Geometric Sequence

1. Given the geometric sequence, $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \ldots,$ the general term is

   $t_n =$ _________.

2. The 19th term of the geometric sequence 6, 18, 54 …. Is
   A) 2 324 522 934
   B) 677 800 132
   C) 45 673 221
   D) 17 098 776

3. If $t_1 = 8.1$ and $t_6 = 259.2$, then t_2 is
   A) 10.5 B) 12.4 C) 16.2 D) 20.8

4. A geometric sequence has a first term of 4, a common ratio of 4, and $t_n = 4096$. The number of terms in the sequence is ____.

5. Use the general term to find t_1 of a geometric sequence given that the common ratio is -2 and the 10th term is -1536. Show work.

6. Is the sequence -4, -1, 2, … geometric? Justify.

7. Given the geometric sequence $\frac{3}{4}$, -3, 12, -48, …, an expression for the general term and the 11th term are

A) $t_n = \frac{3}{4}(-3)^{n-1}$ and $t_{11} = 1\ 048\ 576$

B) $t_n = \frac{3}{4}(-4)^{n-1}$ and $t_{11} = 1\ 048\ 576$

C) $t_n = \frac{3}{4}(-3)^{n-1}$ and $t_{11} = 786\ 432$

D) $t_n = \frac{3}{4}(-4)^{n-1}$ and $t_{11} = 786\ 432$

8. If $t_5 = 1250$ and $t_8 = 156\ 250$, determine t_1 and r. Show all work including two equations in two variables.

9. Is the 7th term of the sequence, $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$, greater or less than $\frac{4}{100}$? Justify.

10. If x + 2, 2x + 1, and 4x – 3, are 3 consecutive terms in a geometric sequence, determine the value of the common ratio and the 3 given terms.

11. How many terms are in the sequence -2, 12, -72, …20 155 392?

Use the following information to answer the next question.

Consider the following two sequences.

A.	$\frac{3}{5}, \frac{-1}{5}$, -1, …
B.	$\frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \ldots$

Consider the following statements.

Statement 1	The common ratio of the geometric sequence is $\frac{1}{2}$.
Statement 2	The common difference of the arithmetic sequence is $\frac{-4}{5}$.
Statement 3	For the geometric sequence, t_9 = 102.4.
Statement 4	For the arithmetic sequence, $t_6 = \frac{-13}{5}$

12. The two true statements are

A) 1 and 2 B) 3 and 4 C) 2 and 3 D) 1 and 4

13. Given an arithmetic sequence, if t_2 = -4 and t_7 = 26, find t_{11}.

14. Given a geometric sequence, if t_3 = 6 and t_9 = 384, determine t_1.

Geometric Sequence Solutions

1. Given the geometric sequence, $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \ldots$, the general term is

$t_n = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{(n-1)}$.

Solution

The common ratio is found by taking any term (other than the first term) and dividing by the previous term. For this example, $\frac{\frac{1}{3}}{\frac{1}{2}}$. Multiply by the reciprocal of the divisor. $\frac{1}{3} X \frac{2}{1} = \frac{2}{3}$. Thus, $r = \frac{2}{3}$.

The first term is $\frac{1}{2}$.

$t_n = t_1\, r^{(n-1)}$

$t_n = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{(n-1)}$

The general term is $t_n = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{(n-1)}$

2. The 19th term of the geometric sequence 6, 18, 54 …. Is
   A) 2 324 522 934
   B) 677 800 132
   C) 45 673 221
   D) 17 098 776

Solution

$t_n = t_1\, r^{(n-1)}$

The common ratio is $\frac{18}{6} = 3$. The first term is 6.

$t_{19} = (6)\,(3)^{(19-1)}$

$t_{19} = (6)(3)^{18}$

$t_{19} = 2\ 324\ 522\ 934$

The correct answer is A.

3. If $t_1 = 8.1$ and $t_6 = 259.2$, then t_2 is

   A) 10.5 B) 12.4 C) 16.2 D) 20.8

Solution

$t_6 = (8.1)(r)^{6-1}$

Substitute 259.2 for t_6.

$259.2 = (8.1)(r)^5$

$32 = r^5$

Take the 5th root of 32.

$r = 2$

Since the first term is 8.1, the second term is (8.1)(2), or 16.2.

The correct answer is C.

4. A geometric sequence has a first term of 4, a common ratio of 4, and t_n = 4096. The number of terms in the sequence is _6_.

Solution

The sequence is 4, 16, 64, …, 4096.

