Common Theads and Nuances of Math 30-1

The breadth and depth of the Math 30-1 content is significant. In order to design a valid 35 question assessment tool (diploma exam) and to prepare students to demonstrate knowledge of said content are challenging tasks.

A nuance is "an expression or appreciation of subtle shades of meaning." The ability to grasp the nuances of the Math 30-1 curriculum is very important.

Although it is impossible to have the same type of question on every exam, based on historical trends, there are certain common threads which appear relatively often. Identifying these common threads is also very important.

This document is an attempt to provide insight into these common threads and nuances.

1. **If a point (x,y) lies on any graph, that point can be substituted to satisfy the equation.**

Often an equation is given that requires a student to find an unknown parameter. Given 1 or more parameters, and a point on the graph, the unknown parameter can be determined.

For example:

The graph below can be written in the form, $y = \mathbf{a}(x + 1)(x - 2)^2$.

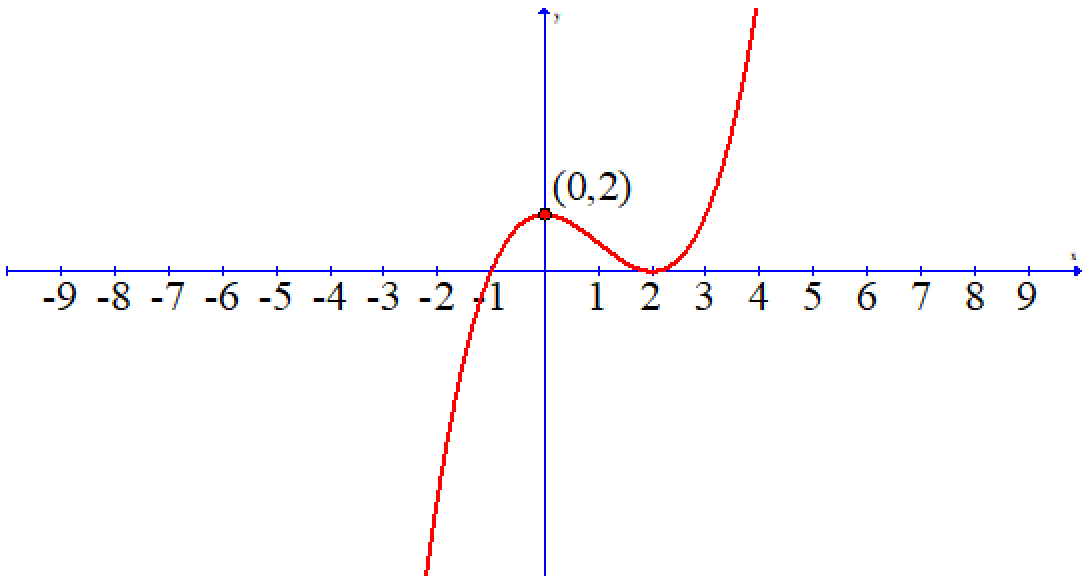
The task is to find the value of **a**, or the vertical stretch factor.

The point given is the y-intercept, or (0,2). Substitute this point into the equation, because we know that it will satisfy this equation.

$2 = \mathbf{a}((0) + 1)((0) - 2)^2$

$2 = \mathbf{a}(1)(4)$

$2 = 4\mathbf{a}$ $\qquad\qquad$ $\mathbf{a} = 0.5$



2. **If a student fully understands a concept, he/she should be able to answer questions requiring both numerical and variable answers. Often times the phrase, 'in terms of' is used.**

For example:

Suppose a student is given a function, $f(x) = a(x + 3)^2 + k$, told that $a < 0$, that the function is translated 5 units up and then asked to state the range 'in terms of' k. This means that the answer has to have a 'k' in it.

Since the parabola opens down, and 'k' was moved 5 units up, the range would be,

$y \leq k + 5$.

