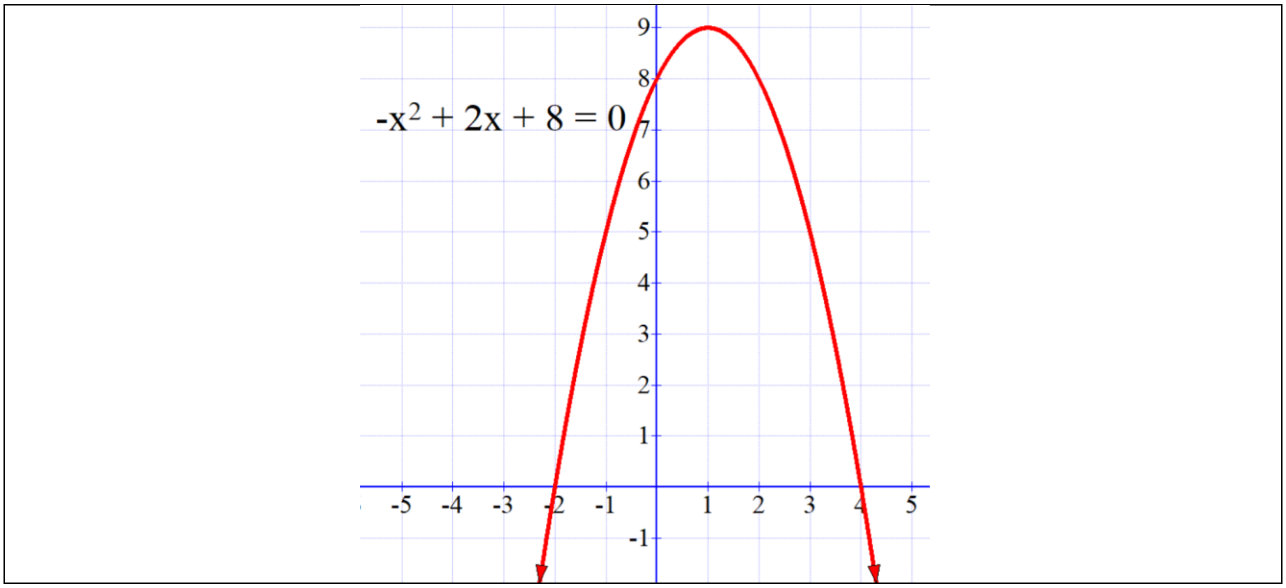
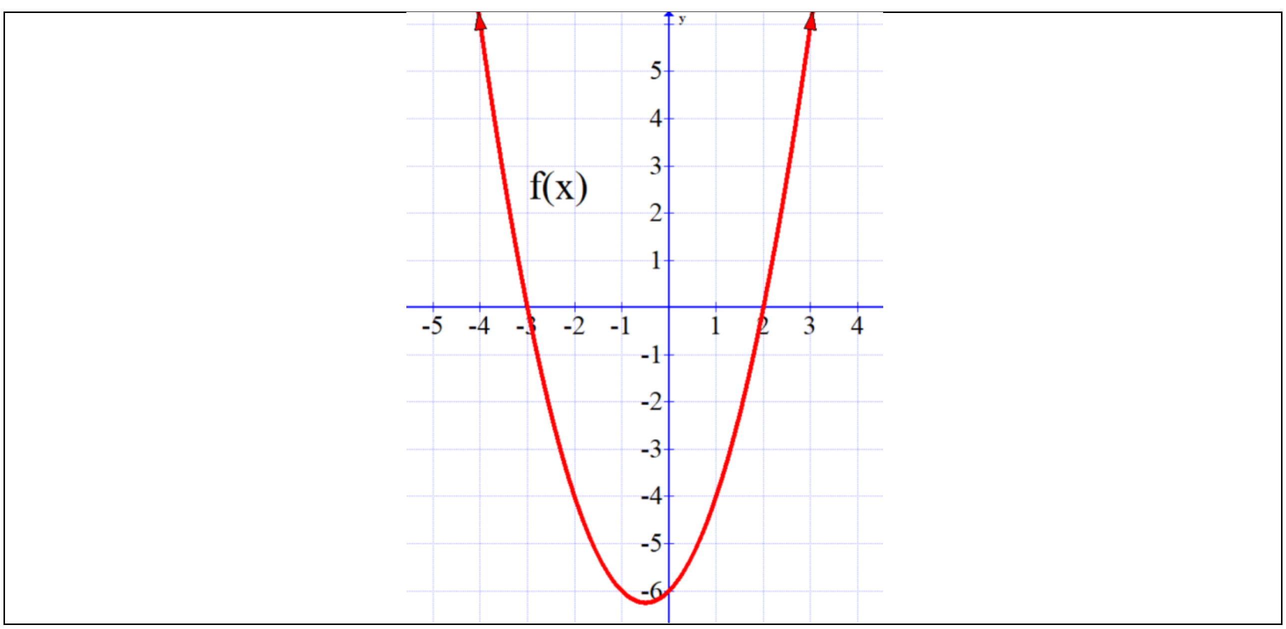
<u>Solving Quadratic Inequalities in One Variable Practice</u>

Use the graph below to answer the first question.



1. The solution to $-x^2 + 2x + 8 > 0$ is
   A) $x < -2$ and $x > 4$ B) $x \le -2$ and $x \ge 4$ C) $-2 < x < 4$ D) $-2 \le x \le 4$

Use the graph below to answer the next question.



