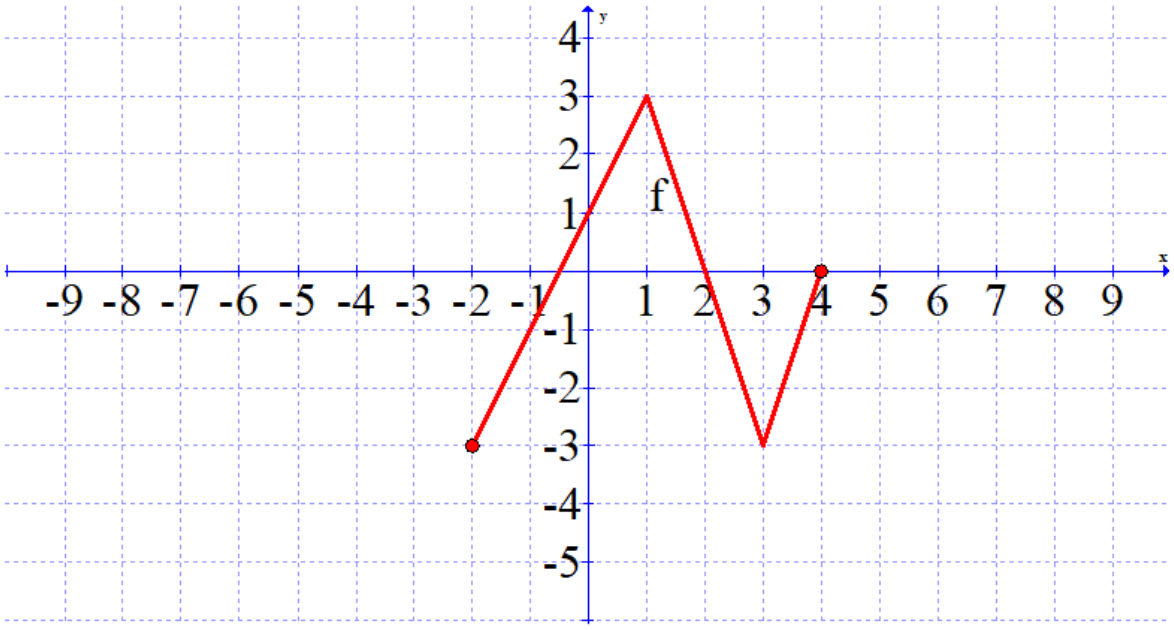
Math 30-1 Released Questions Below 50%

Released 2014

1. The graph of y = f(x) is shown below.



When the graph of y = f(x) is reflected in the line y = x, the number of invariant points is

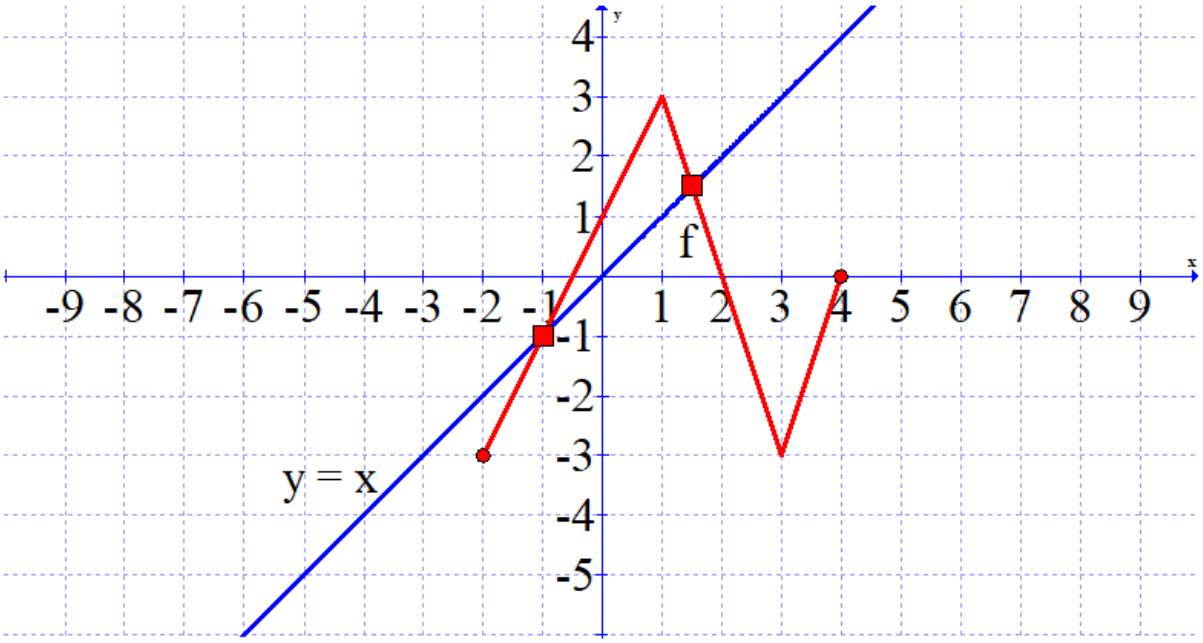
a) 1 b) 2 c) 3 d) 4

Answer

The standard is acceptable and the cognitive level is conceptual.

The correct answer is (b), or 2 invariant points. The graph below shows the 2 points of intersection.

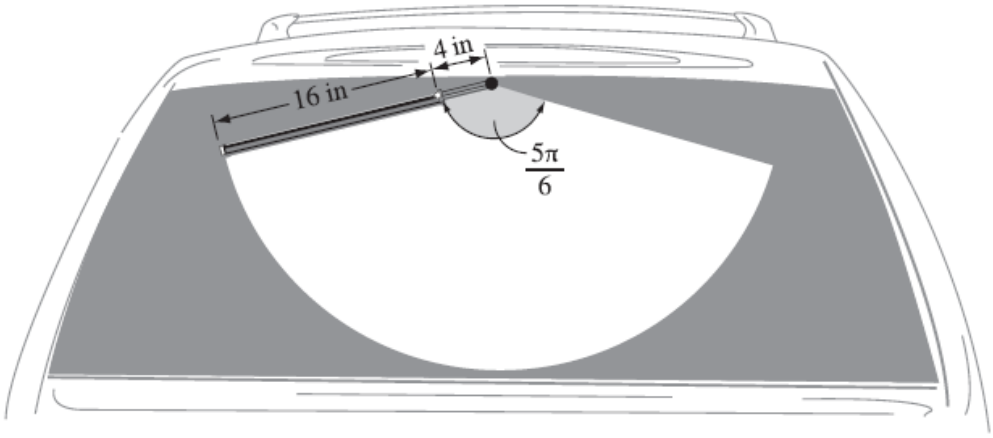
The proportion of students answering this question correctly was **47.6%**.



