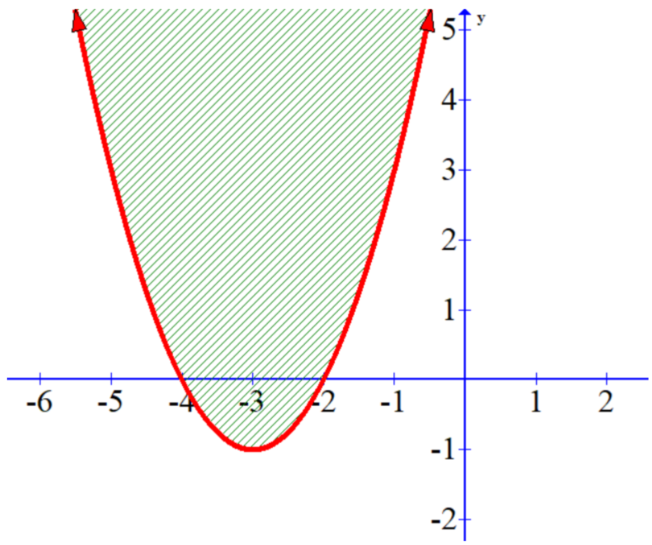
<u>Quadratic Inequalities in Two Variables Practice</u>

Use the following information to answer the first question.

Given the graph below,



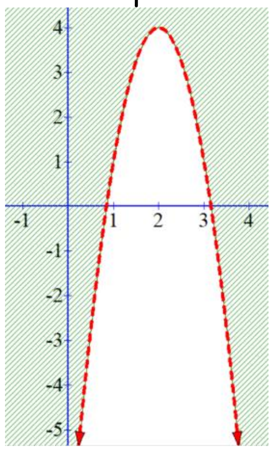
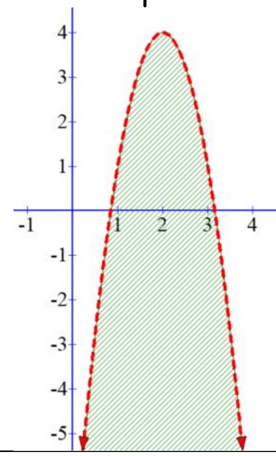
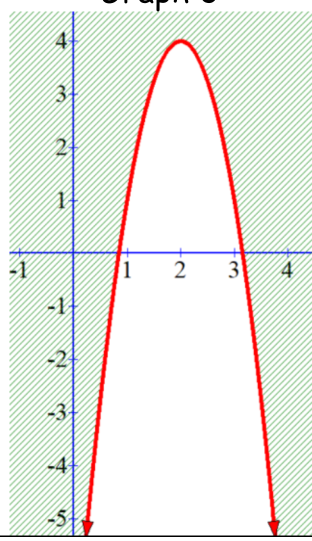
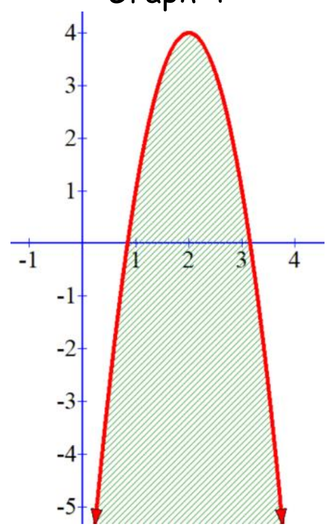
consider the following statements:

Statement 1	A possible inequality to represent this graph is $y \geq (x + 3)^2 - 1$.
Statement 2	A possible inequality to represent this graph is $y < (x + 3)^2 - 1$.
Statement 3	One solution is (-1,1).
Statement 4	One solution is (-2,4).