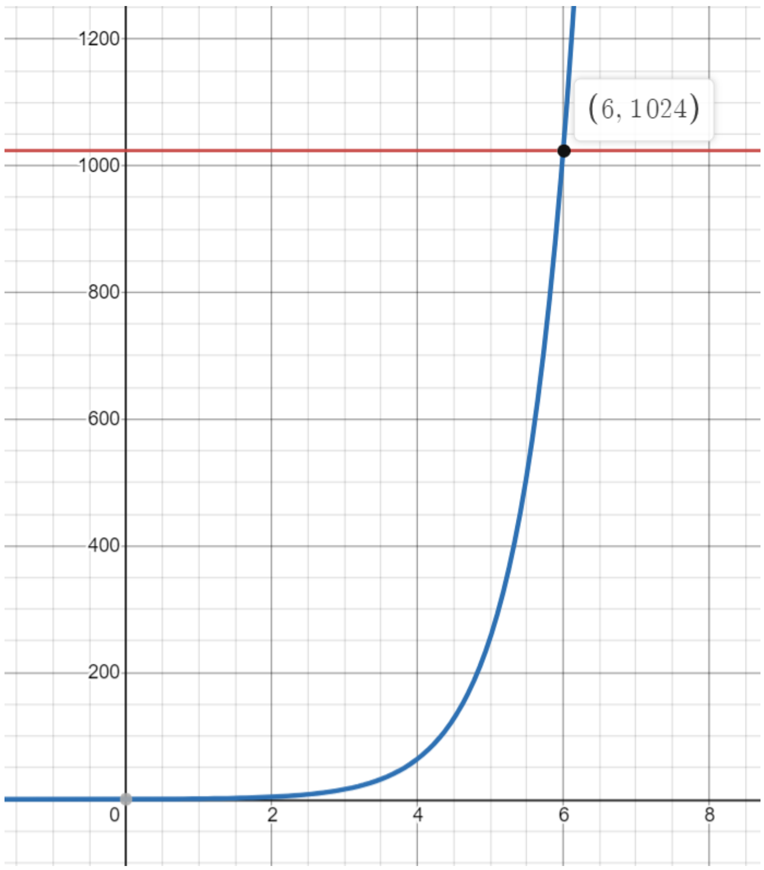
$t_n = t_1 r^{(n-1)}$

$4096 = (4)(4)^{n-1}$

$1024 = 4^{n-1}$

We could guess and test to determine that the value of n is 6.

Or, we could graph $y_1 = 1024$ and $y_2 = 4^{x-1}$ and determine the x-coordinate of the intersection point.



There are 6 terms in the sequence.

5. Use the general term to find t_1 of a geometric sequence given that the common ratio is -2 and the 10th term is -1536. Show work.

Solution

$t_n = t_1 r^{(n-1)}$

$t_{10} = (t_1)(-2)^{10-1}$

$-1536 = (t_1)(-2)^9$

$t_1 = \frac{-1536}{(-2)^9}$.

$t_1 = 3$

The first term of this sequence is 3.

6. Is the sequence -4, -1, 2, … geometric? Justify.

Solution

In order to be a geometric sequence, there must be a common ratio. Any term (other than the first term) divided by the previous term would have to be the same number.

We will determine the values of $\frac{t_2}{t_1}$ and $\frac{t_3}{t_2}$.

$\frac{t_2}{t_1} = \frac{-1}{-4} = \frac{1}{4}$

$\frac{t_3}{t_2} = \frac{2}{-1} = -2$

Since these two values are not equal, there is not a common ratio. Therefore, this is **not** a geometric sequence.

7. Given the geometric sequence $\frac{3}{4}$, -3, 12, -48, …, an expression for the general term and the 11th term are
   A) $t_n = \frac{3}{4}(-3)^{n-1}$ and $t_{11} = 1\,048\,576$
   B) $t_n = \frac{3}{4}(-4)^{n-1}$ and $t_{11} = 1\,048\,576$
   C) $t_n = \frac{3}{4}(-3)^{n-1}$ and $t_{11} = 786\,432$
   D) $t_n = \frac{3}{4}(-4)^{n-1}$ and $t_{11} = 786\,432$

Solution

The common ratio is $\frac{-3}{\frac{3}{4}}$, which is equal to -3 X $\frac{4}{3}$, which is equal to -4.

The general term is $t_n = \frac{3}{4}(-4)^{n-1}$

$t_{11} = \frac{3}{4}(-4)^{11-1}$

$t_{11} = \frac{3}{4}(-4)^{10}$

$t_{11} = 786\ 432$

The correct answer is D.

8. If $t_5 = 1250$ and $t_8 = 156\ 250$, determine t_1 and r. Show all work including two equations in two variables.

Solution

Remember the relationship between the term number and the exponent on the common ratio. The exponent is one less than the term number.

Equation ① $t_5 = (t_1)(r^4)$ and Equation② $t_8 = (t_1)(r^7)$

Substitute the values for t_5 and t_8.

Equation ① $1250 = (t_1)(r^4)$ and Equation② $156\ 250 = (t_1)(r^7)$

Place one equation beneath the other and divide the columns.

$156\ 250 = (t_1)(r^7)$

$\underline{1250 = (t_1)(r^4)}$

$125 = r^3$

$r = 5$

t_1 divided by t_1 is one, and thus this variable is eliminated.

When dividing powers with the same base, keep the base and subtract the exponents.

Substitute r = 5 into either equation to find t_1.

Equation ① $1250 = (t_1)(r^4)$

$1250 = (t_1)(5^4)$

$$t_1 = \frac{1250}{625}$$

$t_1 = 2$

The value of t_1 is 2 and the value of r is 5.