What are possible reasons students struggled with this question?

- The meaning of the word 'invariant' was unclear.
- Invariant points on any reflection will lie on the reflection line. They never thought to draw the reflection line and look for intersection points.
- Students may have had more practice and confidence for invariant points on reflections in the x and y axes.
- Students may have only been looking for integer coordinates.

The wiper on the rear window of a particular car moves through an angle of $\frac{5\pi}{6}$ rad in a single sweep and clears a region that is 16 in wide, as shown in the diagram below.



2. The total perimeter of the cleared region, to the nearest inch, is

a) 52 b) 63 c) 84 d) 95

Answer

The standard is excellence and the cognitive level is problem solving.

The length of the smaller arc is, $\frac{5\pi}{6} = \frac{arc}{4}$

$= \quad arc = \frac{(5\pi)(4)}{6}$

$= \quad 10.5$ in

The length of the larger arc is, $\frac{5\pi}{6} = \frac{arc}{20}$

$$= \quad \text{arc} = \frac{(5\pi)(20)}{6}$$

$$= \quad 52.4 \text{ in}$$

The perimeter is 16 + 16 + 10.5 + 52.4 = 94.9 or 95 in.

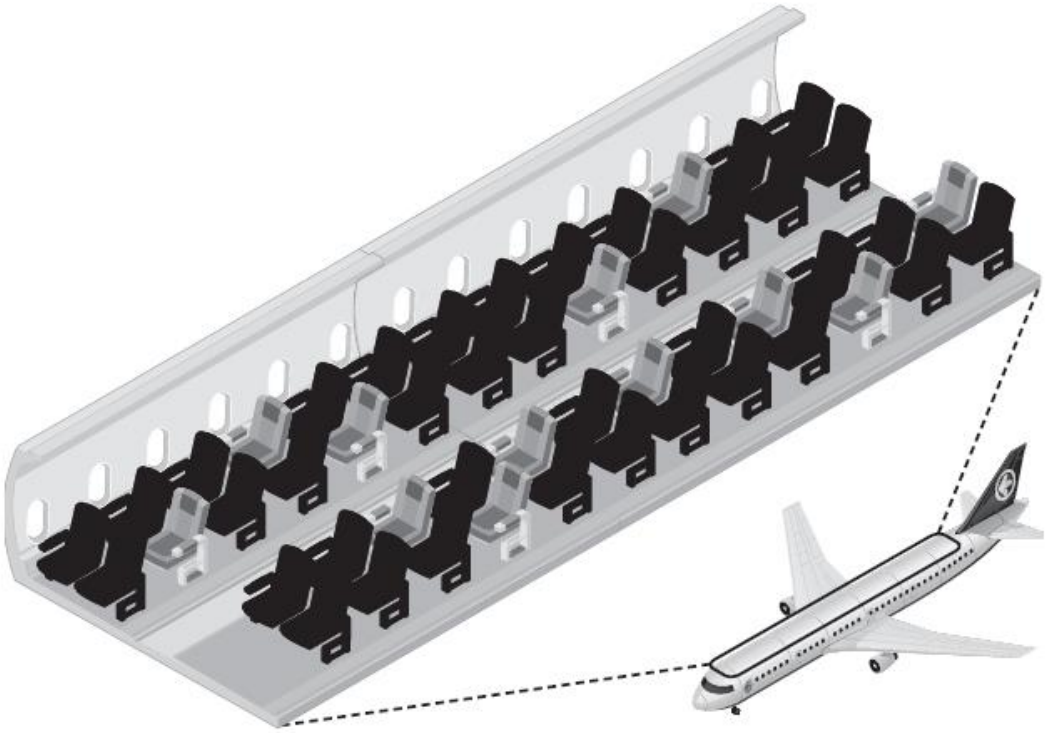
The answer is (d).

The proportion of students answering this question correctly was **21.2%**.

What are possible reasons students struggled with this question?

- Students may have thought that the sides of 4 be included in the perimeter.
- Unfamiliarity of questions requiring the use of finding arc length twice in one question.
- Lack of clarity with the exact composition of the cleared region.
- Not fully understanding how to apply the formula for the definition of a radian.

The diagram below shows a website seating chart for an aircraft. The seats shaded black have been reserved and the lighter-shaded seats are available. There are 12 seats available at this time.



3. If 7 customers are each booking 1 seat, then the number of different ways that they could be assigned a seat is

a) $7!$ b) $\frac{12!}{7!}$ c) ${}_{12}P_7$ d) ${}_{12}C_7$

Answer

The standard is acceptable and the cognitive level is conceptual.

Using the fundamental counting principle, the 7 customers can be thought of as "stages" and the seats will determine the number of "choices" for each person.

12 X 11 X 10 X 9 X 8 X 7 X 6

A randomly selected first customer will have his/her choice of 12 seats. The next customer will only have 11 potential seat choices, and the next customer will only have 10 available seats, etc.

Thinking in terms of permutations, there are 12 total objects (seats), from which 7 people can be arranged or ordered. Thus, it would be ${}_{12}P_7$.

The correct answer is (c).

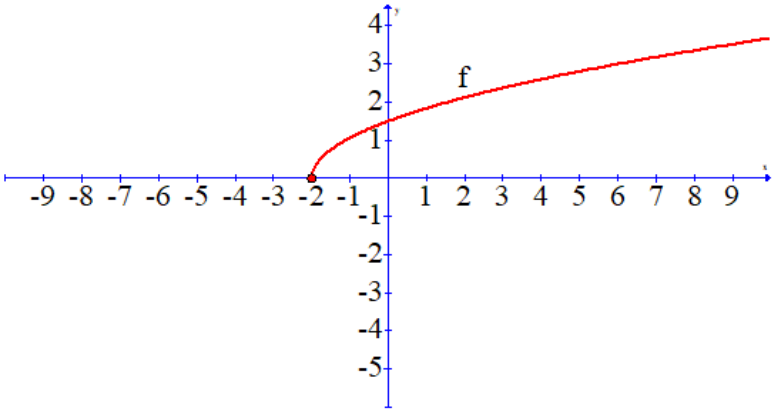
The proportion of students answering this question correctly was **38.4%**.