1. The two true statements are

   A) 1 and 4 B) 2 and 3 C) 1 and 3 D) 2 and 4

2. The graph of $y - 4 < -3(x - 2)^2$ is

Graph 1	Graph 2
	
Graph 3	**Graph 4**
	

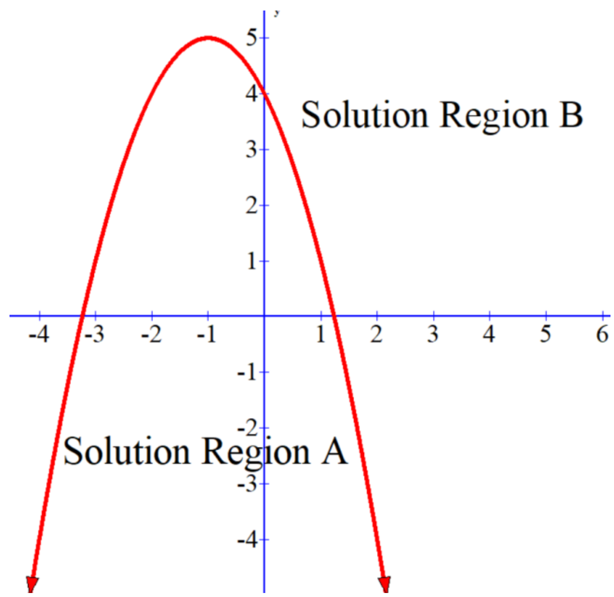
A) 1 B) 2 C) 3 D) 4

3. One solution to the inequality $y + 6 > x^2 + 5x$ is (1, y). A possible value for y is

A) 0 B) 2 C) -4 D) -6

Use the following information to answer the next question.

James was asked to show the solution region for the inequality $y \geq -x^2 - 2x + 4$. He began by drawing $y = -x^2 - 2x + 4$, as shown below.



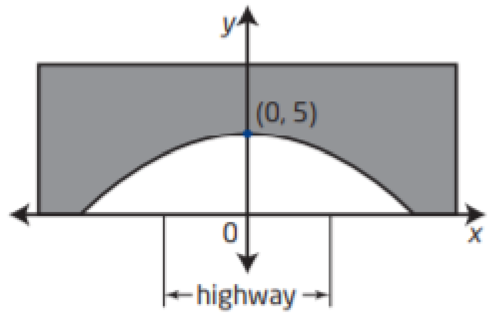
He identified Solution Region A as all the ordered pairs under the parabola, and Solution Region B as all the ordered pairs above the parabola. He picked the test point (0,0) to help him determine which Region to shade.

4. Which of the following statements below is correct?
   A) Since (0,0) satisfies the inequality, shade above the parabola.
   B) Since (0,0) satisfies the inequality, shade below the parabola.
   C) Since (0,0) does not satisfy the inequality, shade above the parabola.
   D) Since (0,0) does not satisfy the inequality, shade below the parabola.

5. One solution of the inequality, $y \leq (x - 3)^2 + k$, that lies **on** the boundary parabola is (5,9). The vertex of this parabola is _____.

Use the following information to answer the next question.

A highway goes under a bridge formed by a parabolic arch, as shown. The highest point of the arch is 5 m high. The road is 10 m wide, and the minimum height of the bridge over the road is 4 m.

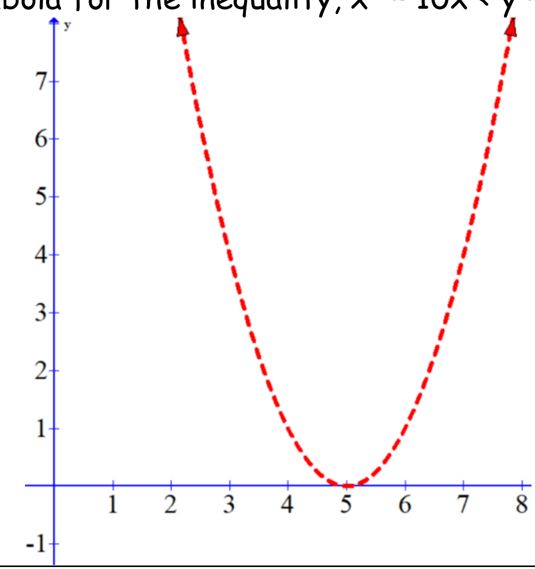


6. a) Determine the quadratic function that models the parabolic arch of the bridge.

   b) What is the inequality that represents the space under the bridge in quadrants 1 and 2?

