Arithmetic Sequence Practice Questions

Use the following information to answer the first question.

Consider the following sequence:

___, -2, 7, ___, ___, 34, …

The following statements are made.

Statement 1	$t_n = -2 + (n - 1)5$
Statement 2	$t_n = -11 + (n - 1)9$
Statement 3	$t_3 = 16$
Statement 4	$t_7 = 43$

1. The two true statements are

   A) 1 and 3 B) 1 and 4 C) 2 and 3 D) 2 and 4

2. In the arithmetic sequence -18, -10, -2, 6, …, which term has the value of 222?

   A) t_{37} B) t_{21} C) t_{19} D) t_{31}

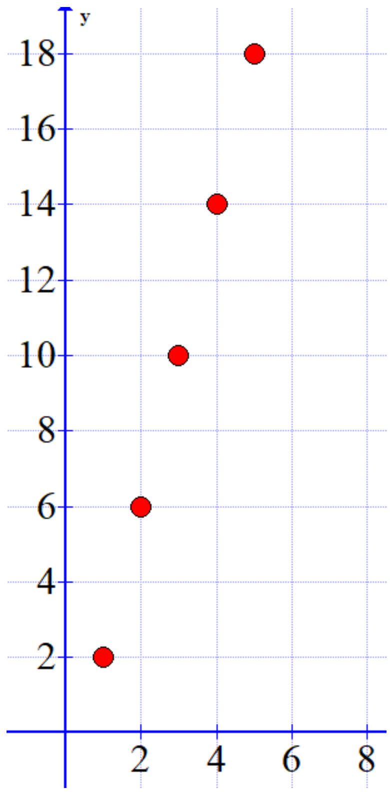
3. Which term below is a term of an arithmetic sequence with $t_9 = 156$ and $t_{17} = 284$?

   A) 411 B) 413 C) 410 D) 412

4. The number of terms in the sequence, $2, \frac{5}{2}, 3, \ldots, \frac{31}{2}$ is _____.

5. In an arithmetic sequence, $t_3 = 16$ and $t_7 = 40$. Determine the common difference and the first term.

6. The graph below shows an arithmetic sequence.



a) Write the general term of this sequence.

b) Determine t_{36}

7. The years in which the Commonwealth Games takes place form an arithmetic sequence with a common difference of 4. In 1978, the Commonwealth Games were held in Edmonton, Alberta. In which of the following years could the Commonwealth Games be held again?

A) 2011 B) 2022 C) 2033 D) 2044

8. A bank of lockers outside a Math 20-1 classroom are numbered 509, 511, 513, …577. Determine the number of lockers in the set.

9. The first 3 terms of an arithmetic sequence are, $x + 7$, $3x$, $4x - 2$, Determine the value of x and state the 3 terms.

Arithmetic Sequence Practice Questions Solutions

Use the following information to answer the first question.

Consider the following sequence:

___, -2, 7, ___, ___, 34, …

The following statements are made.

Statement 1	$t_n = -2 + (n - 1)5$
Statement 2	$t_n = -11 + (n - 1)9$
Statement 3	$t_3 = 16$
Statement 4	$t_7 = 43$

1. The two true statements are

   A) 1 and 3 B) 1 and 4 C) 2 and 3 D) 2 and 4

Solution

We know that the third term (7) is determined by adding 9 to the second term (-2) and therefore the common difference is 9.

By subtracting 9 from -2, we can determine that the first term is -11.

Statement 2, $t_n = -11 + (n - 1)9$, is true because this is the correct general term of this sequence.

$t_3 = -11 + (3 - 1)9$

$t_3 = -11 + 18$

$t_3 = 7$ **Statement 3 is false.**

$t_7 = -11 + (7 - 1)9$

$t_7 = -11 + 54$

$t_7 = 43$ **Statement 4 is true.**

The correct answer is D.

2. In the arithmetic sequence -18, -10, -2, 6, …, which term has the value of 222?

   A) t_{37} B) t_{21} C) t_{19} D) t_{31}

Solution

The common difference is found by taking any term and subtracting the previous term; for example, $t_2 - t_1$

$(-10) - (-18) = 8$

The common difference is 8.

Since $t_1 = -18$, the general term for this sequence is, $t_n = -18 + (n - 1)8$.

Substitute 222 for t_n.

$222 = -18 + (n - 1)8$

$222 = -18 + 8n - 8$

$222 = -26 + 8n$

$248 = 8n$

$n = 31$

222 is the 31st term.

The correct answer is D.

3. Which term below is a term of an arithmetic sequence with $t_9 = 156$ and $t_{17} = 284$?

   A) 411 B) 413 C) 410 D) 412

Solution

$t_{17} = t_1 + (17 - 1)d$ $t_9 = t_1 + (9 - 1)d$

$284 = t_1 + 16d$ $156 = t_1 + 8d$

With two equations in two variables, the equations can be written one beneath the other, and the elimination method can be used.

$284 = t_1 + 16d$

$\underline{156 = t_1 + 8d}$

Subtract the equations to eliminate t_1.

$128 = 8d$

$16 = d$

Substitute this value into either equation to find t_1.

$156 = t_1 + (8)(16)$

$156 = t_1 + 128$

$28 = t_1$

We could use the general term, $t_n = 28 + (n - 1)16$, for each of the given options; or, we could extend the pattern from the highest of the two given terms in the question. Let's do the latter.

$284 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 = 412$.

The correct answer is D.