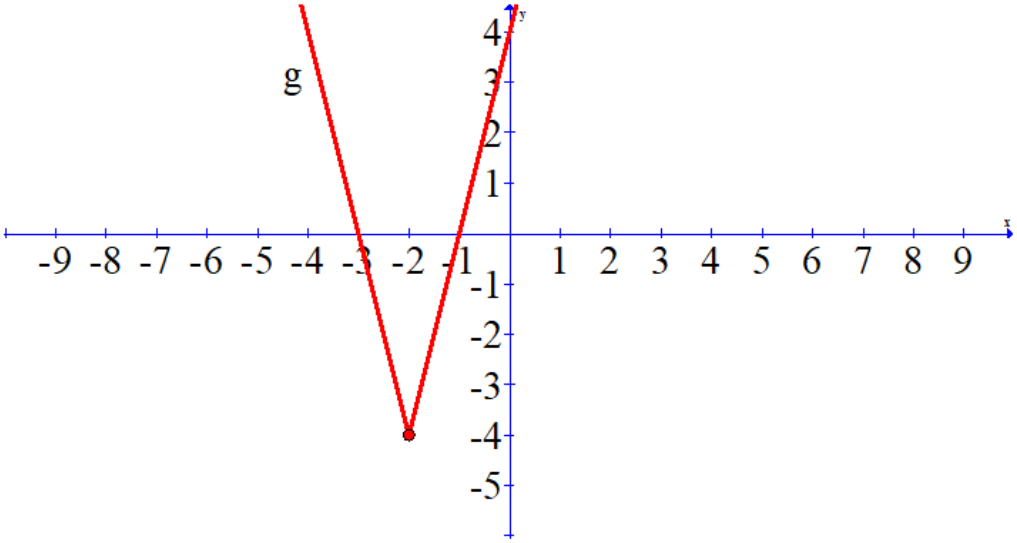
What are possible reasons students struggled with this question?

- Lacking clear understanding between the difference of a permutation and a combination.
- Difficulty with the differentiation between the "stages" and the "choices."
- Since arranging 7 people standing in a straight line is 7!, the fact that 12 is greater than 7 is just perceived as unnecessary excess.

Released 2016

4. The graphs of the functions y = f(x) and y = g(x) are shown below. Each function is then horizontally stretched by a factor of 2 about the y-axis, resulting in the new functions y = p(x) and y = q(x) respectively. The domain and the range of each of the new functions are found in the table below.





Reference #	Possible Domain
1	$x \geq -1$
2	$x \geq -2$
3	$x \geq -4$
4	$x \in R$

Reference #	Possible Range
5	$y \geq 0$
6	$y \geq -2$
7	$y \geq -4$
8	$y \geq -8$
9	$y \in R$

Complete the statements below.

The domain and the range of the new function _____ and _____

y = p(x) are numbered, respectively, Record in the **First** column Record in the **Second** column

The domain and the range of the new function _____ and _____

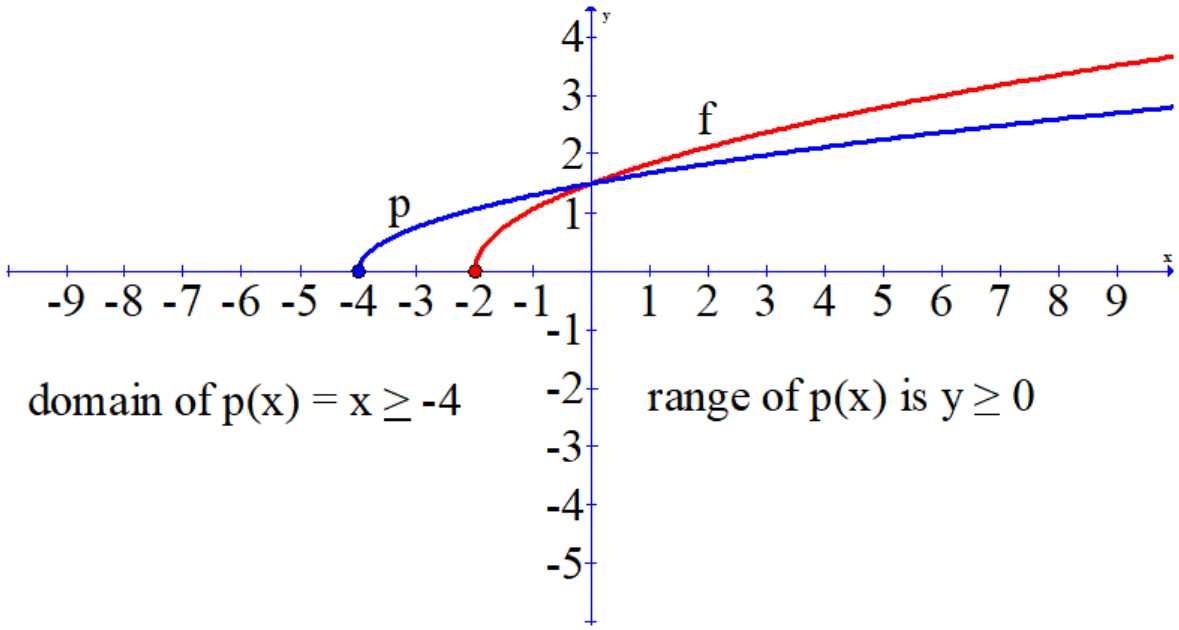
y = q(x) are numbered, respectively, Record in the **Third** column Record in the **Fourth** column