2. Which statement below is **not** true?

   A) If $x < -3$, then $f(x) > 0$.

   B) If $x > 2$, then $f(x) > 0$.

   C) If $x < 2$, then $f(x) < 0$.

   D) If $-3 < x < 2$, then $f(x) < 0$.

Use the following information to answer the next question.

Consider the following quadratic inequality:

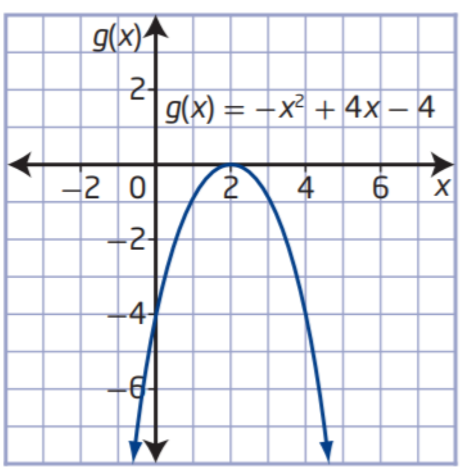
$$x^2 - 3x - 10 \leq 0$$

Some possible solutions to consider are:

5	0	7	-3

3. Of the four possible solutions listed above to consider, the *largest* correct solution is _____.

4. Given the graph below, for which of the following inequalities is there no solution?



A) $-x^2 + 4x - 4 \geq 0$

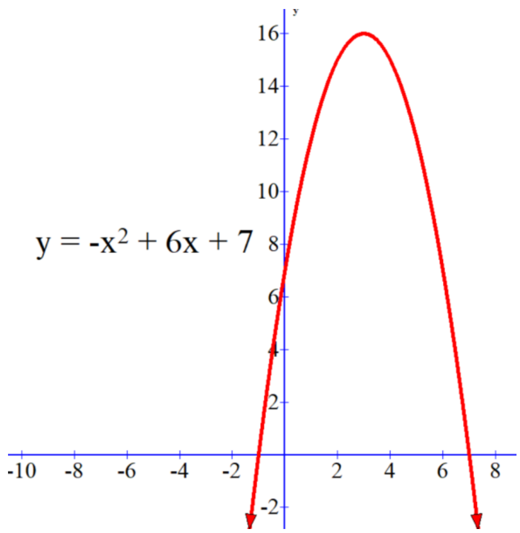
B) $-x^2 + 4x - 4 > 0$

C) $-x^2 + 4x - 4 \leq 0$

D) $-x^2 + 4x - 4 < 0$

Use the following information to answer the next question.

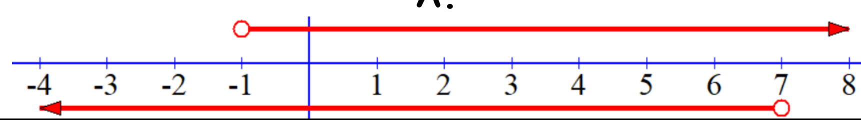
A math student was given the following graph and asked to solve
$-x^2 + 6x + 7 > 0$, using sign analysis.



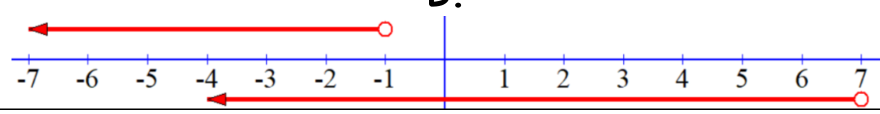
The student factored the equation; $y = -(x + 1)(x - 7)$, and knew that for this function to be greater than zero, one of the binomials has to be positive and the other binomial has to be negative.

Consider the following number line scenarios.

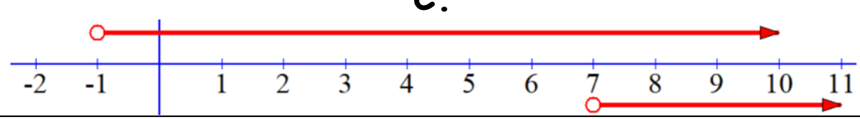
A.



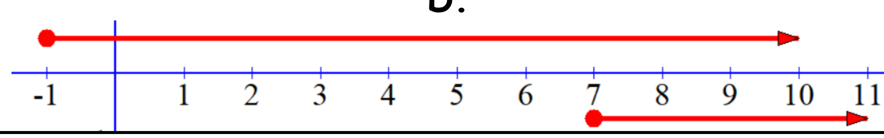
B.



C.



D.

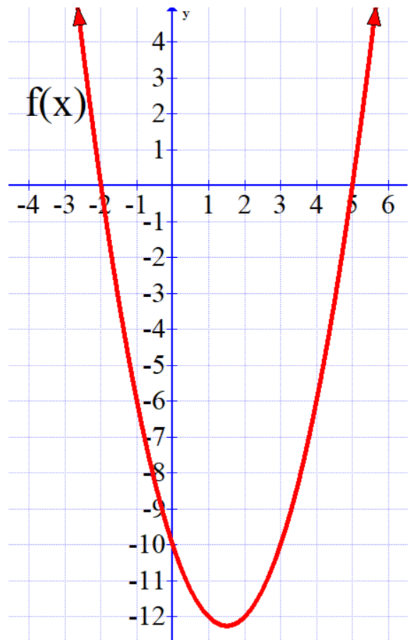


5. The correct number line scenario is

A) A B) B C) C D) D

Use the following information to answer the next question.