3. **In the logarithm and exponent section, being able to move from log form to exponential form and vice versa is critical.**

For example:

(Selecting an example that also incorporates #2 above involving letters)

If $\log_x y = a$, then what is an equivalent for y^2 .

Moving from log form to exponential form, $x^a = y$.

Now raise both sides of the equal sign to an exponent of 2.

$x^{2a} = y^2$

[Note: *This was a diploma exam released question; 23% did not get this correct*]

4. **The coordinates of the points (x,y) on a unit circle, represent (cos,sin), and tan =** $\frac{\sin}{\cos}$**.**

For example:

The terminal arm of an angle of 70^0 in standard position intersects the unit circle at P(x,y). The coordinates of the point P(x,y), rounded to the nearest hundredth, are x = 0.ab and y = 0.cd. What are the values of a,b,c and d. (numerical response question).

By using the calculator and pressing cos70 and sin70, the values for a,b and c,d respectively can be determined.

[Note: *This was a diploma exam released question and only 47.5% got this question correct*]

5. **It is extremely common to see a trig question related to the definition of radian measure. Given any 2 of the 3 (radian measure, arc length, length of the radius), find the third. As well, this is a prime spot to include degree to radian conversion.**

For example:

A central angle of 135^0 is drawn in a circle. If the length of the arc subtended by the angle is 25 cm, what is the diameter of the circle?

In order to use the relationship, radian measure = $\frac{arc}{radius}$, the 135^0 needs to be converted to $\frac{3\pi}{4}$ radians first.

$\frac{3\pi}{4} = \frac{25}{radius}$

Radius = 10.6

The diameter is 21.2 cm.

6. **Given an equation with a series of transformations, determining the new position of a transformed point from an original point, is fairly common.**

For example:

The point A(5,2) is on y = f(x). The equation is transformed by,

$y = -4f(\frac{1}{2}(x - 3)) -7$. What is the new position of point A?

Knowing which number affects which variable is helpful. The front number(vertical stretch) and the back number(vertical translation) affect the **y**. The two middle numbers, (the horizontal stretch and the horizontal translation) affect the **x**.

For the **x** value of 5, multiply it by 2, and then move it 3 units right. Thus, the **x** value of 5 moves to 13.

For the **y** value of 2, multiply it by -4(combine the reflection and the stretch together), and then move it 7 units down. Thus, the **y** value of 2 moves to -15.

The new position of point A is (13, -15).

7. **Some questions involving logarithmic simplification using the logarithmic laws, require the conversion of a number to log form.**

For example:

The logarithmic expression, $\log_2 x + \log_2 12 - 5$, can be simplified to $\log_2\left(\frac{mx}{n}\right)$. Find the value of **m** and **n**.

An equivalent logarithmic expression would be: $\log_2 x + \log_2 12 - \mathbf{\log_2 32}$

Now use the power and quotient laws: $\log_2\left(\frac{12x}{32}\right)$ or $\log_2\left(\frac{3x}{8}\right)$

m = 3 and n = 8

8. **When simplifying logarithmic expressions using the logarithmic laws, apply the power law first (if applicable) and pay close attention to fractional equivalents, especially as they relate to rational exponents.**

For example:

In simplifying, $2\log x - \frac{\log 2}{2} + 3\log y$, students need to know the equivalent,

$2\log x - \left(\frac{1}{2}\right)\log 2 + 3\log y$.

Applying the Power Law first, $\log x^2 - \log 2^{\frac{1}{2}} + \log y^3$

which is equal to: $\log x^2 - \log \sqrt{2} + \log y^3$

which is equal to: $\log\left(\frac{x^2 y^3}{\sqrt{2}}\right)$

9. **Logarithms can be used as exponents. Students may have to deal with an exponent law and a logarithmic law in the same question.**

For example:

Simplify, $(3^{\log x})(3^{\log x})$

The exponent law in this case tells us to keep the base and add the exponents.

$= 3^{\log x + \log x}$

The product law of logarithms tells us to multiply the value of the arguments.