9. Is the 7th term of the sequence, $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$, greater or less than $\frac{4}{100}$? Justify.

Solution

Find the common ratio.

$$r = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{5}{4} \times \frac{2}{5} = \frac{1}{2}$$

Find the 7th term using the general term.

$$t_7 = \left(\frac{5}{2}\right)\left(\frac{1}{2}\right)^{7-1}$$

$$t_7 = \left(\frac{5}{2}\right)\left(\frac{1}{2}\right)^{6}$$

$t_7 = 0.0390625$

The 7th term is less than $\frac{4}{100}$.

10. If x + 2, 2x + 1, and 4x – 3, are 3 consecutive terms in a geometric sequence, determine the value of the common ratio and the 3 given terms.

Solution

Since the common ratio is found by dividing any term (other than the first term) by the previous term, in a geometric sequence, $\frac{t_2}{t_1} = \frac{t_3}{t_2}$.

Therefore, $\frac{2x+1}{x+2} = \frac{4x-3}{2x+1}$. Cross multiply and solve.

$(2x + 1)(2x + 1) = (x + 2)(4x - 3)$

$4x^2 + 4x + 1 = 4x^2 + 5x - 6$

$4x + 1 = 5x - 6$

$7 = x$

Substitute x = 7 in each of the algebraic expressions to find the actual values of the terms.

$(7) + 2, 2(7) + 1, 4(7) - 3$

The first three terms are 9, 15, and 25. The common ratio is $\frac{5}{3}$.

11. How many terms are in the sequence -2, 12, -72, …20 155 392?

Solution

$t_n = t_1 r^{(n-1)}$

$20\ 155\ 392 = (-2)(-6)^{(n-1)}$

Divide both sides by -2 to isolate the power.

$-10\ 077\ 696 = (-6)^{(n-1)}$

Either guess and test, or graph $y_1 = -10\ 077\ 696$ and $y_2 = (-6)^{(x-1)}$ and determine the x-coordinate of the intersection point.

The value of n is 10.

There are 10 terms in the sequence.

Use the following information to answer the next question.

Consider the following two sequences.

A.	$\frac{3}{5}, \frac{-1}{5}, -1, \ldots$
B.	$\frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \ldots$

Consider the following statements.

Statement 1	The common ratio of the geometric sequence is $\frac{1}{2}$.
Statement 2	The common difference of the arithmetic sequence is $\frac{-4}{5}$.
Statement 3	For the geometric sequence, $t_9 = 102.4$.
Statement 4	For the arithmetic sequence, $t_6 = \frac{-13}{5}$

12. The two true statements are

B) 1 and 2 B) 3 and 4 C) 2 and 3 D) 1 and 4

Solution

Based on the statements, we know that one sequence is arithmetic and the other is geometric.

Sequence A is arithmetic since it has a common difference of $-\frac{4}{5}$. This value can be determined by taking any term (other than the first term) and subtracting the previous term.

Sequence B is geometric since it has a common ratio of 2. The value can be determined by taking any term (other than the first term) and dividing by the previous term.

Statement 1 is false, since the common ratio is 2.

Statement 2 is true.

Determine the general term for the geometric sequence.

Since $t_1 = \frac{2}{5}$ and $r = 2$,

$t_9 = (\frac{2}{5})\ (2)^8$

$t_9 = 102.4$

Statement 3 is true.

Determine the general term for the arithmetic sequence.

Since $t_1 = \frac{3}{5}$ and $r = -\frac{4}{5}$,

$t_6 = \frac{3}{5} + (6 - 1) - \frac{4}{5}$

$t_6 = -\frac{17}{5}$

Statement 4 is false.

The correct answer is *C*.

13. Given an arithmetic sequence, if $t_2 = -4$ and $t_7 = 26$, find t_{11}.

Solution

Equation ①	Equation ②
$-4 = t_1 + (n - 1)d$	$26 = t_1 + (n - 1)d$
$-4 = t_1 + d$	$26 = t_1 + 6d$

$26 = t_1 + 6d$

$\underline{-4 = t_1 + d}$ Subtract to eliminate t_1.

$30 = 5d$

$d = 6$

Since $d = 6$ and $t_2 = -4$, then we know that $t_1 = -10$.

$t_{11} = -10 + (n - 1)6$

$t_{11} = -10 + 6n - 6$ Substitute $n = 11$.

$t_{11} = -10 + 6(11) - 6$

$t_{11} = -10 + 60$

$t_{11} = 50$

14. Given a geometric sequence, if $t_3 = 6$ and $t_9 = 384$, determine t_1.

Solution

Equation ① Equation ②

$384 = (t_1)\, r^8$ $6 = (t_1)\, r^2$

$384 = (t_1)\, r^8$

$\underline{6 = (t_1)\, r^2}$ Divide to eliminate t_1

$64 = r^6$

$r = 2$

Since 6 is the third term, divide it by 2 to get the second term, which is 3. Divide this by 2 to get the first term, which is 1.5.

The first term, t_1, is 1.5.