Reciprocal Functions Practice

1. If the point $(\frac{1}{2}, 8)$ is on y = f(x), then which of the following points is on the graph of $y = \dfrac{1}{f(x)}$?
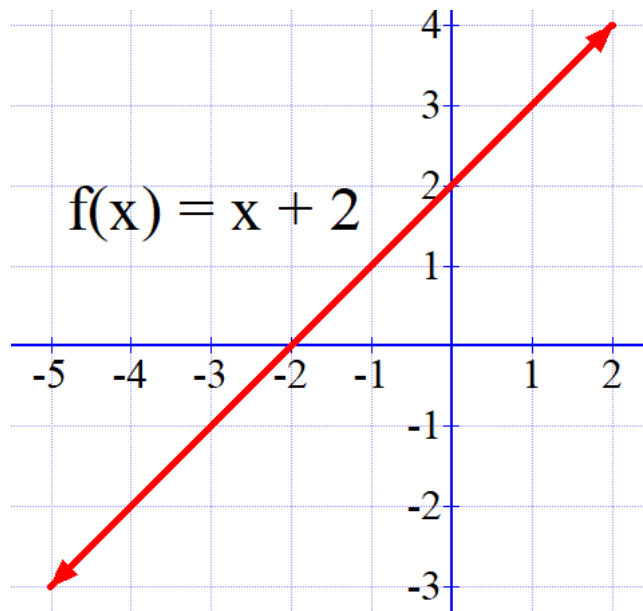
   A) (2, 8) B) $(2, \frac{1}{8})$ C) $(\frac{1}{2}, -8)$ D) $(\frac{1}{2}, \frac{1}{8})$.
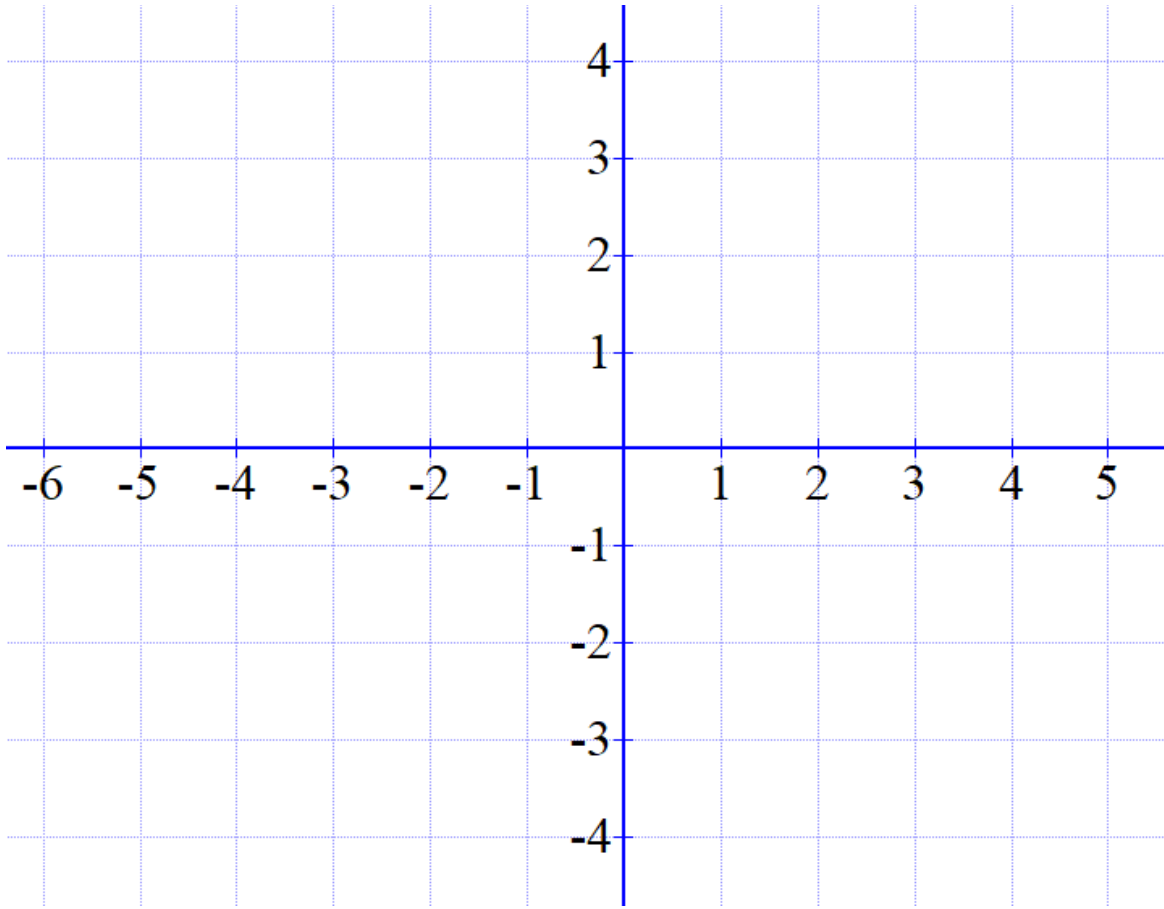
Use the following information to answer the next question.