4. The number of terms in the sequence, $2, \frac{5}{2}, 3, \ldots, \frac{31}{2}$ is _<u>28</u>__.

Solution

The common difference is found by taking any term and subtracting the previous term; for example, $t_2 - t_1$

$\frac{5}{2} - 2 = \frac{1}{2}$

The common difference is $\frac{1}{2}$.

The first term is 2.

Use the general term:

$t_n = 2 + (n - 1)\,\frac{1}{2}$.

Substitute $\frac{31}{2}$ for t_n.

$\frac{31}{2} = 2 + (n - 1)\,\frac{1}{2}$.

Multiply each term by 2 to clear the fraction.

$31 = 4 + (n - 1)$

$31 = 3 + n$

$28 = n$

The number of terms in the sequence is 28.

5. In an arithmetic sequence, $t_3 = 16$ and $t_7 = 40$. Determine the common difference and the first term.

Solution

$t_3 = t_1 + (n - 1)d$	and	$t_7 = t_1 + (n - 1)d$
$16 = t_1 + ((3) - 1)d$	and	$40 = t_1 + ((7) - 1)d$
$16 = t_1 + 2d$	and	$40 = t_1 + 6d$

Line up the two equations above and subtract to eliminate t_1

$40 = t_1 + 6d$

$\underline{16 = t_1 + 2d}$

$24 = 4d$

$d = 6$

Substitute this value of d into either equation to find t_1.

$16 = t_1 + 2d$

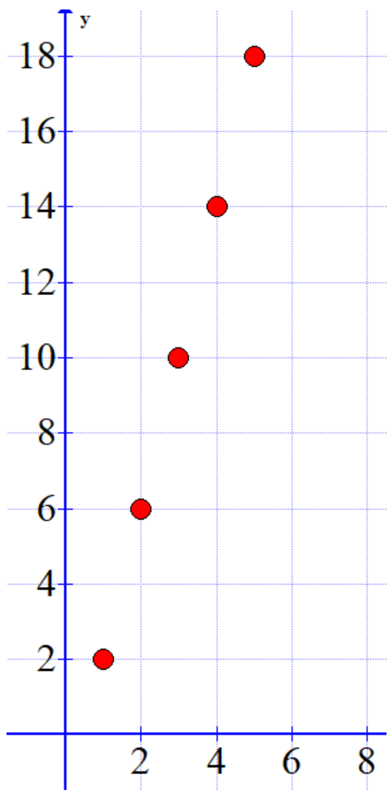
$16 = t_1 + 2(6)$

$16 = t_1 + 12$

$t_1 = 4$

The common difference is 6 and the first term is 4.

6. The graph below shows an arithmetic sequence.



a) Write the general term of this sequence.

$t_n = 2 + (n - 1)4$

b) Determine t_{36}

$t_{36} = 142$

Solution

The term number (t_1, t_2, t_3, …) is represented on the x-axis and the actual value of the term is represented on the y-axis. For example, $t_1 = 2$; $t_2 = 6$, and so on. The sequence could also be shown as (2, 6, 10, 14, 18).

a) With a first term of 2 and a common difference of 4, the general term is:

$t_n = 2 + (n - 1)4$

b) $t_{36} = 2 + (36 - 1)4$

$t_{36} = 2 + 140$

$t_{36} = 142$

7. The years in which the Commonwealth Games takes place form an arithmetic sequence with a common difference of 4. In 1978, the Commonwealth Games were held in Edmonton, Alberta. In which of the following years could the Commonwealth Games be held again?

A) 2011 B) 2022 C) 2033 D) 2044

Solution

The first term in 1978 and the common difference is 4.

The general term is $t_n = 1978 + (n - 1)4$

Consider each option. We know that n must be an integer.

<u>If $t_n = 2011$</u>

$2011 = 1978 + (n - 1)4$

$2011 = 1978 + 4n - 4$

$2011 = 4n + 1974$

$37 = 4n$

Since n is not an integer, option A is not correct.

<u>If $t_n = 2022$</u>

$2022 = 1978 + (n - 1)4$

$2022 = 1978 + 4n - 4$

$2022 = 1974 + 4n$

$48 = 4n$

$n = 12$

Since n is an integer, option B is correct.

<u>If $t_n = 2033$</u>

$2033 = 1978 + (n - 1)4$

$2033 = 1978 + 4n - 4$

$2033 = 4n + 1974$

$59 = 4n$

Since n is not an integer, option C is not correct.

<u>If $t_n = 2044$</u>

$2044 = 1978 + (n - 1)4$

$2044 = 1978 + 4n - 4$

$2044 = 4n + 1974$

$70 = 4n$

Since n is not an integer, option D is not correct.

The correct answer is B.

8. A bank of lockers outside a Math 20-1 classroom are numbered 509, 511, 513, …577. Determine the number of lockers in the set.

Solution

The first term is 509 and the common difference is 2.

The general term is $t_n = 509 + (n - 1)2$

Substitute 577 for t_n.

$577 = 509 + (n - 1)2$

$577 = 509 + 2n - 2$

$577 = 507 + 2n$

$70 = 2n$

$n = 35$

There are 35 lockers in the set.

9. The first 3 terms of an arithmetic sequence are, $x + 7, 3x, 4x - 2$, Determine the value of x and state the 3 terms.

Solution

The common difference is found by taking any term and then subtracting the previous term from it.

$t_2 - t_1 = t_3 - t_2$

$(3x) - (x + 7) = (4x - 2) - (3x)$

$2x - 7 = x - 2$

$x = 5$

The value of x is 5 and the three terms are 12, 15, 18.