A quadratic equation $y = x^2 - 3x - 10$, related to a particular inequality, is shown below.

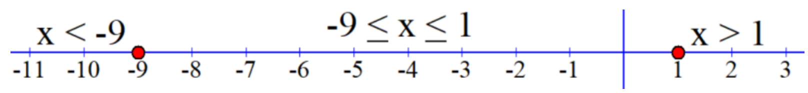


When a test point of $x = 1$ is used to determine the solution set, the inequality is **not** satisfied.

6. If 5 is a solution, then place the correct inequality sign in the blank,
   $x^2 - 3x - 10$ ____ 0.

Use the following information to answer the next question.

A math student was asked to solve $x^2 - 8x - 9 \geq 0$ using roots and test points.
His partial work is shown below.

Step 1	$(x + 9)(x - 1) \geq 0$
Step 2	Place the roots on a number line and identify three regions. 
Step 3	Pick a test point, for example $x = 2$. $((2) + 9)((2) - 1) \geq 0$ $11 \geq 0$
Step 4	Since a true statement is made, the region $x > 1$ is part of the solution.

7. Analyze this student's work. If there are errors, identify and correct the mistakes.

8. The solution to the inequality $-x^2 - 4x < -6$ is

   A) $-2 - \sqrt{10} < x < -2 + \sqrt{10}$

   B) $x < -2 - \sqrt{10}$ and $x > -2 + \sqrt{10}$

   C) $x > -2 - \sqrt{10}$

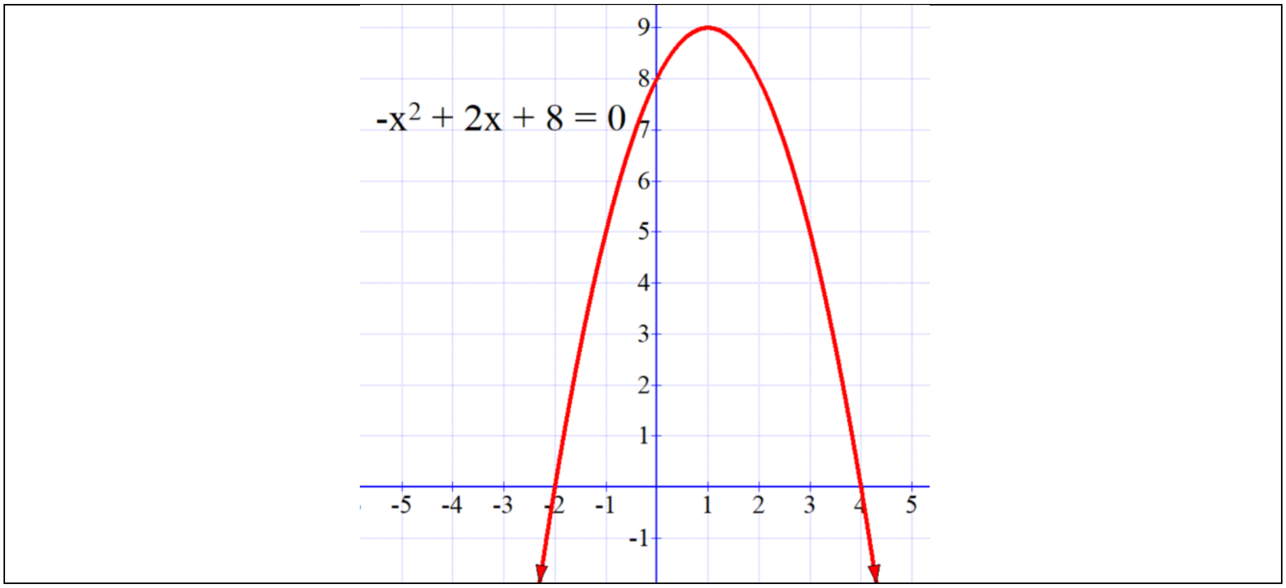
   D) $x < -2 + \sqrt{10}$

9. One leg of a right angle triangle is 3 cm longer than the other leg. How long should the shorter leg be to ensure that the area of the triangle is less than or equal to 14 cm^2?

10. A kicked soccer ball can be modelled by the quadratic equation $h(d) = -0.045d^2 + 1.3d$, where h is the height of the ball in metres and d is the horizontal distance travelled by the ball in metres.
    a) Write an inequality to represent the interval for horizontal distance when the ball is above or equal to a height of 8 m.
    b) Solve the inequality, to the nearest tenth of a metre.

Solving Quadratic Inequalities in One Variable Practice Solutions

Use the graph below to answer the first question.