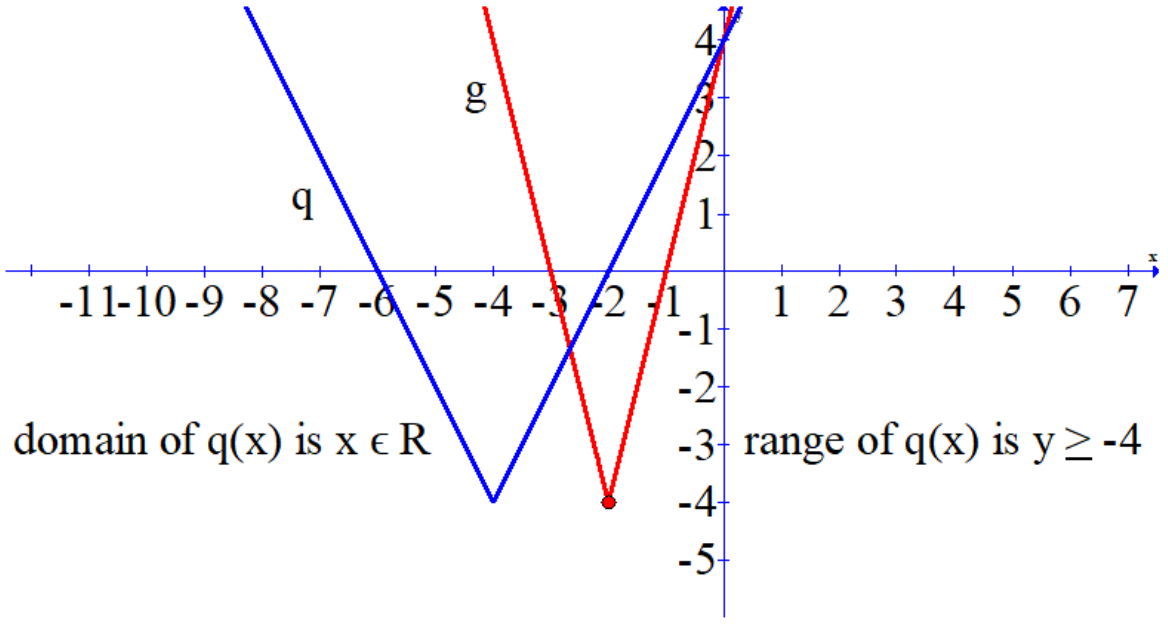
Answer

The standard is acceptable and the cognitive level is conceptual.

The correct numerical response is 3547,

The proportion of students answering this question correctly was **43.4%**.


f
p
domain of p(x) = x ≥ -4
range of p(x) is y ≥ 0


g
q
domain of q(x) is x ϵ R
range of q(x) is y ≥ -4

The domain and the range of the new function y = p(x) are numbered, respectively,	3	and	5
	Record in the **First** column		Record in the **Second** column
The domain and the range of the new function y = q(x) are numbered, respectively,	4	and	7
	Record in the **Third** column		Record in the **Fourth** column

What are possible reasons students struggled with this question?

- Information overload; students not used to analyzing the transformation of 2 different functions, as well as determining their new domains and ranges.
- Unable to accurately sketch the result of a single stretch applied to a graph.
- Not connecting the ability to state domain and range to an accurate visual representation.

Use the following information to answer the next question

The point A(3,-5) lies on the graph of the function y = f(x). The ordered pairs below represent possible coordinates of the new point corresponding to point A after f(x) undergoes a single transformation.

Point 1 (3,5)

Point 2 (-3,5)

Point 3 (-3,-5)

Point 4 (-5,-3)

Point 5 (-5,3)

Point 6 (5,-3)

5. The corresponding point when the graph of y = f(x) is reflected about the line,
   x = y is Point _____ (Record in the first column)
   x = 0 is Point _____ (Record in the second column)
   y = 0 is Point _____ (Record in the third column)

Answer

The standard is acceptable and the cognitive level is conceptual.

The answer is 531.

The proportion of students answering this question correctly was **35.5%**.

What are possible reasons students struggled with this question?

- The more common terminology, [reflection in the x-axis or reflection in the y-axis] was incorrectly translated when using the terms, x = 0 and y = 0, to describe the reflection lines.
- The connection between a reflection and the coordinate (the x or y) it affects was not totally understood.

Use the following information to answer the next question

A student is asked to solve the equation $\frac{125^{x(x+1)}}{5^{(3x-4)}} = 25^{(x-5)}$ using an algebraic process. She is able to simplify the equation to $3x^2 + bx + c = 0$

6. The value of c is

a) 6 b) 9 c) 14 d) 40

Answer

The standard is acceptable and the cognitive level is problem solving.

The answer is (c).

The proportion of students answering this question correctly was **33.1%**.

$\frac{125^{x(x+1)}}{5^{(3x-4)}} = 25^{(x-5)}$

$\frac{(5^3)^{x(x+1)}}{5^{(3x-4)}} = (5^2)^{(x-5)}$

$\frac{5^{3x^2+3x}}{5^{3x-4}} = 5^{2x-10}$

$5^{3x^2+4} = 5^{2x-10}$

Since the bases are equal, the exponents must be equal.

$3x^2 + 4 = 2x - 10$

$= \quad 3x^2 - 2x + 14 = 0$

The value of c is 14.

What are possible reasons students struggled with this question?

- Unable to equate the bases.
- A lack of complete understanding and execution of the exponent laws (not knowing when to add, multiply or subtract exponents).
- No previous experience of dealing with a quadratic in the exponential position.

Use the following information to answer the next question

The loudness of a sound is related to the logarithm of the ratio of the measured intensity, I, to a reference intensity, I_o. The loudness, L, of a sound measured in decibels, dB, and can be determined using the following formula.

$$L = 10\log_{10}\left(\frac{I}{I_o}\right)$$

During an international soccer tournament in 2010, a noisemaker called a vuvuzela had a measured loudness of 127 dB at full volume.

7. If the intensity of the sound of the vuvuzela is 5000 times greater than the intensity of the sound of a lawn mower, then the measured soundness of the lawn mower, to the nearest decibel, is

a) 3 b) 37 c) 90 d) 123

Answer

The standard is excellence and the cognitive level is problem solving.

The answer is (c).

The proportion of students answering this question correctly was **32.8%**.

Since the logarithmic scale is in bels, convert decibels to bels as the exponent for the power of 10.

$\frac{10^{12.7}}{10^{x}}$ = 5000

$10^{12.7 - x}$ = 5000

(12.7 – x) log10 = log5000

12.7 – x = log5000

12.7 – log5000 = x

x = 9 bels.

Converting back to decibels, the answer is 90 decibels.

What are possible reasons students struggled with this question?

- Thinking that the logarithmic scale is in decibels.
- Not knowing how to properly set up the powers of 10 when comparing intensities of loudness.
- Unable to solve an exponential equation using logarithms.

8. For the polynomial function $P(x) = 2x^5 + 3x^4 - 10x^3 - 21x^2 + kx$, two of the zeros are -1 and -2. It can be determined that the largest zero of this function, to the nearest tenth, is _____.

Answer

The standard is acceptable and the cognitive level is problem solving.

The answer is 2.5.

The proportion of students answering this question correctly was **47.0%**.