Consider the graph of f(x) = x + 2 shown below.

$$f(x) = x + 2$$

The questions that follow serve as a guide to sketching the reciprocal of this function.

2. A) Determine the x-intercept of f(x).

   B) Determine the invariant points.

   C) Identify the equations of the asymptotes.

D) Sketch the graph of $y = \dfrac{1}{f(x)}$

E) State the domain and range of $y = \dfrac{1}{f(x)}$.

3. If f(x) = -6x – 5, which of the following is an invariant point for $y = \dfrac{1}{f(x)}$?
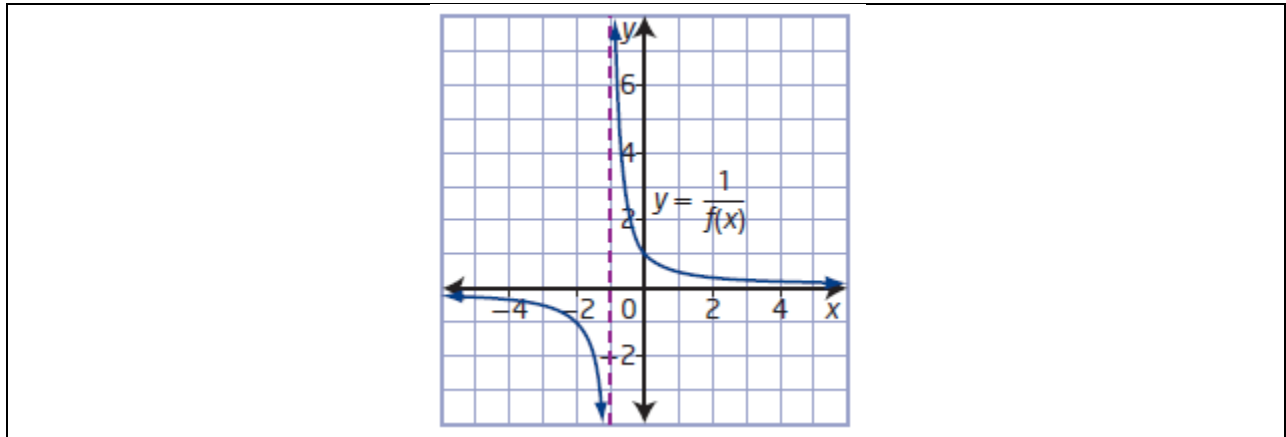
A) $\left(-\dfrac{2}{3}, -1\right)$ B) $\left(\dfrac{2}{3}, 1\right)$ C) (-1,-1)   D) (1,1)

4. When the reciprocal of y = 10x – 50 is graphed, the equation for the vertical asymptote is x = ___.

5. What is the y-intercept of y = f(x)?

6. What is the range of y = f(x)?

Consider the following statements made regarding the reciprocal function:

$$y = \frac{1}{3x - 5}$$

Statement 1	The domain is $x \neq \frac{5}{3}$.
Statement 2	The non-permissible value is 5.
Statement 3	The equation of the vertical asymptote is $x = -\frac{5}{3}$.
Statement 4	The y-intercept is -0.2.