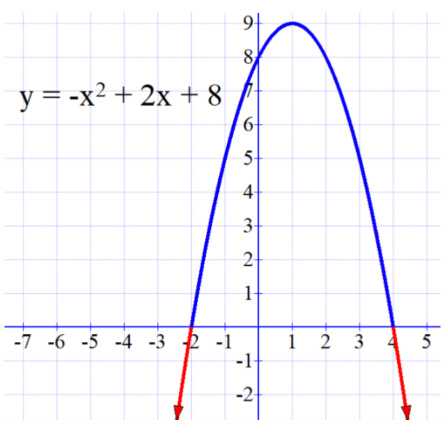


1. The solution to $-x^2 + 2x + 8 > 0$ is

   A) $x < -2$ and $x > 4$ B) $x \leq -2$ and $x \geq 4$ C) $-2 < x < 4$ D) $-2 \leq x \leq 4$

Solution

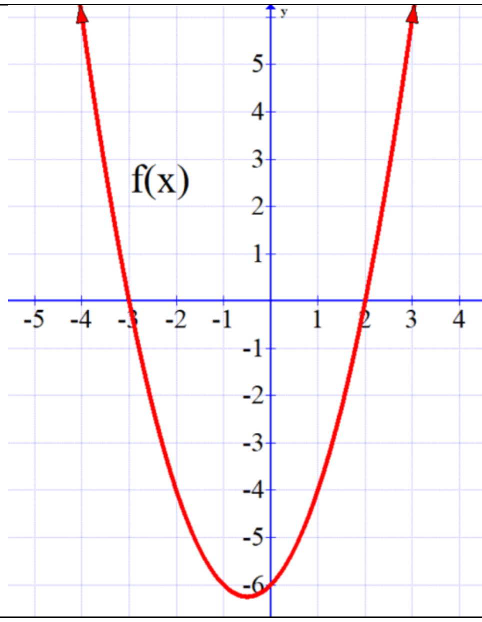
The function, $f(x) = -x^2 + 2x + 8$, is the same as y. We can think of the inequality as finding values for x when $y > 0$. The portion of the graph when $y > 0$ is above the x-axis, as shown by the blue part of the parabola below.



The values of x to generate all of these positive y values would be any real number in between the x-intercepts of -2 and 4. Since the inequality sign is > and not ≥, the x-intercepts are not part of the solution.

The correct answer is C.

Use the graph below to answer the next question.



2. Which statement below is **not** true?

A) If x < -3, then f(x) > 0.

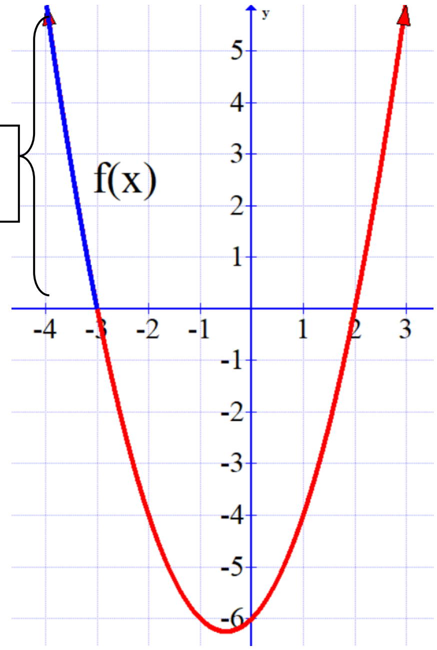
B) If x > 2, then f(x) > 0.

C) If x < 2, then f(x) < 0.

D) If -3 < x < 2, then f(x) < 0.

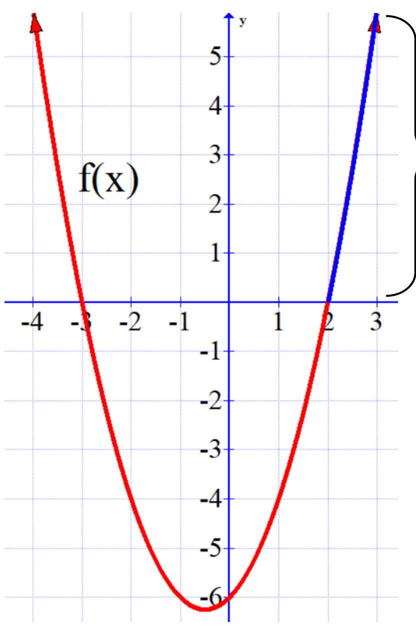
Solution

Recalling that f(x) and y are the same, when x < -3, f(x) will have positive values for y, or show the graph above the x-axis, which it does. Statement A is **true.**