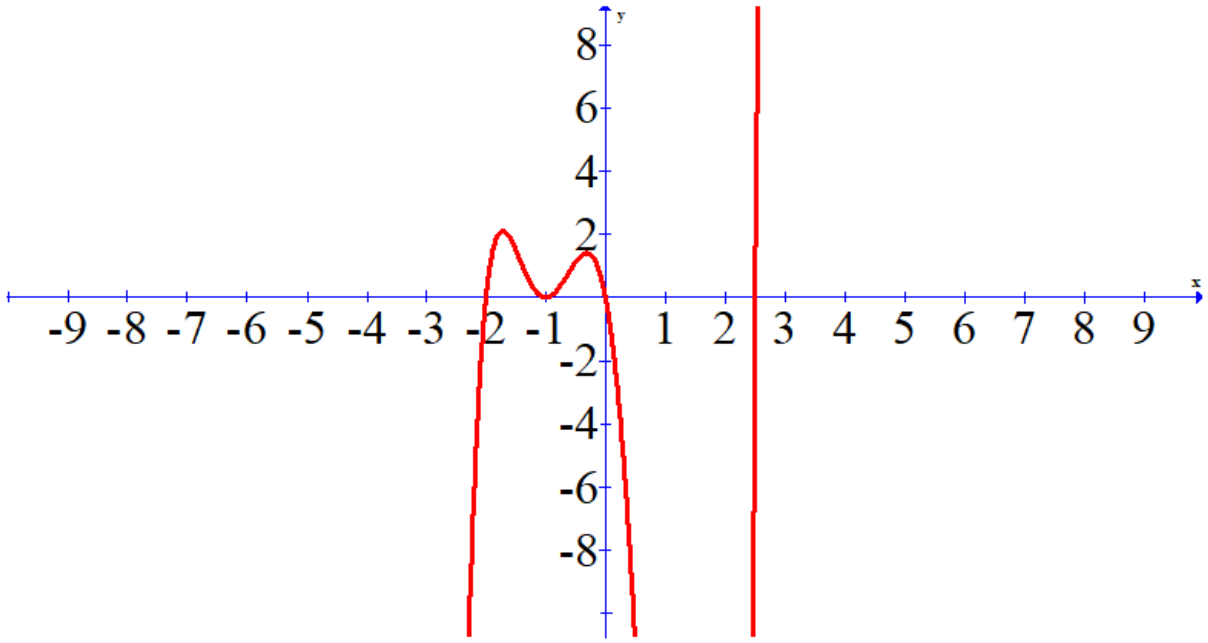
Substitute the value of -1 or -2 into the equation to solve for k.

$2(-1)^5 + 3(-1)^4 - 10(-1)^3 - 21(-1)^2 + K(-1) = 0$

$-2 + 3 + 10 - 21 - K = 0.$

$-10 = K$

A partial graph of P(x) is shown below. The zeros can be seen as the x-intercepts. The largest zero is the one furthest to the right on the x-axis, which is 2.5.

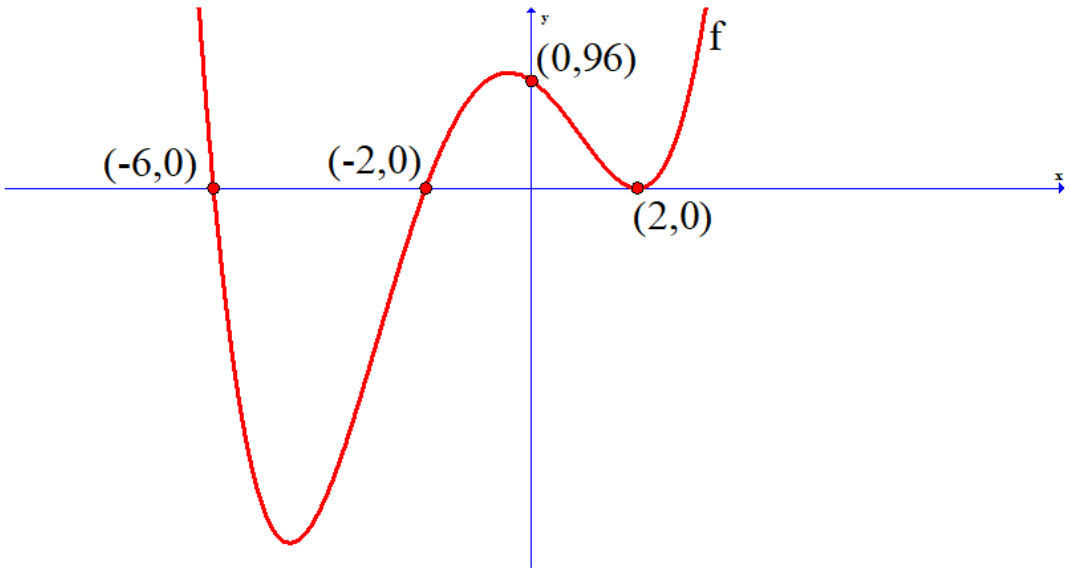


What are possible reasons students struggled with this question?

- Unable to solve the value for k; either with an incorrect substitution, or a manual execution problem.
- Instead of graphing, continuing to find the factors using synthetic division. Since this would require multiple steps, there is a greater chance of error.

Use the following information to answer the next question.

The graph of a particular function is shown below.



9. If the equation of the function graphed above is written in the form $f(x) = ax^4 + bx^3 + cx + d$, then the value of a is

a) 1 b) 2 c) 4 d) 96

Answer

The standard is acceptable and the cognitive level is problem solving.

The answer is (b).

The proportion of students answering this question correctly was **42.2%**.

In factored form, the equation of the graph is $f(x) = a(x + 6)(x + 2)(x - 2)^2$

Substituting the point (0, 96) into the equation, enables us to determine the value of a.

$96 = a((0) + 6)((0) + 2)((0) - 2)^2$

$96 = a(6)(2)(4)$

$96 = 48a$

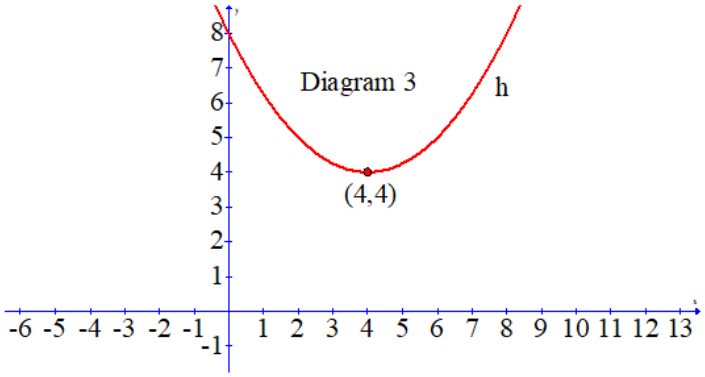
$a = 2$

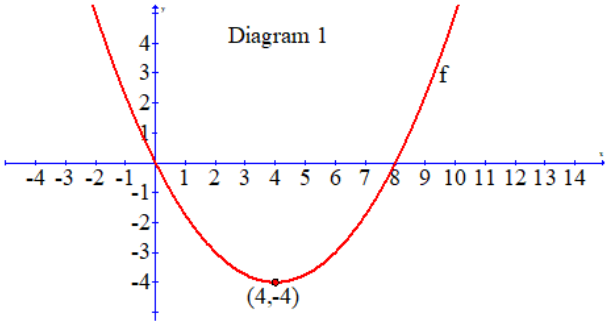
What are possible reasons students struggled with this question?

