Arithmetic Series Practice

1. The sum of the first 20 terms of the arithmetic series 204 + 212 + 220 + … is

   A) 11 200 B) 7120 C) 5680 D) 5600

2. The sum of the 17 terms in the series, ___ + ___ + _-1_ + ….+ _27_ is 187. The first term can be written in the form –K, where K is an integer. The value of K is _____.

3. Determine the sum of the sequence -1, -5, -9, …, -77.

   A) -690 B) -780 C) -810 D) -908

4. The first term of an arithmetic series is 21. If $S_{12} = 582$, then d = ____.

Use the following information to answer the next question.

The sum of the first three terms of an arithmetic series is -3 and the sum of the first eight terms is 132. The following statements are made:

Statement 1	$t_1 = -5$
Statement 2	$d = 7$
Statement 3	$S_4 = 13$
Statement 4	$S_{10} = 235$

5. The two true statements are:

   A) 1 and 3 B) 2 and 4 C) 1 and 4 D) 2 and 3

Use the following information to answer the next question.

The seating chart for a concert hall shows that a section of seats has 10 rows. Tickets in the first section sell for \$95 each. Tickets in each consecutive section are \$6 cheaper than the tickets in the preceding section. Mary wants to buy 1 ticket from each section to give to charity. She spends \$680.

6. The number of rows in this concert hall is ____.

7. An arithmetic series has $t_{11} = 17$ and $S_{11} = 132$. The first term, t_1, is
   A) -7 B) 7 C) 9 D) -129

Use the following information to answer the next question.

Suppose you started a business that sold \$18 000 worth of sports injury products during the first year. You have set a goal of increasing annual sales by \$4000 each year for the next 15 years. Assume the goal is met every year.

8. The correct formula to use to calculate the accumulated total of sales **and** the total amount of sales after 16 years is
   A) $S_{16} = \frac{16}{2}(18\,000 + 4\,000)$ **and** a total sales of \$176 000.
   B) $S_{16} = \frac{16}{2}(18\,000 + 4\,000)$ **and** a total sales of \$768 000.
   C) $S_{16} = \frac{16}{2}[2(18\,000) + (15)(4000)]$ **and** a total sales of \$176 000.
   D) $S_{16} = \frac{16}{2}[2(18\,000) + (15)(4000)]$ **and** a total sales of \$768 000.

Arithmetic Series Practice Solutions

1. The sum of the first 20 terms of the arithmetic series 204 + 212 + 220 + … is

   A) 11 200 B) 7120 C) 5680 D) 5600

Solution

We know the first term but not the last term. This means that we cannot use the formula: $S_n = \frac{n}{2}(t_1 + t_n)$.

The common difference is 212 – 204, or 8. Since we know the first term, the number of terms, and the common difference, we can use the formula:

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{20} = \frac{20}{2}[2(204) + (20-1)8]$.

$S_{20} = 10[408 + 152]$.

$S_{20} = 5600$.

The correct answer is D.

2. The sum of the 17 terms in the series, ____ + ____ + _-1_ + ….+ _27_ is 187. The first term can be written in the form –K, where K is an integer. The value of K is _5_.

Solution

Since we know a given sum, the number of terms and the last term, we can use the formula, $S_n = \frac{n}{2}(t_1 + t_n)$, to find the first term.

$S_n = \frac{n}{2}(t_1 + t_n)$.

$187 = \frac{17}{2}(t_1 + 27)$.

Multiply both sides of the equal sign by 2 to clear the fraction.

374 = $17(t_1 + 27)$

374 = $17t_1$ + 459

-85 = $17t_1$

t_1 = -5.

The value of K is 5.

3. Determine the sum of the sequence -1, -5, -9, …, -77.

A) -690 B) -780 C) -810 D) -908

Solution

We know the first term and the last term, and we can calculate the common difference, but we do not know the number of terms. The number of terms is needed for both formulas. Use the general term for an arithmetic sequence to determine the number of terms.

The common difference is -5 – (-1), or -4.

$t_n = t_1 + (n - 1)d$

-77 = -1 + (n – 1)-4

-77 = -1 – 4n + 4

-77 = -4n + 3

-80 = -4n

n = 20

$S_n = \frac{n}{2}(t_1 + t_n).$

$S_{20} = \frac{20}{2}(-1 + -77).$

$S_{20} = -780$

The correct answer is B.

4. The first term of an arithmetic series is 21. If S_{12} = 582, then d = _5_.

Solution

_21_, ____, ____, …, _t_n_

We know the first term; we know that there are 12 terms and we know the sum of these 12 terms. We do not know the last term, so we cannot use the formula:

$S_n = \frac{n}{2}(t_1 + t_n).$

However, we know everything but *d* given the formula:

$S_n = \frac{n}{2}[2t_1 + (n-1)d].$

$S_{12} = \frac{12}{2}[2(21) + (12-1)d].$

582 = 6(42 + 11d)

582 = 252 + 66d

330 = 66d

5 = d

The value of d is 5.