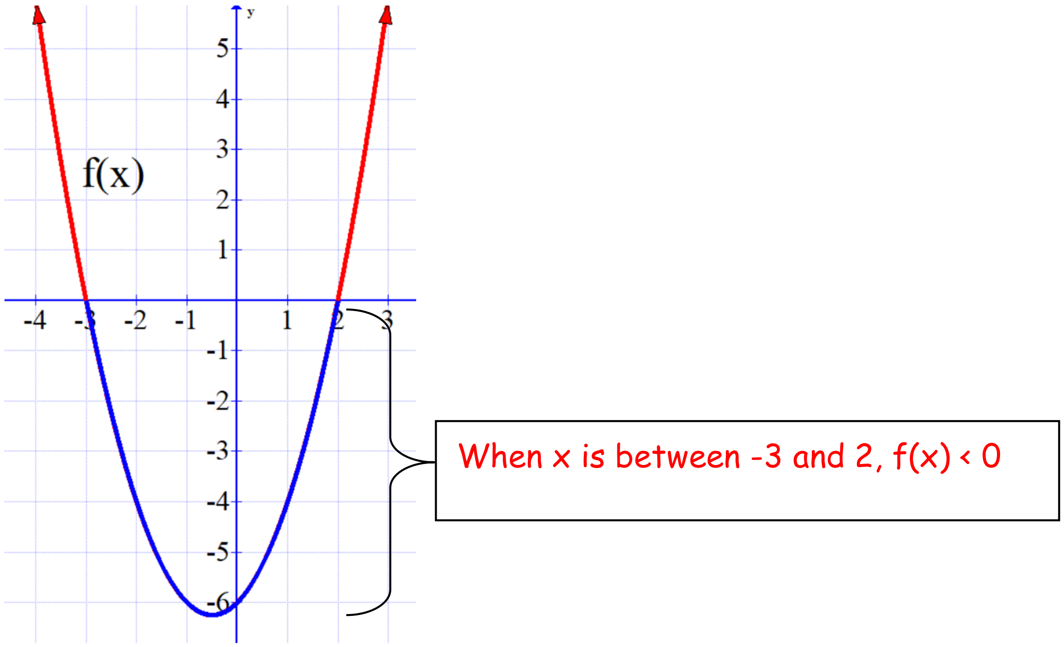
When x is less than -3, f(x) > 0

When x is greater than 2, f(x) will have positive values for y, or show the graph above the x-axis, which it does. Statement B is **true.**



When x is greater than 2, f(x) > 0.

When x is between -3 and 2, f(x) will have negative values for y, or show the graph below the x-axis, which it does. Statement D is **true.**



Statement C is **not true.** When x is less than 2, part of f(x) is less than zero and part of f(x) is greater than zero.

The correct answer is C.

Use the following information to answer the next question.

Consider the following quadratic inequality:

$x^2 - 3x - 10 \leq 0$

Some possible solutions to consider are:

5	0	7	-3

3. Of the four possible solutions listed above to consider, the *largest* correct solution is _<u>5</u>__.

Solution

If a number is a solution, the number will satisfy the inequality.

<u>Substitute x = 5</u>

$(5)^2 - 3(5) - 10 \leq 0$

$25 - 15 - 10 \leq 0$

$0 \leq 0$. This value satisfies the inequality and is therefore a solution.

<u>Substitute x = 0</u>

$(0)^2 - 3(0) - 10 \leq 0$

$0 - 0 - 10 \leq 0$

$-10 \leq 0$. This value satisfies the inequality and is therefore a solution.

<u>Substitute x = 7</u>

$(7)^2 - 3(7) - 10 \leq 0$

$49 - 21 - 10 \leq 0$

18 is not less than or equal to 0. This value does not satisfy the inequality and therefore is not a solution.

<u>Substitute x = -3</u>

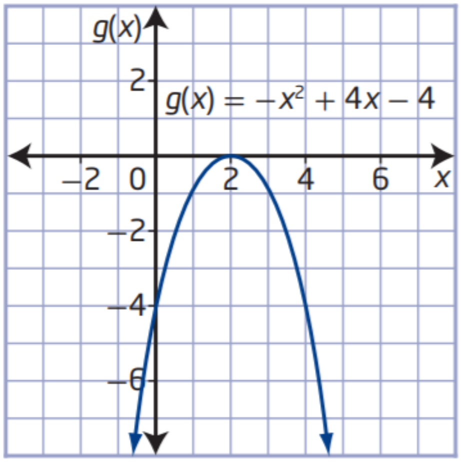
$(-3)^2 - 3(-3) - 10 \leq 0$

$9 + 9 - 10 \leq 0$

8 is not less than or equal to 0. This value does not satisfy the inequality and therefore is not a solution.

From the given list, the two solutions are 0 and 5. Of these, 5 is the largest.

4. Given the graph below, for which of the following inequalities is there no solution?



A) $-x^2 + 4x - 4 \geq 0$
B) $-x^2 + 4x - 4 > 0$
C) $-x^2 + 4x - 4 \leq 0$
D) $-x^2 + 4x - 4 < 0$

Solution

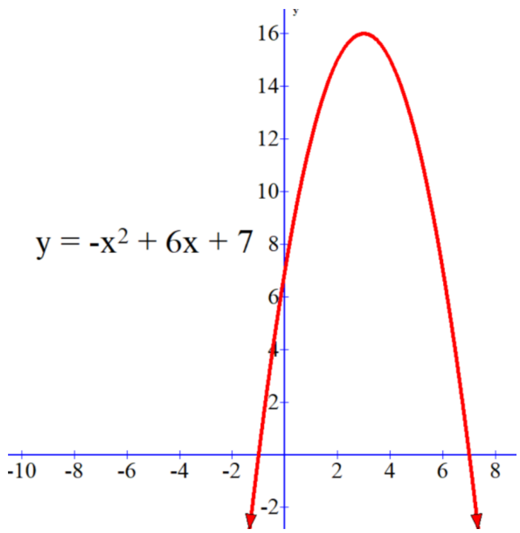
When $x = 2$, these two inequalities, $-x^2 + 4x - 4 \geq 0$ and $-x^2 + 4x - 4 \leq 0$, will both be satisfied and thus have one solution. For all other values of x, the value for y will be negative since the balance of the graph is below the x-axis. Therefore, there will be an infinite number of solutions for $-x^2 + 4x - 4 < 0$.

There is no value of x that would generate a value for y greater than 0. There is no solution for $-x^2 + 4x - 4 > 0$.

The correct answer is B.

Use the following information to answer the next question.

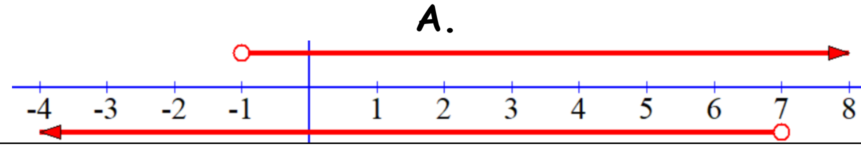
A math student was given the following graph and asked to solve $-x^2 + 6x + 7 > 0$, using sign analysis.