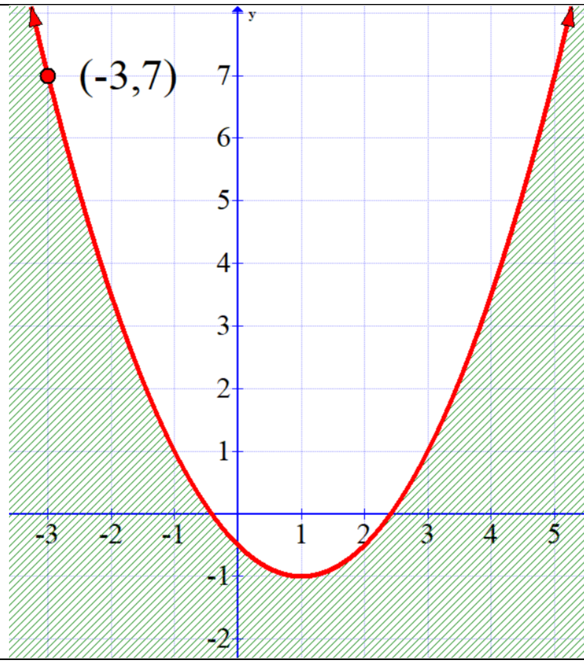
Use the following information to answer the next question.

The boundary parabola for the inequality, $x^2 - 10x < y - 25$, is shown below.



7. Should the shading be above or below the parabola? Explain.

Use the following information to answer the next question.



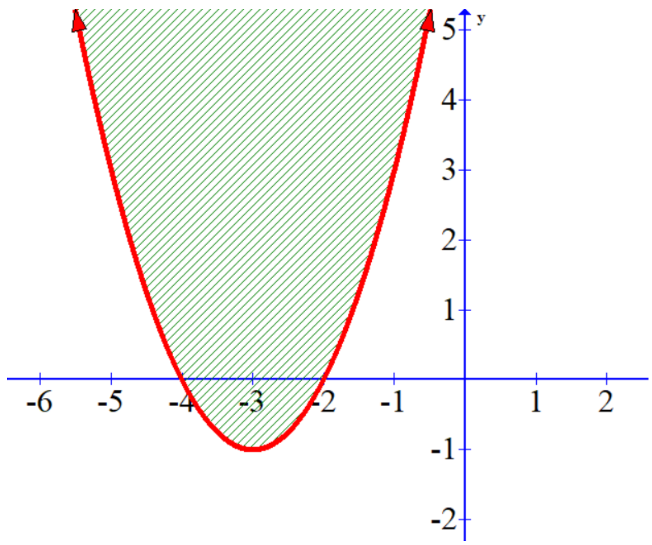
8. Determine the inequality associated with the graph above.

9. A manila rope used for rappelling down a cliff can safely support a weight (W) in pounds provided $W \leq 1375d^2$, where d is the diameter (in inches) of the rope. Graph the inequality and interpret the solution.

Quadratic Inequalities in Two Variables Practice Solutions

Use the following information to answer the first question.

Given the graph below,



consider the following statements:

Statement 1	A possible inequality to represent this graph is $y \geq (x + 3)^2 - 1$.
Statement 2	A possible inequality to represent this graph is $y < (x + 3)^2 - 1$.
Statement 3	One solution is (-1,1).
Statement 4	One solution is (-2,4).

1. The two true statements are

A) 1 and 4 B) 2 and 3 C) 1 and 3 D) 2 and 4

Solution

Statement 1

Select a point in the solution region, for example (-3,0).

Use the inequality given in statement 1 to determine if a true statement is made.

