Geometric Series Practice

1. The sum of the first 20 terms of the geometric series, $\frac{5}{2}, \frac{5}{6}, \frac{5}{18}, \ldots$ can be found using

   A) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{1}{3}\right)^{20} - 1\right)}{1-\left(\frac{1}{3}\right)}$

   B) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{2}{3}\right)-1\right)^{20}}{1-\left(\frac{2}{3}\right)}$

   C) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{2}{3}\right)-1\right)^{20}}{\left(\frac{2}{3}\right)-1}$

   D) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{1}{3}\right)^{20} - 1\right)}{\left(\frac{1}{3}\right)-1}$

2. If $t_1 = 32$ and $r = \frac{3}{4}$, the sum of the first 6 terms is LMK.21875, where L, M and K are integers. The values of L, M and K respectively, are ____, ____, and ____.

3. The sum of the series 7 + 14 + 28 + … + 7168 is

   A) 12 455 B) 14 329 C) 15 508 D) 18 702

4. The number of terms in the series 2 + (-10) + 50 + …. that will yield a sum of -81 380 208 is ______.

5. The common ratio of a geometric series is $\frac{1}{2}$ and the sum of the first 7 terms is 508. Using the appropriate formula, determine the value of t_1.

6. The second term of a geometric series is 3 and the common ratio is $\frac{4}{5}$. Which statement below best describes the sum of the first 23 terms of this series?
   A) The sum is between 10 and 15.
   B) The sum is between 15 and 20.
   C) The sum is between 20 and 25.
   D) The sum is between 25 and 30.

7. A gardener wanted to reward a girl for her strong work ethic by giving a small bonus. He gave the girl two choices. She could either have $10 at once or she could get 1 cent on the first day, 2 on the second day,4 on the third day,8 cents on the fourth day and so on for ten days. Which option should the girl choose to get the maximum amount of money?

8. A student is constructing a family tree. He is hoping to trace back through 11 generations to calculate the total number of ancestors he has. Determine the total number of ancestors after the 11th generation.

9. If $S_8 = 340$, $t_n = 512$ and $r = (-2)$, determine t_1.

Use the following information to answer the next question.

Micah was asked to sum the first 15 terms of the geometric sequence:

$\frac{1}{9}, \frac{1}{3}, 1....$

His work is shown below.

Step 1	Identify $t_1 = \frac{1}{9}$ and $r = 3$.
Step 2	$S_{15} = \dfrac{\frac{1}{9}\left((3)^{15} - 1\right)}{3-1}$
Step 3	$S_{15} = \left(\frac{2}{9}\right)((3^{15}) - 1)$
Step 4	$S_{15} = 3\ 188\ 645.778$

10. Unfortunately Micah made an error. Identify and correct his error.

Geometric Series Practice Solutions

1. The sum of the first 20 terms of the geometric series, $\frac{5}{2}, \frac{5}{6}, \frac{5}{18}, \ldots$ can be found using

   A) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{1}{3}\right)^{20} - 1\right)}{1 - \left(\frac{1}{3}\right)}$

   B) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{2}{3}\right) - 1\right)^{20}}{1 - \left(\frac{2}{3}\right)}$

   C) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{2}{3}\right) - 1\right)^{20}}{\left(\frac{2}{3}\right) - 1}$

   D) $S_{20} = \dfrac{\frac{5}{2}\left(\left(\frac{1}{3}\right)^{20} - 1\right)}{\left(\frac{1}{3}\right) - 1}$

Solution

We are applying the formula:

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

We know:

- $t_1 = \frac{5}{2}$
- $r = \frac{\frac{5}{6}}{\frac{5}{2}} = \frac{5}{6} X \frac{2}{5} = \frac{2}{6} \ or \ \frac{1}{3}$
- n = 20

The correct answer is D.

2. If $t_1 = 32$ and $r = \frac{3}{4}$, the sum of the first 6 terms is LMK.21875, where L, M and K are integers. The values of L, M and K respectively, are __1__, __0__, and __5__.

Solution

Substitute the appropriate values into the formula:

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_6 = \frac{32\left(\left(\frac{3}{4}\right)^6 - 1\right)}{\left(\frac{3}{4}\right) - 1}$$

$$S_6 = \frac{32(-0.822\ \ldots)}{\left(-\frac{1}{4}\right)}$$

$$S_6 = 105.21875$$

The values of L, M, and K respectively are 1, 0, and 5.

