressions.

Expression I	$\dfrac{x}{2x+1}$
Expression II	$\dfrac{x+1}{x^2-x}$
Expression III	$\dfrac{x}{x-1}$
Expression IV	$\dfrac{x-1}{x}$

1. The expression having non-permissible values of 0 and 1 is

   A) I B) II C) III D) IV

2. The non-permissible values of the expression $\frac{x-3}{x^2-x-56}$ are

   A) 0 and 3 B) -7, 0, and 8 C) -7, 3, and 8 D) -7 and 8

3. There are two non-permissible values for the rational expression

$$\frac{3x+5}{2x^2+5x-3}.$$

   The NPV that is **not** an integer, accurate to one decimal, is ______.

Use the following information to answer the next question.

Susan and James were each asked to simplify the following rational expression and state the non-permissible values.

$$\frac{5-x}{x^2-25}$$

Their responses are shown in the table.

	Simplification	NPVs, Stated as Restrictions
Susan	$\frac{-1}{x+5}$	$x \neq -5$
James	$\frac{1}{x-5}$	$x \neq \pm 5$

4. One person got the correct simplification and one person stated the NPVs correctly. Identify the person who correctly answered their component of the question. Justify.

5. The simplified expression of $\frac{2x^3-200x}{x^2+9x-10}$, $x \neq -10, 1$, is

A) $\frac{2x(x+10)}{x-1}$ B) $\frac{2(x+10)}{x+1}$ C) $\frac{2x(x-10)}{x-1}$ D) $\frac{2(x-10)}{x+1}$

Use the following information to answer the next question.

Given the rational expression $\frac{16x^2-9y^2}{8x-6y}$,

a math student made the following statements.

Statement 1	The simplification is $\frac{4x+3y}{2}$.
Statement 2	The simplification is $\frac{4x-3y}{2}$.
Statement 3	The expression representing the non-permissible values for x is $x = \left(\frac{4}{3}\right)y$.
Statement 4	The expression representing the non-permissible values for x is $x = \left(\frac{3}{4}\right)y$.
Statement 5	An example of a non-permissible value is (15,20).
Statement 6	An example of a non-permissible value is (20,15).

6. The three true statements are ____, ____, and ____.

7. In the rational expression $\frac{x-c}{x^2(x+k)}$, the non-permissible value(s) is/are

   A) –k B) 0, -k C) 0, k D) 0, c, k

Use the following information to answer the next question.

An expression equivalent to $\frac{2x-5}{x+1}$, x ≠ -1, 0, is written in the form $\frac{Ax^{[B]}-20x^2}{Cx^3+Cx^2}$, where A, B, and C represent single-digit whole numbers.

8. The value of A is ____, the value of B is ____, and the value of C is _____.

9. Which of the following expressions is equivalent to $\frac{6}{4x+7}$, $x \neq \frac{-7}{4}, 0, \frac{7}{4}$?

A) $\frac{6}{x(4x-7)(4x+7)}$ B) $\frac{6x(4x-7)}{x(4x-7)(4x+7)}$ C) $\frac{6(4x-7)}{x(4x-7)(4x+7)}$ D) $\frac{6x(4x+7)}{x(4x-7)(4x+7)}$

Use the following information to answer the next question.

Consider the following rational expressions.

Expression 1	$\frac{1}{x^2+6}$	Expression 2	$\frac{x^3-2x}{x^2-16}$
Expression 3	$\frac{7x}{x^2+5x+6}$	Expression 4	$\frac{3(x+2)}{8x(x-1)}$

10. The expression that does **not** have a non-permissible value, **and** the expression having two negative integers as NPVs, respectively, are

A) 1 and 3 B) 2 and 4 C) 1 and 4 D) 2 and 3

11. John said, that when given the rational expression $\frac{x+1}{7x+7}$, he would divide an 'x' out of the numerator and the denominator, and simplify the expression to $\frac{1}{14}$. Do you agree or disagree? Explain.

12. Sarah said, that when given the rational expression $\frac{x+3x}{x}$, it will always simplify to 4, regardless of the value of x. Do you agree or disagree? Explain.

Rational Expressions – Simplifying and NPV Practice Solutions

Use the following information to answer the first question.

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Expression I	$\dfrac{x}{2x+1}$
Expression II	$\dfrac{x+1}{x^2-x}$
Expression III	$\dfrac{x}{x-1}$
Expression IV	$\dfrac{x-1}{x}$

1. The expression having non-permissible values of 0 and 1 is

   A) I B) II C) III D) IV

Solution

For expression I, set the denominator equal to zero and solve to find the NPV.

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

For expression II, first factor the denominator.

$x^2 - x = x(x - 1)$

Set the factored form equal to zero and use the zero product principle to solve.