$y \geq (x + 3)^2 - 1$

$0 \geq ((-3) + 3)^2 - 1$

$0 \geq -1$

Since this point satisfies the inequality, Statement 1 is **true.**

Statement 2

Based on the work shown above for Statement 1, the inequality sign is not correct. Statement 2 is **false.**

Statement 3

Substitute the point (-1,1) in the inequality, $y \geq (x + 3)^2 - 1$

$1 \geq ((-1) + 3)^2 - 1$

$1 \geq 4 - 1$

Since 1 is not greater than 3, Statement 3 is **false.**

Statement 4

Substitute the point (-2,4) in the inequality, $y \geq (x + 3)^2 - 1$

$4 \geq ((-2) + 3)^2 - 1$

$4 \geq (1)^2 - 1$

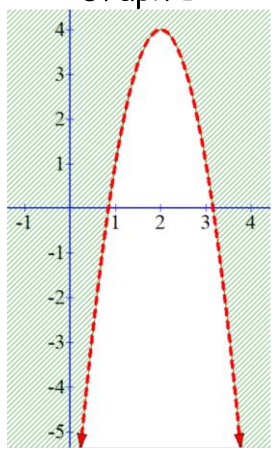
$4 \geq 0.$

Since a true statement is made, (-2,4) is a solution. Statement 4 is **true**.

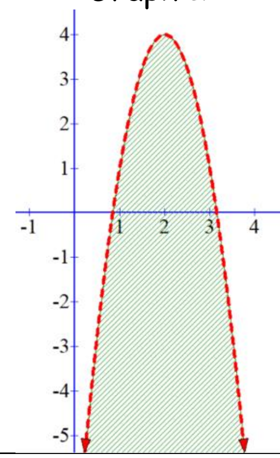
The correct answer is A.

2. The graph of $y - 4 < -3(x - 2)^2$ is

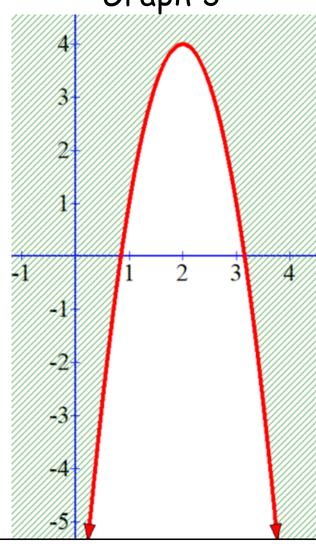
Graph 1



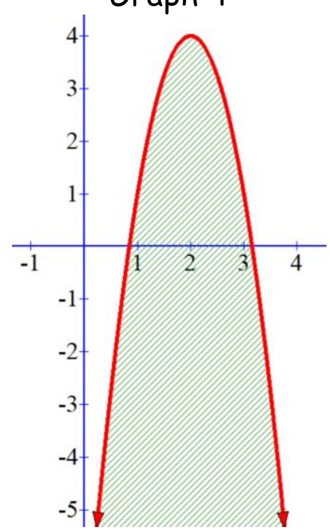
Graph 2



Graph 3



Graph 4



A) 1 B) 2 C) 3 D) 4

Solution

Isolate 'y' in $y - 4 < -3(x - 2)^2$.

$$y < -3(x - 2)^2 + 4$$

All 4 graphs display a parabola opening down and having a vertex of (2,4). Since the inequality sign is <, the boundary parabola should be dotted. As such, our choices are reduced to either Graph 1 or Graph 2.

Select a test point, for example (0,0), and determine which inequality sign will make a true statement.

<u>Left Side</u>		<u>Right Side</u>
0		$-3(0 - 2)^2 + 4$
0		$-3(4) + 4$
0		-8
0	$>$	-8

Since the inequality in this question is the opposite sign (<), the correct region to shade would be under the parabola.

The correct answer is B.

3. One solution to the inequality $y + 6 > x^2 + 5x$ is (1, y). A possible value for y is

A) 0 B) 2 C) -4 D) -6

Solution

Test each y coordinate to see if a true statement is made.

<u>Test the point (1,0)</u>

$(0) + 6 > (1)^2 + 5(1)$

$6 > 6$

Since 6 is not greater than 6, a y coordinate of 0 is not possible.