The student factored the equation; $y = -(x + 1)(x - 7)$, and knew that for this function to be greater than zero, one of the binomials has to be positive and the other binomial has to be negative.

Consider the following number line scenarios.

A.



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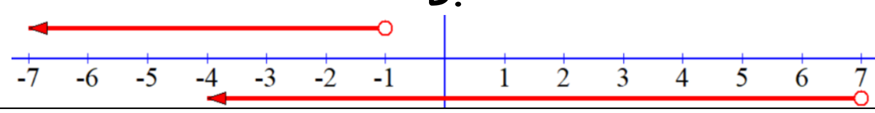
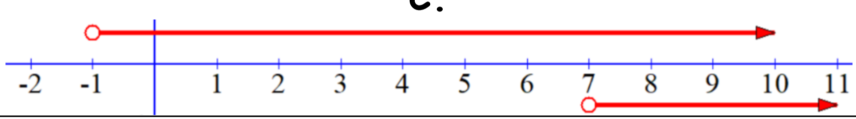
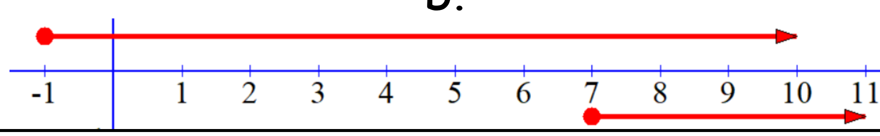


C.



D.

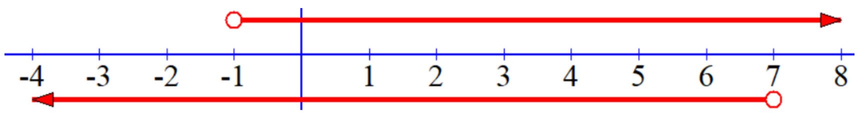


5. The correct number line scenario is

A) A B) B C) C D) D

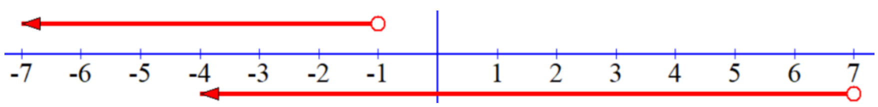
Solution

Option A



This option illustrates when x > -1 and x < 7. When x is greater than -1, the binomial (x + 1) is positive. When x is less than 7, the binomial (x – 7) is negative. Thus, the sign requirements are satisfied. This line scenario is correct.

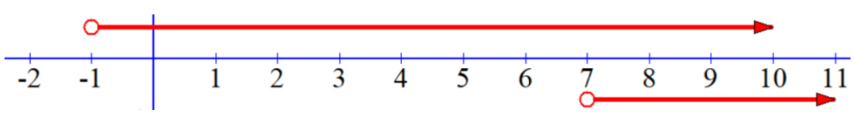
Option B



This option illustrates when x < -1 and x < 7. When x is less than -1, the binomial

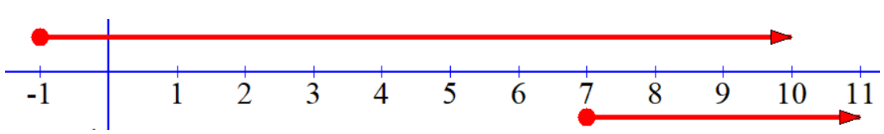
(x + 1) is negative. When x is less than 7, the binomial (x – 7) is negative. Since both binomials are negative, the sign requirements are not satisfied. This line scenario is not correct.

Option C



This option illustrates when x > -1 and x > 7. When x is greater than -1, the binomial (x + 1) is positive. When x is greater than 7, the binomial (x – 7) is positive. Since both binomials are positive, the sign requirements are not satisfied. This line scenario is not correct.

Option D



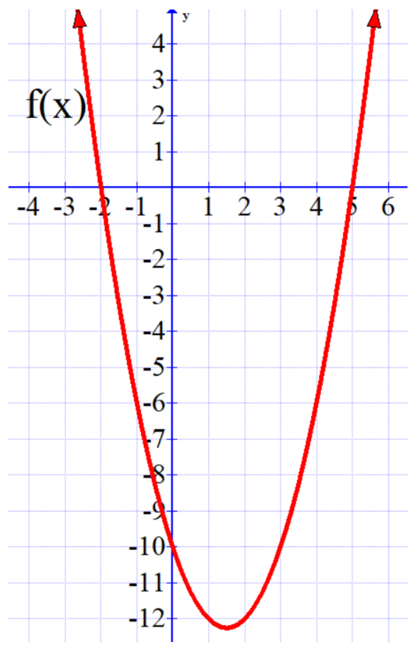
This option is similar to Option C above. Both binomials are positive and do not meet the sign requirements. The difference between these two options is that one has a solid dot and one has an open dot. It is still nonetheless, not a correct line scenario.

The correct answer is A.

[*The solution is the overlap of the two number lines in A; which is -1 < x < 7*]

Use the following information to answer the next question.

A quadratic equation $y = x^2 - 3x - 10$, related to a particular inequality, is shown below.



When a test point of x = 1 is used to determine the solution set, the inequality is **not** satisfied.