$x(x - 1) = 0$

Either x = 0, or x = 1.

For expression III, with a denominator of x – 1, the NPV is 1, because 1 – 1 = 0.

For expression IV, the NPV is 0.

The correct answer is B.

2. The non-permissible values of the expression $\frac{x-3}{x^2-x-56}$ are

   A) 0 and 3 B) -7, 0, and 8 C) -7, 3, and 8 D) -7 and 8

Solution

Begin by factoring the denominator.

$$\frac{x-3}{(x-8)(x+7)}$$

Set this factored form of the denominator equal to zero and use the zero product principle to solve.

(x – 8)(x + 7) = 0

Either x – 8 = 0; or x + 7 = 0

x = 8 or x = -7

The correct answer is D.

3. There are two non-permissible values for the rational expression

$$\frac{3x+5}{2x^2+5x-3}.$$

   The NPV that is **not** an integer, accurate to one decimal, is __0.5__.

Solution

Factor the denominator.

$$\frac{3x+5}{(2x-1)(x+3)}$$

Set this factored form of the denominator equal to zero and use the zero product principle to solve.

$(2x - 1)(x + 3) = 0$

Either $2x - 1 = 0$; or $(x + 3) = 0$

There are two non-permissible values, $x = \frac{1}{2}$ or $x = -3$.

The NPV that is not an integer, accurate to one decimal, is 0.5.

Use the following information to answer the next question.

Susan and James were each asked to simplify the following rational expression and state the non-permissible values.

$$\frac{5-x}{x^2-25}$$

Their responses are shown in the table.

	Simplification	NPVs, Stated as Restrictions
Susan	$\frac{-1}{x+5}$	$x \neq -5$
James	$\frac{1}{x-5}$	$x \neq \pm 5$

4. One person got the correct simplification and one person stated the NPVs correctly. Identify the person who correctly answered their component of the question. Justify.

Solution

Re-write the expression by factoring the denominator.

$$\frac{5-x}{(x-5)(x+5)}$$

Since the binomial in the numerator (5 – x) and one of the binomials in the denominator (x – 5) have the same terms with opposite signs, negative one can divided out of either to create a common factor.

$$\frac{-1(x-5)}{(x-5)(x+5)}$$

Determine the non-permissible values **prior** to simplification.

The NPVs are -5 and 5, or they can be written as± 5.

As restrictions, it is stated as x $\neq \pm 5$.

The simplification is:

$$\frac{-1}{x+5}$$

Susan has the correct simplification and James has the correct NPVs stated as restrictions.

5. The simplified expression of $\frac{2x^3-200x}{x^2+9x-10}$, $x \neq -10, 1$, is

A) $\frac{2x(x+10)}{x-1}$ B) $\frac{2(x+10)}{x+1}$ C) $\frac{2x(x-10)}{x-1}$ D) $\frac{2(x-10)}{x+1}$

Solution

The numerator will be factored, by first taking a common factor, followed by difference of squares. Remember, the **first** thing to do in any factoring situation is to look for a **common factor**.

The denominator will be factored by the sum/product method. We require 2 numbers that multiply to -10, and add to 9.

$$\frac{2x(x+10)(x-10)}{(x+10)(x-1)}$$

The non-permissible values are -10 and 1. Divide out the common binomial factor

(x + 10)

= $\frac{2x(x-10)}{x-1}$, $x \neq -10, 1$

The correct answer is C.

Use the following information to answer the next question.

Given the rational expression $\frac{16x^2-9y^2}{8x-6y}$,

a math student made the following statements.

Statement 1	The simplification is $\frac{4x+3y}{2}$.
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Statement 4	The expression representing the non-permissible values for x is $x = \left(\frac{3}{4}\right)y$.
Statement 5	An example of a non-permissible value is (15,20).
Statement 6	An example of a non-permissible value is (20,15).

6. The three true statements are _1_, _4_, and _5_.

Solution

Factor the numerator by difference of squares and the denominator by common factor.

$\frac{16x^2-9y^2}{8x-6y}$,

$$\frac{(4x+3y)(4x-3y)}{2(4x-3y)}$$

Divide out the common binomial in the numerator and the denominator.

$$\frac{(4x+3y)}{2}$$

Statement 1 is true and statement 2 is false.

To determine the expression for the non-permissible values of x, set 4x – 3y = 0, and solve for x.

$$x = \left(\frac{3}{4}\right)y.$$

Statement 4 is true and statement 3 is false.

Given the denominator of 8x – 6y, substitute the points (15,20) and (20,15). The ordered pair that makes the denominator equal to zero, is an example of a non-permissible value.

8(15) – 6(20)

120 – 120 = 0.

8(20) – 6(15)

160 – 90 = 70.