<u>Test the point (1,2)</u>

$(2) + 6 > (1)^2 + 5(1)$

$8 > 6$

Since 8 is greater than 6, a y coordinate of 2 is possible.

Test the point (1,-4)

$(-4) + 6 > (1)^2 + 5(1)$

$2 > 6$

Since 2 is not greater than 6, a y coordinate of -4 is not possible.

Test the point (1,-6)

$(-6) + 6 > (1)^2 + 5(1)$

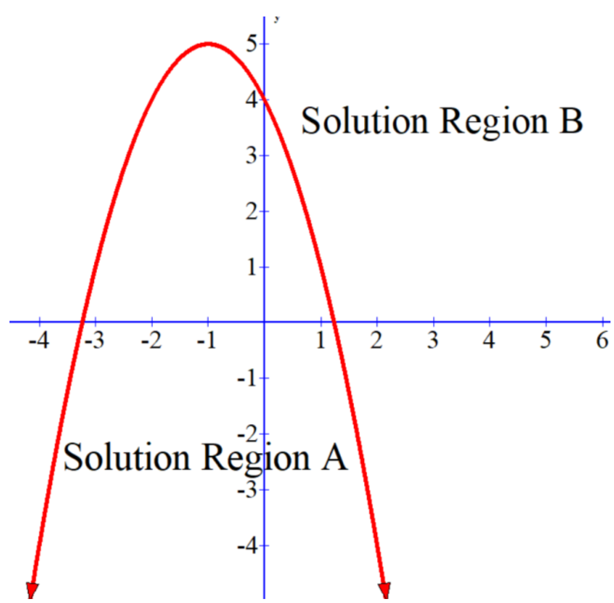
$0 > 6$

Since 0 is not greater than 6, a y coordinate of -6 is not possible.

The correct answer is B.

Use the following information to answer the next question.

James was asked to show the solution region for the inequality $y \geq -x^2 - 2x + 4$. He began by drawing $y = -x^2 - 2x + 4$, as shown below.



He identified Solution Region A as all the ordered pairs under the parabola, and Solution Region B as all the ordered pairs above the parabola. He picked the test point (0,0) to help him determine which Region to shade.

4. Which of the following statements below is correct?
   A) Since (0,0) satisfies the inequality, shade above the parabola.
   B) Since (0,0) satisfies the inequality, shade below the parabola.
   C) Since (0,0) does not satisfy the inequality, shade above the parabola.
   D) Since (0,0) does not satisfy the inequality, shade below the parabola.

Solution

$0 \geq -(0)^2 - 2(0) + 4$

$0 \geq 4$

Since this statement is not correct, (0,0) does not satisfy the inequality.

The test point (0,0) is in Solution Region A. Therefore, the correct shading should be in Solution Region B, which is above the parabola.

The correct answer is C.

5. One solution of the inequality, $y \leq (x - 3)^2 + k$, that lies **on** the boundary parabola is (5,9). The vertex of this parabola is _(3,5)__.

Solution

Since we have a point that lies on the boundary parabola, we know that (5,9) satisfies the equation, $y = (x - 3)^2 + k$. This fact can be used to determine the value of k. Substitute this point into the equation.

$(9) = ((5) - 3)^2 + k$

$9 = (2)^2 + k$

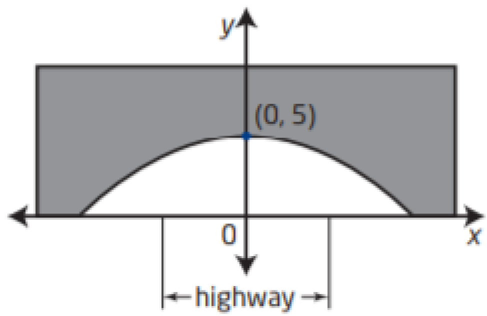
$9 = 4 + k$

$k = 5$

The vertex of this parabola is _(3,5)__.