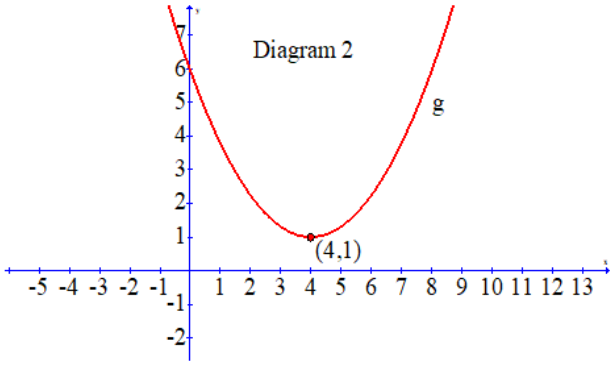
- Difficulty in writing the equation of a graph in factored form.
- Not understanding that to find a missing value in an equation, substituting any point on the graph into the equation will enable the value to be determined.

Use the following information to answer the next question.

The graphs of the functions y = f(x), y = g(x), y = h(x) are shown below. Each function is transformed into y = $\sqrt{f(x)}$, y = $\sqrt{g(x)}$, y = $\sqrt{h(x)}$ respectively







10. The number of invariant points on the original graph and its transformed graph for

    Diagram 1 is _____ (Record in the first column)
    Diagram 2 is _____ (Record in the second column)
    Diagram 3 is _____ (Record in the third column)

Answer

The standard is acceptable and the cognitive level is conceptual.

The answer is 410.

The proportion of students answering this question correctly was **29.0%**.

Invariant points will occur where y = 0 or y = 1.

In diagram 1, the graph crosses the x-axis at 2 points (i.e. y = 0) and a horizontal line drawn at y = 1, will intersect the graph at 2 points. Thus, in diagram 1, there are 4 invariant points.

In diagram 2, there are no x-intercepts and thus no invariant points for y = 0. If a horizontal line were to be drawn at y = 1, the line would intersect the graph at 1 point. Thus diagram 2 has 1 invariant point.

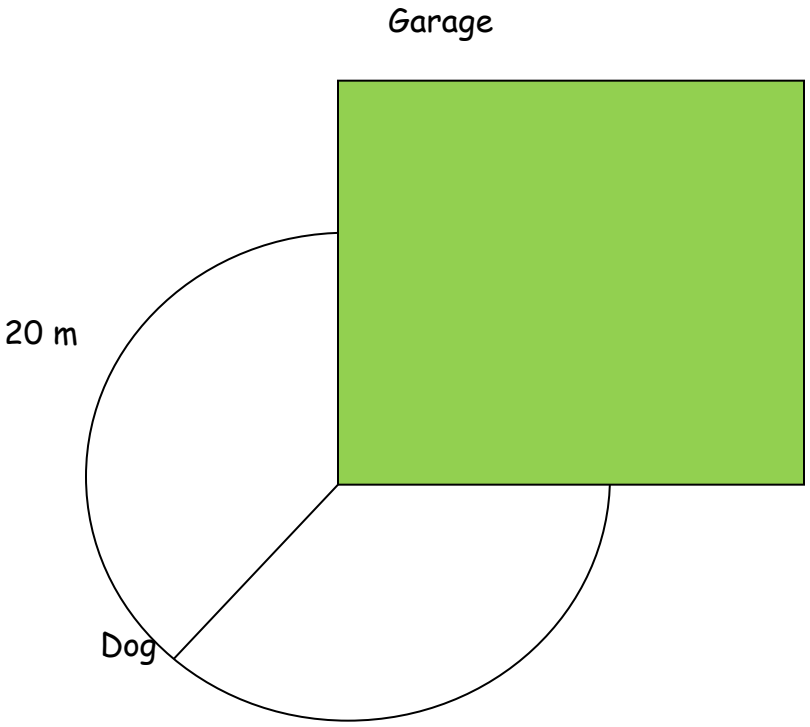
In diagram 3, there are no x-intercepts and the graph is totally above the horizontal line, y = 1. Thus, there are no invariant points for diagram 3.

What are possible reasons students struggled with this question?

- When y = f(x) is transformed by y = $\sqrt{f(x)}$ it means that for a given x value all corresponding **y values** will have their square roots taken. The square root of 0 is 0. The square root of 1 is 1. Hence by drawing horizontal lines at y = 0 and y = 1 will determine any points of potential intersection, in other words, invariant points. Students may not understand the connection between the square root of the function and the square root of y (they are the same thing).
- Difficulty in analyzing and comparing multiple graphs.

Use the following information to answer the next question.

A dog is tied to the corner of a rectangular garage. He is given enough leash to run along a 20m circular path, completing $\frac{3}{4}$ of a circle, as shown in the diagram below.



11. The length of the dog's leash, to the nearest 10^{th} of a metre is, _____.

Answer

The standard is acceptable and the cognitive level is problem solving.

The answer is 4.2.

The proportion of students answering this question correctly was **47.7%**.

Radian Measure = $\frac{arc}{radius}$

$\frac{3\pi}{2} = \frac{20}{x}$

x = 4.2

What are possible reasons students struggled with this question?

- A lack of recognition that the question required use of the radian, arc length, radius formula.
- The fact that $\frac{3}{4}$ of a circle is the same as $\frac{3\pi}{2}$ radians.

Use the following information to answer the next question

The terminal arm of an angle of 70^0 in standard position intersects the unit circle at the point P(x,y). The coordinates of point P(x,y), rounded to the nearest hundredth, are x = 0.ab and y = 0.cd.

12. The value of x is 0. _____ Record in the **first** and **second** columns
    The value of y is 0. _____ Record in the **third** and **fourth** columns

Answer

The standard is acceptable and the cognitive level is procedural.

The answer is 3494.

The proportion of students answering this question correctly was **47.5%**.

Cos(70) = 0.3420201433

Sin (70) = 0.9396926208

The coordinates of points on the unit circle are (cos, sin).

Since the x coordinate represents cosine of 70^0, **a** and **b** respectively are 3 and 4.

Since the y coordinate represents sine of 70^0, **c** and **d** respectively are 9 and 4.

What are possible reasons students struggled with this question?