Use the following information to answer the next question.

The sum of the first three terms of an arithmetic series is -3 and the sum of the first eight terms is 132. The following statements are made:

Statement 1	$t_1 = -5$
Statement 2	$d = 7$
Statement 3	$S_4 = 13$
Statement 4	$S_{10} = 235$

5. The two true statements are:

A) 1 and 3 B) 2 and 4 C) 1 and 4 D) 2 and 3

Solution

We know the sum of 3 terms and the sum of 8 terms. Without knowing the first term, the last term or the common difference, it is necessary to set up two equations in two variables.

Equation 1

$$S_n = \frac{n}{2}[2t_1 + (n-1)d].$$

$$S_3 = \frac{3}{2}[2t_1 + (3-1)d].$$

$$-3 = \frac{3}{2}[2t_1 + 2d].$$

$$-3 = 3t_1 + 3d.$$

Divide every term by 3 to simplify.

$$-1 = t_1 + d.$$

Equation 2

$S_n = \frac{n}{2}[2t_1 + (n-1)d].$

$S_8 = \frac{8}{2}[2t_1 + (8-1)d].$

$132 = 4[2t_1 + 7d].$

$132 = 8t_1 + 28d.$

Given these two equations:

$$132 = 8t_1 + 28d$$

$$-1 = t_1 + d$$

Multiply the second equation by 8 to eliminate t_1.

$$132 = 8t_1 + 28d \qquad 132 = 8t_1 + 28d$$

$$8(-1 = t_1 + d) \qquad \underline{-8 = 8t_1 + 8d}$$

Subtract to eliminate t_1

$140 = 20d$

$d = 7$

Substitute $d = 7$ into either equation to determine the first term.

$-1 = t_1 + d$

$-1 = t_1 + (7)$

$-8 = t_1$

Statement 1

Statement 1 is **false** because $t_1 \neq -5$.

Statement 2

Statement 2 is **true** because $d = 7$.

Statement 3

$S_4 = \frac{4}{2}[2(-8) + (4 - 1)7].$

$S_4 = 2[-16 + 21].$

$S_4 = 2[5].$

$S_4 = 10$

Statement 3 is **false**.

Statement 4

$S_{10} = \frac{10}{2}[2(-8) + (10 - 1)7].$

$S_{10} = 5[-16 + 63].$

$S_{10} = 235$

Statement 4 is **true**.

The correct answer is B.

Use the following information to answer the next question.

The seating chart for a concert hall shows that a section of seats has 10 rows. Tickets in the first section sell for $95 each. Tickets in each consecutive section are $6 cheaper than the tickets in the preceding section. Mary wants to buy 1 ticket from each section to give to charity. She spends $680.

6. The number of rows in this concert hall is _100_.

Solution

We know the first term is 95 and the common difference is -6. We also know that there is a sum of 680, but we do not know the number of terms. Each term represents a section, and each section has 10 rows. Use the formula to solve for n.

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$680 = \frac{n}{2}[2(95) + (n-1) - 6]$.

$680 = \frac{n}{2}[190 - 6n + 6]$.

$680 = \frac{n}{2}[196 - 6n]$.

Clear the fraction by multiplying each side of the equal sign by 2.

1360 = n(196 – 6n)

$1360 = 196n - 6n^2$

$-6n^2 + 196n - 1360 = 0$.

Graph this equation to find the x-intercepts. There are 2 positive x-intercepts, but only x = 10 makes sense in this context.

There are 10 sections and with each section having 10 rows, there are 100 rows.

7. An arithmetic series has $t_{11} = 17$ and $S_{11} = 132$. The first term, t_1, is

A) -7 B) 7 C) 9 D) -129

Solution

Use the formula,

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_{11} = \frac{11}{2}(t_1 + 17)$$

$$132 = \frac{11}{2}(t_1 + 17)$$

Multiply both sides by 2/11.

$24 = t_1 + 17$

$t_1 = 7$

The correct answer is B.

Use the following information to answer the next question.

Suppose you started a business that sold \$18 000 worth of sports injury products during the first year. You have set a goal of increasing annual sales by \$4000 each year for the next 15 years. Assume the goal is met every year.

8. The correct formula to use to calculate the accumulated total of sales **and** the total amount of sales after 16 years is

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   B) $S_{16} = \frac{16}{2}(18\ 000 + 4\ 000)$ **and** a total sales of \$768 000.

   C) $S_{16} = \frac{16}{2}[2(18\ 000) + (15)(4000)]$ **and** a total sales of \$176 000.

   D) $S_{16} = \frac{16}{2}[2(18\ 000) + (15)(4000)]$ **and** a total sales of \$768 000.

Solution

The first two options given above are linked to the sum formula that requires both the first term and the last term. We know the first term (18 000) but not the last. Therefore, the answer has to be either C or D.

For option C, the formula is correct, but the total sales is not correct.

The correct answer is D.