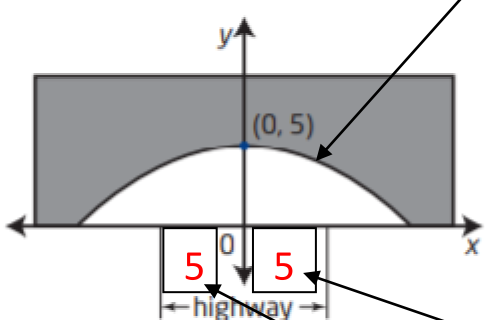
Use the following information to answer the next question.

A highway goes under a bridge formed by a parabolic arch, as shown. The highest point of the arch is 5 m high. The road is 10 m wide, and the minimum height of the bridge over the road is 4 m.



6. a) Determine the quadratic function that models the parabolic arch of the bridge.

Solution



Since the minimum height of the bridge over the road is 4 m, a point on the parabola is (5,4).

With the width of the road being 10 m, there is 5 m to the left and right of the zero point on the x-axis.

With the parabola opening down, and having a maximum value of 5 on the y-axis, the equation can be written in the form, $y = -ax^2 + 5$

Use the point on the parabola, (5,4), to find the value of a.

$4 = a(5)^2 + 5$

$-1 = 25a$

$a = -0.04$

The quadratic function that models the parabolic arch of the bridge is

$y = -0.04x^2 + 5$.

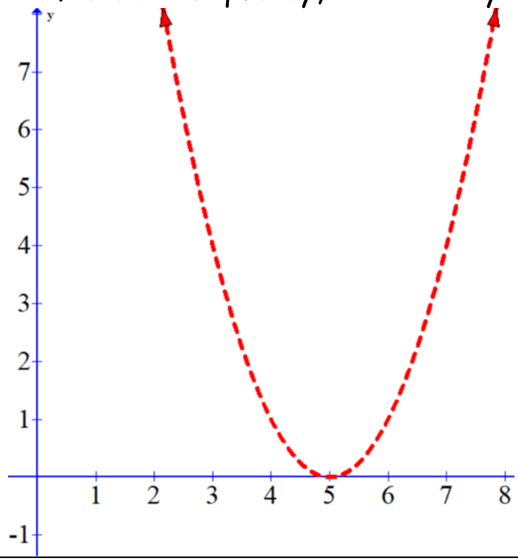
b) What is the inequality that represents the space under the bridge in quadrants 1 and 2?

Solution

$y \le -0.04x^2 + 5$.

Use the following information to answer the next question.

The boundary parabola for the inequality, $x^2 - 10x < y - 25$, is shown below.



7. Should the shading be above or below the parabola? Explain.

Solution

Given $x^2 - 10x < y - 25$, add 25 to both sides of the inequality sign to isolate y.

$x^2 - 10x + 25 < y$

An equivalent form is:

$y > x^2 - 10x + 25$

In this form, when the symbol is either > or ≥, shade the region above the parabola.

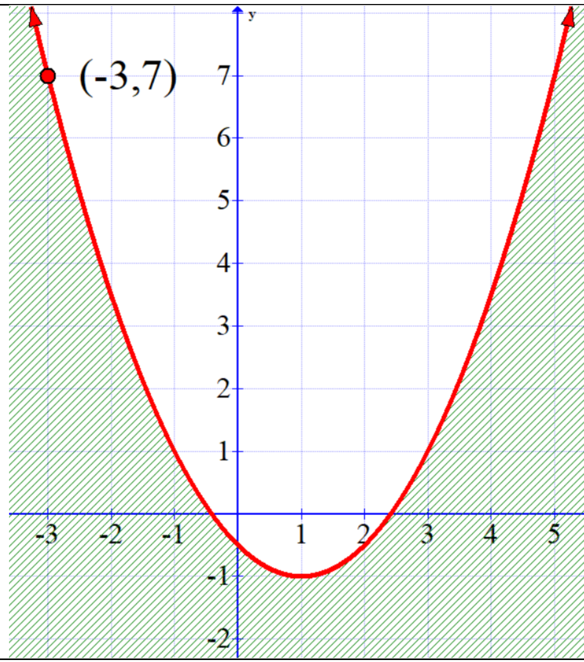
Or, we can take a test point to determine the region to shade. Select the test point (0,0).