$= 3^{\log x^2}$

10. **Solving exponential equations where bases can be made the same, requires a solid understanding of the exponent laws. It is very common to have a fractional base, and a base with an exponent of 1.**

For example:

Solve, $32^{3x} = (2)\left(\frac{1}{16}\right)^{x-1}$

Make all the bases equivalent, $(2^5)^{3x} = (2)^1\left(\frac{1}{2^4}\right)^{x-1}$

Reciprocating the fraction will change the sign on the exponent,

$2^{15x} = (2^1)(2^{-4})^{(x-1)}$

Apply the exponent power law, $2^{15x} = (2^1)(2^{-4x+4})$

Apply the exponent product law, $2^{15x} = 2^{-4x+5}$

Since the bases are the same, the exponents must be equal.

$15x = -4x + 5$

$19x = 5$ $\quad x = 5/19$

11. **Due to the enormous amount of course content, it is not unusual to see questions requiring skills and knowledge from more than 1 unit to determine a correct answer.**

For example:

Simplify, $\log_m(\sin\left(\frac{\pi}{2}\right))$

Going inside the brackets first, $\log_m(1)$

$\log_m(1) = 0$

[Note: *$\log_x 1$ is always zero regardless of the value of x, and*

$\log_x x^v$ is always ***v*** *regardless of the value of x*]

12. **There is a strong emphasis in this course on the analysis of change. Given a graph, an equation, a situation, etc, change of some parameter is often applied, which then leads to the analysis of the change, in terms of a variety of things including, solutions, intercepts, asymptotes, domain, range, key points, etc.**

For example:

The graphs of $y = 2^x$ and $y = \log_2(x + 6)$ intersect at one point in quadrant 2 and at one point in quadrant 1. Which of the transformations below will result in these 2 graphs having no points of intersection?

a) $y = 2^x - 2$
b) $y = \log_2(x + 6) + 2$
c) $y = 2^x + 7$
d) $y = \log_2(x + 6) - 1$

The answer is c.

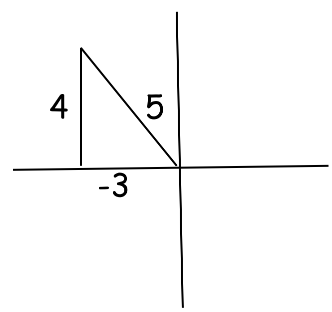
13. **In the trig unit, it is common to see questions requiring the determination of a ratio, given a ratio in fractional form, or a point on the unit circle (in both cases providing information for 2 of the 3 sides in the triangle to be known). Pythagorean Theorem is then applied.**

For example:

If cot(θ) = $\frac{-3}{4}$, and $\frac{\pi}{2}$ ≤ θ ≤ π, what is the value of sinθ?

Using the CAST rule, cot is negative in quadrants 2 and 4. The domain given in the question indicates the rotation angle is in quadrant 2.

Using Pythagorean Theorem, the hypotenuse, or radius of the circle is 5.



Thus, sin(θ) = $\frac{4}{5}$

14. **Mapping notation is a way to express the operations performed on all x and y coordinates when an original equation is transformed by a new equation.**

For example:

When y = f(x) is transformed to y – 9 = -3f((2)(x + 4)), what is the mapping notation?

(x,y) $\longrightarrow$ ($\left(\frac{1}{2}\right)$x – 4, -3y + 9)

15. **Understanding mathematical terminology is critical. Most students likely understand most terms, at least to a reasonable degree. But emphasizing the meaning of words can go a long way in helping students be successful. The following list is not exhaustive, but should include most of the key terms.**

Domain, range,intercepts, asymptote, invariant, function, inverse, solve, equation, expression, point of discontinuity, terminal arm, co-terminal, standard position, exact vs approximate values, identity, verify, general term, permutation, combination, non-permissible values, restriction, logarithm, synthetic division, radian, stretch, translation and that <u>zeros, x-intercepts, roots and solutions all mean the same thing.</u>