- Students did not know that the coordinates on the unit circle represent cos and sin.
- Perhaps the coordinates were reversed, i.e. sin and cos.
- The calculator was in radian mode instead of degree mode.

13. If sin(θ) = -2cos(θ), and $\frac{\pi}{2} \leq \theta \leq \pi$, then the exact value of tan(2θ) is

a) $\frac{4}{3}$ b) $\frac{4}{5}$ c) $-\frac{4}{3}$ d) $-\frac{4}{5}$

Answer

The standard is excellence and the cognitive level is conceptual.

The answer is (a) $\frac{4}{3}$.

The proportion of students answering this question correctly was **28.3%**.

By dividing both sides of the equation by cos(θ), the equivalent is:

$\frac{\sin(\theta)}{\cos(\theta)} = -2$, or $\tan(\theta) = -2$

In radians, $\tan^{-1}(-2) = -1.107148718$ (which gives the angle measure in quadrant 4)

Add π radians to find the angle measure in quadrant 2, as per the question restriction.

θ = 2.034443936

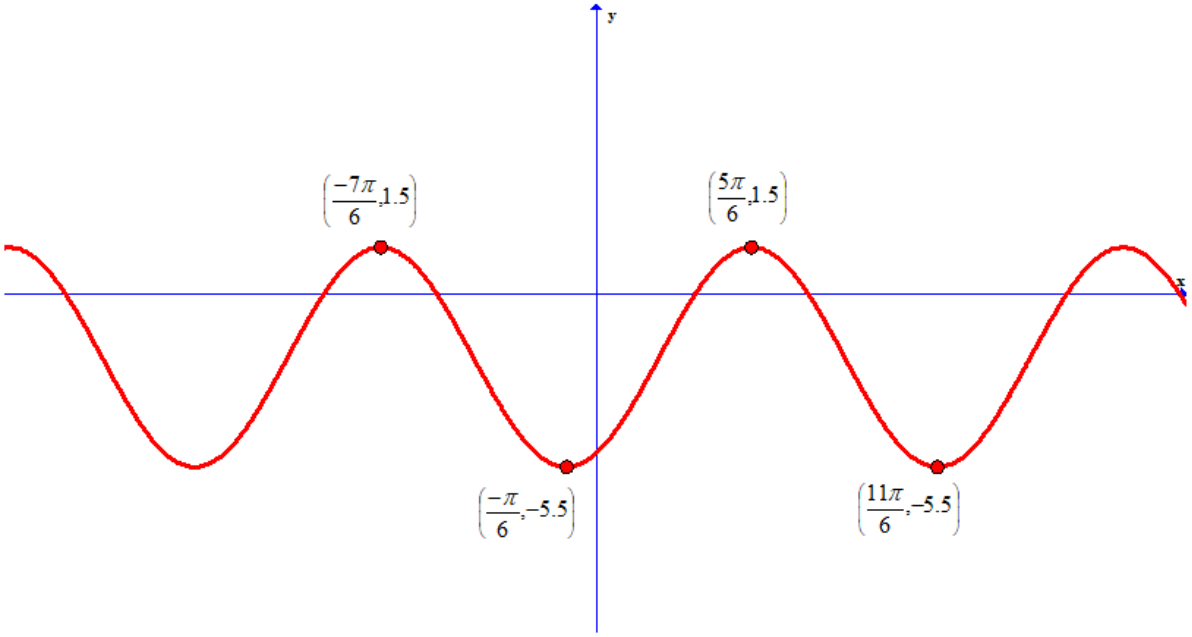
tan (2(2.034443936)) = $\frac{4}{3}$

What are possible reasons students struggled with this question?

- Not knowing how to deal with sin(θ) and cos(θ) in the same equation.
- The question can also be solved by graphing and not realizing this fact.
- Unsure of how to find the correct angle measure in radians in quadrant 2.

Use the following information to answer the next question

The partial graph of $f(x) = a \sin[b(x - c)] + d$, where $a > 0$ and x is in radians, is shown below. Two of its maximum points and two of its minimum points are labelled.



14. The minimum positive value of c, to the nearest hundredth of a radian, is

a) 1.05 b) 1.57 c) 2.09 d) 2.62

Answer

The standard is acceptable and the cognitive level is problem solving.

The answer is (c) 1.05.

The proportion of students answering this question correctly was **39.2%**.

What are possible reasons students struggled with this question?

- Students were unable to determine the period given a graph.
- Students were unable to determine the amplitude and vertical displacement given a graph.
- Students didn't know that any of the given points could be substituted into the equation to find the value of c.

Use the following information to answer the next question.

The steps shown below were used to determine the complete solution set to the equation, $3\sin^2 x + \cos^2 x + 5\sin x - 4 = 0$, where $0 \leq x < 2\pi$.

$$3\sin^2 x + \cos^2 x + 5\sin x - 4 = 0$$

Step 1 $\quad 2\sin^2 x + \sin^2 x + \cos^2 x + 5\sin x - 4 = 0$

Step 2 $\quad 2\sin^2 x + 1 + 5\sin x - 4 = 0$

Step 3 $\quad 2\sin^2 x + 5\sin x - 3 = 0$

Step 4 $\quad (2\sin x - 1)(\sin x + 3) = 0$

Step 5 $\quad \sin x = \frac{1}{2}$ or $\sin x = -3$

Step 6 $\quad x = \left\{\frac{\pi}{6}\right\}$

15. The **first** recorded error was in step

a) 1 b) 2 c) 4 d) 6

Answer

The standard is excellence and the cognitive level is problem solving.

The answer is (d) 6.

The proportion of students answering this question correctly was **27.8%**.

What are possible reasons students struggled with this question?

- Students didn't realize that there is another positive sin solution in quadrant 2, i.e., $\frac{5\pi}{6}$, given the domain in the question of $0 \leq x < 2\pi$.
- They may have thought that there was an error in the substitution or an error in the factoring, which there was not.

16. The non-permissible values of θ for the identity $\frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$ are

a) $n\pi, n \in I$
b) $2n\pi, n \in I$
c) $\frac{\pi}{2} + n\pi, n \in I$
d) $\pi + 2n\pi, n \in I$

Answer

The standard is excellence and the cognitive level is procedural.