$0 > (0)^2 - 10(0) + 25$

$0 > 25$

Since this statement is not true, and this test point is in the region below the parabola, it is then necessary to shade the region above the parabola (also known as the region within the boundary of the parabola).

Use the following information to answer the next question.



8. Determine the inequality associated with the graph above.

Solution

The vertex is (1, -1).

The vertex form of the equation is $y = a(x - 1)^2 - 1$

Use the given point (-3,7) to determine the value of a.

$7 = a((-3) - 1)^2 - 1$

$8 = 16a$

$\frac{1}{2} = a$

The equation of the boundary parabola is $y = \frac{1}{2}(x - 1)^2 - 1$.

Pick a test point to determine the correct inequality sign.

Choose (0,0).

Left Side	Right Side
0	$\frac{1}{2}(0 - 1)^2 - 1$
0	$\frac{1}{2}(1) - 1$
0	$\frac{1}{2} - 1$
0	$-\frac{1}{2}$

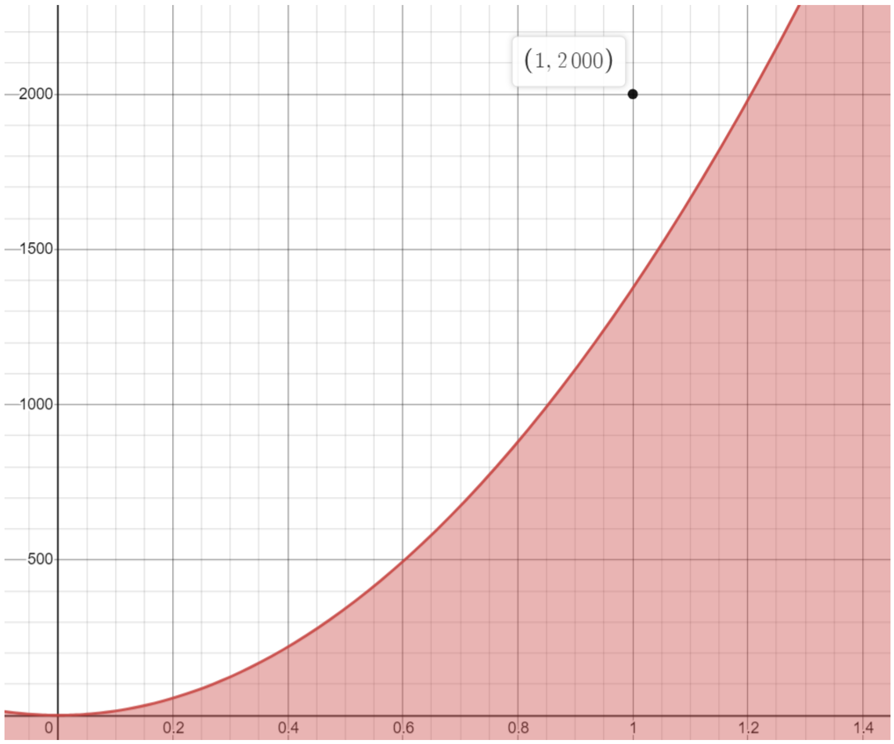
Since $0 \geq -\frac{1}{2}$ and the solution region in the question is not in the test point region, the correct inequality is $\leq$.

The inequality associated with this graph is $y \leq \frac{1}{2}(x - 1)^2 - 1$.

9. A manila rope used for rappelling down a cliff can safely support a weight (W) in pounds provided $W \leq 1375d^2$, where d is the diameter (in inches) of the rope. Graph the inequality and interpret the solution.

Solution

Graph $W = 1375d^2$ for non-negative values of d. Because the inequality symbol is $\leq$, make the boundary parabola solid. Pick a test point inside the parabola, such as (1, 2000).



W ≤ $1375d^2$

(2000) ≤ $1375(1)^2$

2000 ≰ 1375

Since (1, 2000) is not a solution, shade the region outside the parabola.

The shaded region represents weights that can be supported by ropes with various diameters.