3. The sum of the series 7 + 14 + 28 + … + 7168 is

A) 12 455 B) 14 329 C) 15 508 D) 18 702

Solution

Since we know the last term, or t_n (nth), is 7168, and do not know the number of terms, we will use the formula:

$$S_n = \frac{rt_n - t_1}{r - 1}$$

To find the common ratio, r, take any term (other than the first term) and divide by the previous term. For example, 14/7 = 2.

The common ratio is 2.

$$S_n = \frac{(2)(7168) - (7)}{(2) - 1}$$

S_n = 14 329

The correct answer is B.

4. The number of terms in the series 2 + (-10) + 50 + …. that will yield a sum of -81 380 208 is _12___.

Solution

The formula that has a variable n, which represents the number of terms, is

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

We know

- t_1 = 2
- $r = \frac{-10}{2}\ or -5$
- S_n = -81 380 208

Substitute these values into the formula.

$$-81\ 380\ 208 = \frac{2((-5)^n - 1)}{(-5) - 1}$$

Clear the fraction by multiplying both sides of the equal sign by -6.

488 281 248 = $2((-5)^n - 1)$

Divide both sides of the equal sign by 2.

244 140 624 = $(-5)^n - 1$

Add one to both sides to isolate the power.

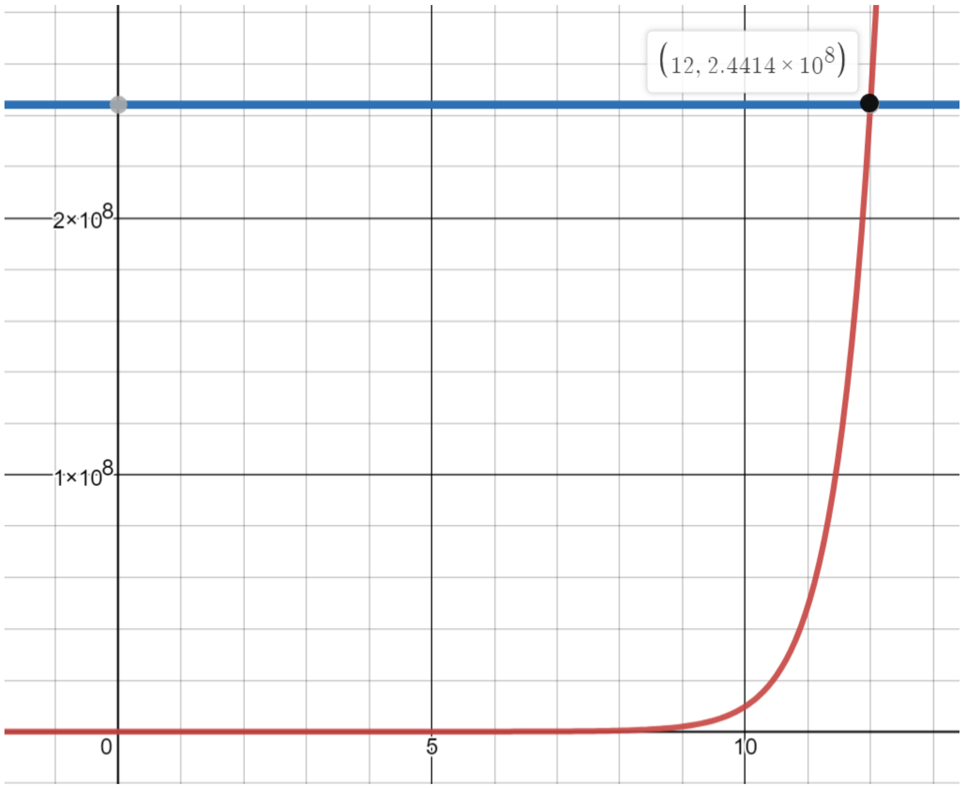
244 140 624 = $(-5)^n$

We can guess and test. By doing so, the value of n is 12.

Or we can solve by graphing. Change the maximum y value to 300 000 000.

Graph $y_1 = 5^n$ and $y_2 = 244\ 140\ 625$

The x-coordinate of the intersection point is the solution.



The number of terms in the series 2 + (-10) + 50 + … to yield a sum of -81 380 208 is 12.