6. If 5 is a solution, then place the correct inequality sign in the blank,

$x^2 - 3x - 10$ _$\geq$_ 0.

Solution

Substitute x = 1 and determine which inequality sign makes a true statement.

$(1)^2 - 3(1) - 10$ ____ 0

$1 - 3 - 10$ ___ 0

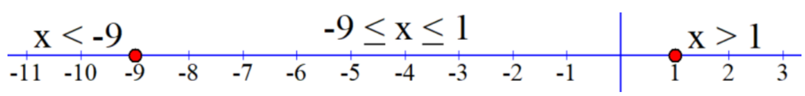
$-12 < 0$

Since we are told that $x = -1$ does **not** satisfy the inequality, the sign must be either $>$ or $\geq$.

Since 5 is an x-intercept and we are told that it is a solution, the symbol required is $\geq$.

Use the following information to answer the next question.

A math student was asked to solve $x^2 - 8x - 9 \geq 0$ using roots and test points. His partial work is shown below.

Step 1	$(x + 9)(x - 1) \geq 0$
Step 2	Place the roots on a number line and identify three regions. 
Step 3	Pick a test point, for example $x = 2$. $((2) + 9)((2) - 1) \geq 0$ $11 \geq 0$
Step 4	Since a true statement is made, the region $x > 1$ is part of the solution.

7. Analyze this student's work. If there are errors, identify and correct the mistakes.

Solution

In step one, the factoring is incorrect. It should be $(x - 9)(x + 1) \geq 0$.

For step two, the x-intercepts should be 9 and -1. Therefore, the three regions for possible solution sets are:

$x < -1$, and

$-1 \leq x \leq 9$, and

$x > 9$.

In step three, the test point does create a true statement, but it is based on incorrect factoring. If the factoring was correct, step four would be correct.

Using a test point of $x = 2$ and the correct factoring of $(x - 9)(x + 1)$:

$((2) - 9)((2) + 1) \geq 0$

$(-7)(3) \geq 0$

-21 is not greater than or equal to 0.

Since the test point was in the solution region of $-1 \leq x \leq 9$ and did not satisfy the inequality, the solution is $x \leq -1$ and $x \geq 9$.

8. The solution to the inequality $-x^2 - 4x < -6$ is

A) $-2 - \sqrt{10} < x < -2 + \sqrt{10}$

B) $x < -2 - \sqrt{10}$ and $x > -2 + \sqrt{10}$

C) $x > -2 - \sqrt{10}$

D) $x < -2 + \sqrt{10}$

Solution

Begin by adding 6 to both sides of the inequality.

$-x^2 - 4x + 6 < 0$.

Find the x-intercepts of the related quadratic equation, $-x^2 - 4x + 6 = 0$.

Use the quadratic formula, where $a = -1$, $b = -4$ and $c = 6$.

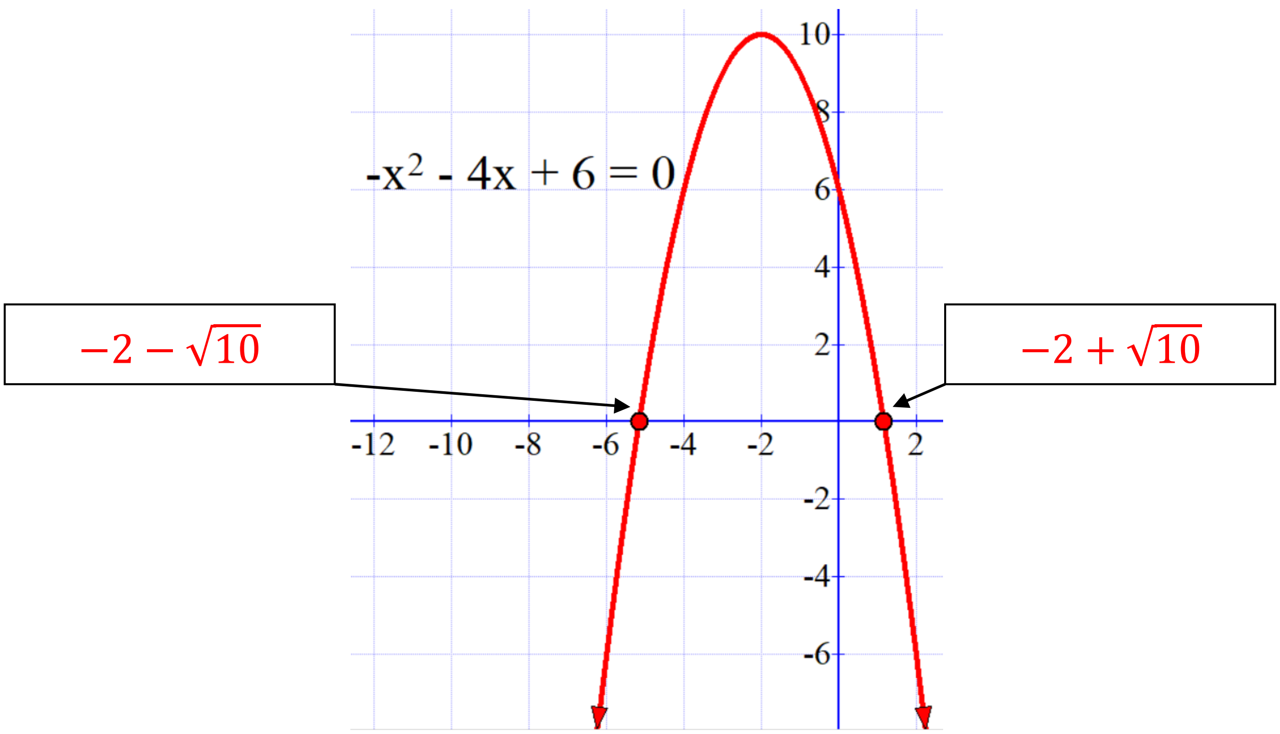
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-1)(6)}}{2(-1)}$$