Since the point (15,20) results in a denominator equal to zero, this is an example of a non-permissible value.

Statement 5 is true and statement 6 is false.

The true statements are 1, 4, and 5.

7. In the rational expression $\frac{x-c}{x^2(x+k)}$, the non-permissible value(s) is/are

A) -k B) 0, -k C) 0, k D) 0, c, k

Solution

The denominator consists of two components multiplied together, x^2 and $(x + k)$.

Set each component equal to zero.

$x^2 = 0.$

$x = 0$

$x + k = 0$

$x = -k$

The correct answer is B.

Use the following information to answer the next question.

An expression equivalent to $\frac{2x-5}{x+1}$, $x \neq -1, 0$, is written in the form $\frac{Ax^{[B]}-20x^2}{Cx^3+Cx^2}$, where A, B, and C represent single-digit whole numbers.

8. The value of A is ____, the value of B is ____, and the value of C is _____.

Solution

The key to this question is to look at the second term in the numerator for the expression $\frac{2x-5}{x+1}$ and the expression $\frac{Ax^B-20x^2}{Cx^3+Cx^2}$. How is -5 related to $-20x^2$? When -5 is divided into $20x^2$, the result is $4x^2$.

This means that $4x^2$ was divided out of every term.

Multiply $4x^2$ by every term in $\frac{2x-5}{x+1}$.

$\frac{8x^3 - 20x^2}{4x^3 + 4x^2}$

Compare this expression to $\frac{Ax^B - 20x^2}{Cx^3 + Cx^2}$.

We see that *A* = 8, B = 3, and *C* = 4.

The value of *A* is 8, the value of B is 3 and the value of *C* is 4.

9. Which of the following expressions is equivalent to $\frac{6}{4x+7}$, $x \neq \frac{-7}{4}, 0, \frac{7}{4}$?

$A)$ $\frac{6}{x(4x-7)(4x+7)}$ B) $\frac{6x(4x-7)}{x(4x-7)(4x+7)}$ C) $\frac{6(4x-7)}{x(4x-7)(4x+7)}$ D) $\frac{6x(4x+7)}{x(4x-7)(4x+7)}$

Solution

By simplifying each of the options, A, B, C, and D, we can determine the expression equivalent to $\frac{6}{4x+7}$, $x \neq \frac{-7}{4}, 0, \frac{7}{4}$.

For option B, $\frac{6x(4x-7)}{x(4x-7)(4x+7)}$, by dividing out the common factors, x and (4x - 7), the expression is equivalent to $\frac{6}{4x+7}$, $x \neq \frac{-7}{4}, 0, \frac{7}{4}$.

The correct answer is B.

Use the following information to answer the next question.

Consider the following rational expressions.

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Expression 3	$\frac{7x}{x^2+5x+6}$	Expression 4	$\frac{3(x+2)}{8x(x-1)}$

10. The expression that does **not** have a non-permissible value, **and** the expression having two negative integers as NPVs, respectively, are

A) 1 and 3 B) 2 and 4 C) 1 and 4 D) 2 and 3

Solution

Expression 1 does not have a non-permissible value. Set the denominator equal to zero, $x^2 + 6 = 0$.

$x^2 = -6$

Whether x is negative or positive, squaring that value will always result in a positive number. Thus there is no way that we can get a result of -6.

Expression 2 has non-permissible values of ± 4; Expression 3 has non-permissible values of -2 and -3; Expression 4 has non-permissible values of 0 and 1. It is expression 3 that has two negative NPVs.

The correct answer is A.

11. John said, that when given the rational expression $\frac{x+1}{7x+7}$, he would divide an 'x' out of the numerator and the denominator, and simplify the expression to $\frac{1}{14}$. Do you agree or disagree? Explain.

Solution

I disagree. In the rational expression $\frac{x+1}{7x+7}$, there are two terms in the denominator and two terms in the numerator. In order to divide out common factors in a rational expression, the factor must be a factor in each and every term. Since there is not an x in each of the four terms, a common x cannot be divided out of just the two terms.

To simplify, first factor the denominator.

$\frac{x+1}{7x+7} = \frac{(x+1)}{7(x+1)}$

Now factor a common binomial, (x + 1), out of the numerator and the denominator.

$= \quad \frac{1}{7}.$

12. Sarah said, that when given the rational expression $\frac{x+3x}{x}$, it will always simplify to 4, regardless of the value of x. Do you agree or disagree? Explain.

Solution

Simplify the expression first.

$\frac{x+3x}{x} = \frac{x(1+3)}{x} = 4$

This simplification indicates that whatever value is substituted for x, will result in a simplification of 4. This will work for every value except one, when x = 0.

I partially agree with Sarah; the value will always simplify to 4, except if x = 0.