5. The common ratio of a geometric series is $\frac{1}{2}$ and the sum of the first 7 terms is 508. Using the appropriate formula, determine the value of t_1.

Solution

Since we know the common ratio, the number of terms (7), and the sum of the terms, the formula to use is:

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$508 = \frac{t_1\left(\left(\frac{1}{2}\right)^7 - 1\right)}{\left(\frac{1}{2}\right) - 1}$$

$$508 = \frac{t_1(-0.9921875)}{\left(\frac{-1}{2}\right)}$$

$-254 = t_1 (-0.9921875)$

Divide both sides of the equal sign by -0.9921875

$256 = t_1$

The value of t_1 is 256.

6. The second term of a geometric series is 3 and the common ratio is $\frac{4}{5}$. Which statement below best describes the sum of the first 23 terms of this series?
   A) The sum is between 10 and 15.
   B) The sum is between 15 and 20.
   C) The sum is between 20 and 25.
   D) The sum is between 25 and 30.

Solution

Recall that determining the common ratio, any term (except the first term) is divided by the previous term.

$$r = \frac{t_2}{t_1}$$

$$\frac{4}{5} = \frac{3}{t_1}$$

Cross multiply.

$(4)(t_1) = (3)(5)$

$4t_1 = 15$

$$t_1 = \frac{15}{4}$$

The first term is $\frac{15}{4}$.

Use the formula:

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{23} = \frac{\left(\frac{15}{4}\right)\left(\left(\frac{4}{5}\right)^{23} - 1\right)}{\left(\frac{4}{5}\right) - 1}$$

$S_{23} = 18.639...$

The correct answer is B.

7. A gardener wanted to reward a girl for her strong work ethic by giving a small bonus. He gave the girl two choices. She could either have $10 at once or she could get 1 cent on the first day, 2 on the second day,4 on the third day,8 cents on the fourth day and so on for ten days. Which option should the girl choose to get the maximum amount of money?

Solution

This situation can be represented by a geometric series:

1 + 2 + 4 + 8 + ….

where $t_1 = 1$ and the common ratio is 2.

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{1(2^{10} - 1)}{2 - 1}$$

$$S_{10} = 1023$$

After 10 days, there is an accumulation of 1023 cents, or $10.23.

Option 2 is the better choice.

8. A student is constructing a family tree. He is hoping to trace back through 11 generations to calculate the total number of ancestors he has. Determine the total number of ancestors after the 11th generation.

Solution

This situation can be represented by a geometric series:

2 + 4 + 8 + ….

where $t_1 = 2$ and the common ratio is 2.

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{11} = \frac{2(2^{11} - 1)}{2 - 1}$$

$$S_{11} = 4094$$

The total number of ancestors after the 11th generation is 4094.

9. If $S_8 = 340$, $t_n = 512$ and $r = (-2)$, determine t_1.

Solution

Since we know the nth term ($t_n = 512$), use the formula:

$$S_n = \frac{rt_n - t_1}{r - 1}$$

$$S_8 = \frac{(-2)(512) - t_1}{(-2) - 1}$$

$$340 = \frac{(-2)(512) - t_1}{(-2) - 1}$$

$-1020 = -1024 - t_1$

$t_1 = -4$

The value of the first term, $t_1 = -4$.

Use the following information to answer the next question.

Micah was asked to sum the first 15 terms of the geometric sequence:

$$\frac{1}{9}, \frac{1}{3}, 1\ldots.$$

His work is shown below.

Step 1	Identify $t_1 = \frac{1}{9}$ and $r = 3$.
Step 2	$S_{15} = \dfrac{\frac{1}{9}\left((3)^{15} - 1\right)}{3-1}$
Step 3	$S_{15} = \left(\frac{2}{9}\right)\left((3^{15}) - 1\right)$
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10. Unfortunately Micah made an error. Identify and correct his error.

Solution

The number to the right of the equal sign in step 3 is incorrect.

In step 2, the denominator is 2. The simplification is step 3 should be 1/9 divided by 2.

$$\frac{\frac{1}{9}}{2} = \frac{1}{9} X \frac{1}{2} = \frac{1}{18}$$

The number immediately to the right of the equal sign in step 3 should be $\frac{1}{18}$.

Thus, S_{15} = 797 161. 444…