$$x = \frac{4 \pm \sqrt{16 + 24}}{-2}$$

$$x = \frac{4 \pm \sqrt{40}}{-2}$$

$$x = \frac{4 \pm 2\sqrt{10}}{-2}$$

$$x = -2 \pm \sqrt{10}$$



The correct answer is B.

9. One leg of a right angle triangle is 3 cm longer than the other leg. How long should the shorter leg be to ensure that the area of the triangle is less than or equal to 14 cm^2?

Solution

Let the length of the smaller leg = x

Let the length of the larger leg = x + 3

The area for a triangle is found by using the formula:

$$A = \frac{bh}{2}$$

$$14 \geq \frac{x(x+3)}{2}$$

$28 \geq x^2 + 3x$

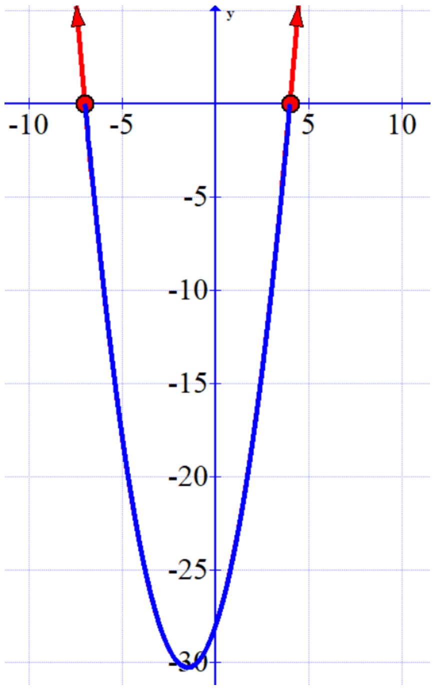
$0 \geq x^2 + 3x - 28$

If 0 must be greater than the expression, an equivalent is that the expression must be less than 0.

$x^2 + 3x - 28 \leq 0$

Factor the find the zeros.

$(x + 7)(x - 4) \leq 0$ The zeros are -7 and 4.



Any value of x between -7 and 4 will generate a value of y less than zero. Since negative values do not make sense in this context, the solution is $0 < x \leq 4$.

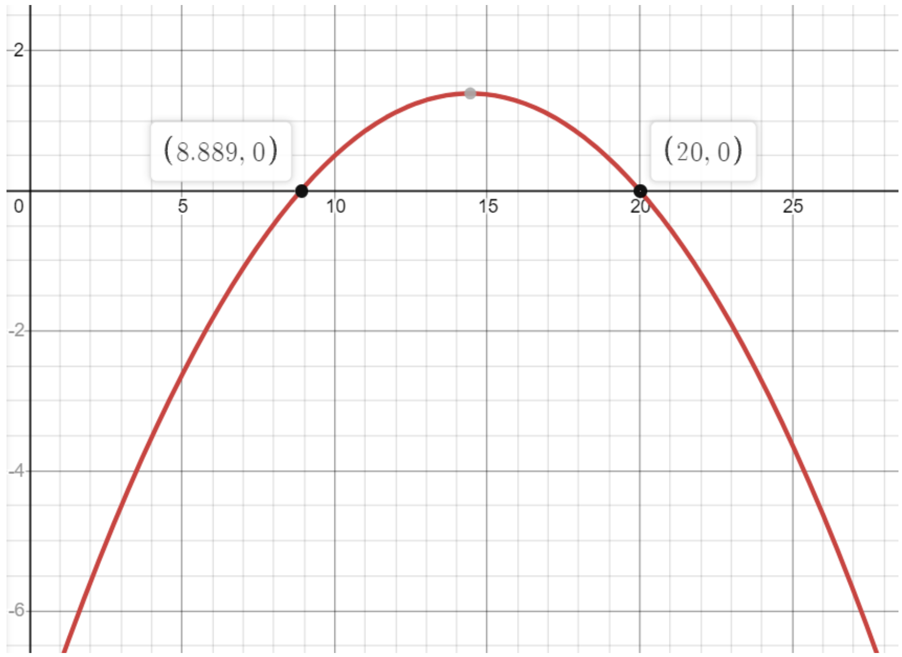
10. A kicked soccer ball can be modelled by the quadratic equation $h(d) = -0.045d^2 + 1.3d$, where h is the height of the ball in metres and d is the horizontal distance travelled by the ball in metres.

a) Write an inequality to represent the interval of horizontal distance for when the ball is above or equal to a height of 8 m.
b) Solve the inequality, to the nearest tenth of a metre.

Solution

a) $-0.045d^2 + 1.3d \geq 8$
$-0.045d^2 + 1.3d - 8 \geq 0$

b) Draw the graph.



The function is greater than or equal to zero, at or above the x-axis. The solution is the interval between the x-intercepts.

The solution set to the problem is:

$\{d \mid 8.889 \leq d \leq 20, d \in R\}$