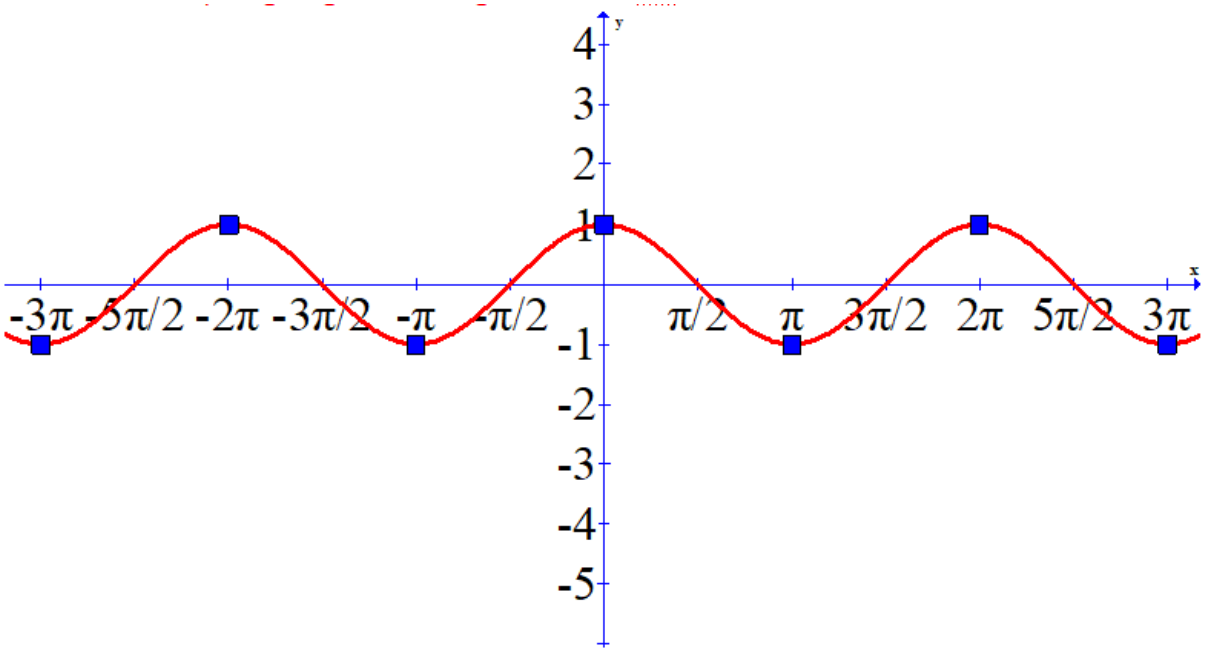
The answer is (a).

The proportion of students answering this question correctly was **45.6%**.

Beginning with the first denominator and setting it equal to zero; $1 - \cos^2\theta = 0$

$\cos^2\theta = 1$
$\cos\theta = \pm 1$

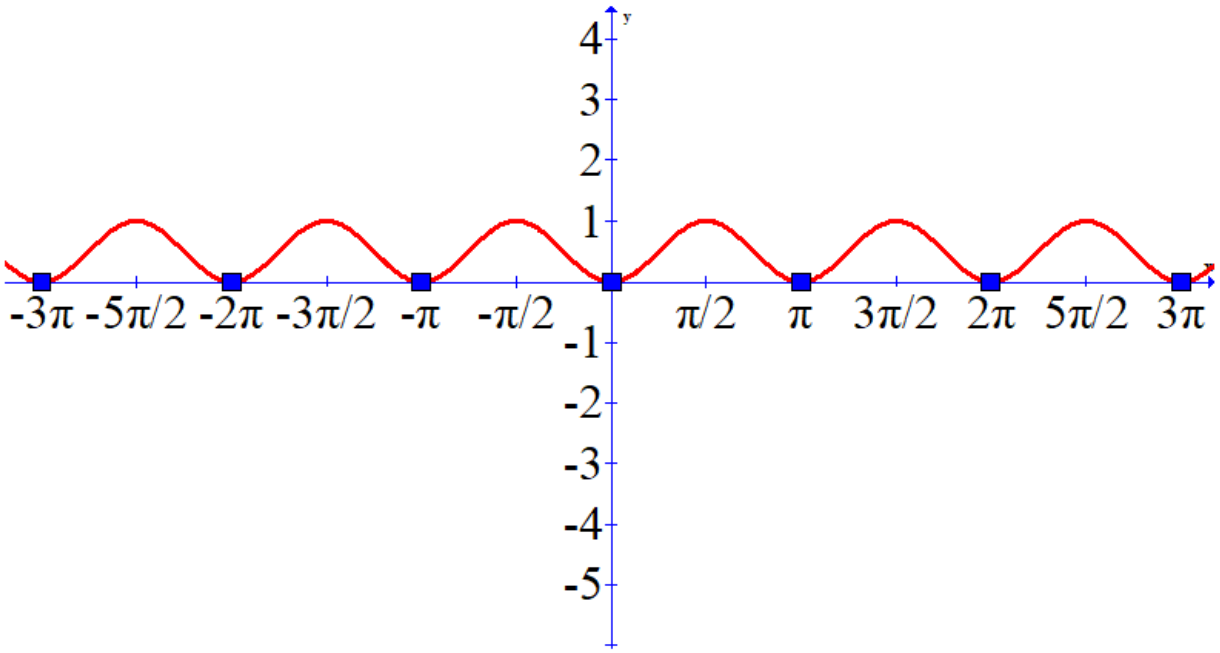
The cosine graph below shows the points where $\cos\theta$ = 1 or -1. These points occur at Π and every Π going left or right; hence nΠ.



Looking at the second denominator and setting it equal to zero, $\sin^2\theta$ = 0,

$\sin\theta$ = 0.

The sine graph below shows all the points where $\sin\theta$ = 0. These points occur at Π and every Π going left or right; hence nΠ.



What are possible reasons students struggled with this question?

- Not understanding that NPV are values which make the denominator equal to zero.
- Not knowing how to find the NPV.
- Not knowing how to state the final answer when having to deal with 2 separate equations.

17. In the expansion of $\left(x^3+\frac{1}{2x^2}\right)^8$ written in descending powers of x, the 6th term is

a) $\frac{28}{x}$ b) $\frac{7}{4x}$ c) $\frac{1}{64x^6}$ d) $\frac{7}{16x^6}$

Answer

The standard is excellence and the cognitive level is procedural.

The answer is (b).

The proportion of students answering this question correctly was **48.4%**.

For the 6th term, k = 5.

$${}_8C_5\,(x^3)^3\left(\frac{1}{2x^2}\right)^5$$

$$= \quad 56(x^9)\left(\frac{1}{32x^{10}}\right)$$

$$= \quad \frac{7}{4x}$$

What are possible reasons students struggled with this question?

- Not knowing how to relate the general term to value of k.
- Unsure of the relationship between k and the values of the exponents.
- Unclear of exponent law application, and simplification.