7. The two true statements are

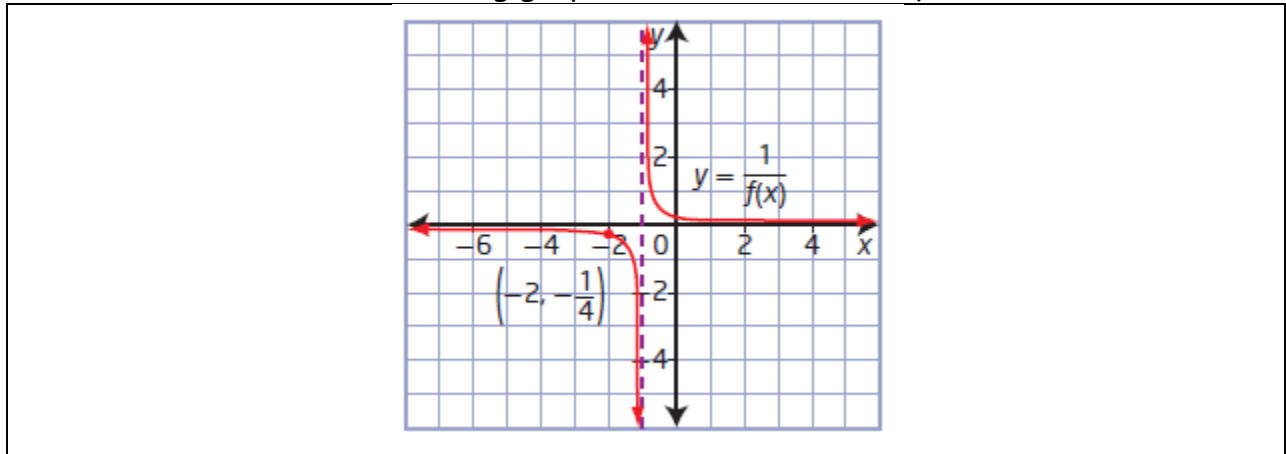   A) 1 and 2   B) 3 and 4   C) 1 and 4   D) 2 and 3

8. The intensity, I, in watts per square metre (W/m2), of a sound equals 0.004 multiplied by the reciprocal of the square of the distance, d, in metres, from the source of the sound.
   a) Write a function for I, in terms of d, to represent this relationship.
   b) What is the intensity of a car horn for a person standing 5 m from the car?

Use the following graph to answer the next question.

9. Determine $y = f(x)$. Justify.

1. If the point $(\frac{1}{2}, 8)$ is on y = f(x), then which of the following points is on the graph of $y = \dfrac{1}{f(x)}$?

   A) (2, 8)   B) $(2, \frac{1}{8})$ C) $(\frac{1}{2}, -8)$ D) $(\frac{1}{2}, \frac{1}{8})$.

Solution

For a given x-coordinate on an original function, take the reciprocal of the **y-coordinate.** For the point $(\frac{1}{2}, 8)$, the given x-coordinate is $\frac{1}{2}$. The y-coordinate on the reciprocal function is $\frac{1}{8}$. The point $(\frac{1}{2}, \frac{1}{8})$ is on the graph of $y = \dfrac{1}{f(x)}$.

The correct answer is D.

Use the following information to answer the next question.

Consider the graph of f(x) = x + 2 shown below.

$$f(x) = x + 2$$
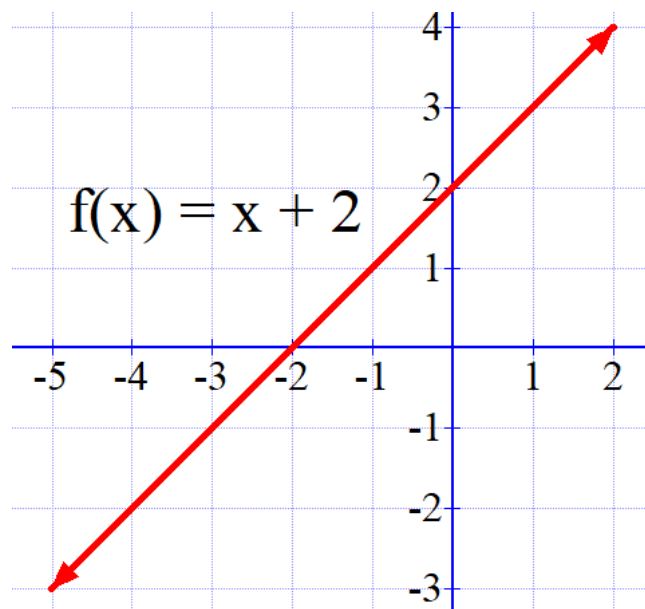
The questions that follow serve as a guide to sketching the reciprocal of this function.

2. A) Determine the x-intercept of f(x).

Solution

The x-intercept is -2. It can either be read from the graph or solved algebraically.

To find the x-intercept, set y = 0 and solve for x.

$\quad$ y = x + 2

$\quad$ (0) = x + 2

$\quad$ -2 = x

B) Determine the invariant points.

Solution

Invariant points are points that do not change when a transformation is applied. They are the same on the original function and the transformed function.

Remembering that it is the **y-coordinate** that is reciprocated, when y = 1, the reciprocal is still 1. When y = -1, the reciprocal is still -1.

To determine invariant points, substitute y = 1 into the original equation and solve the equation for x. Then, substitute y = -1 into the original equation and solve the equation for x.

y = x + 2

(1) = x + 2

-1 = x

y = x + 2

(-1) = x + 2

-3 = x

The two invariant points are (-1, 1) and (-3,-1).

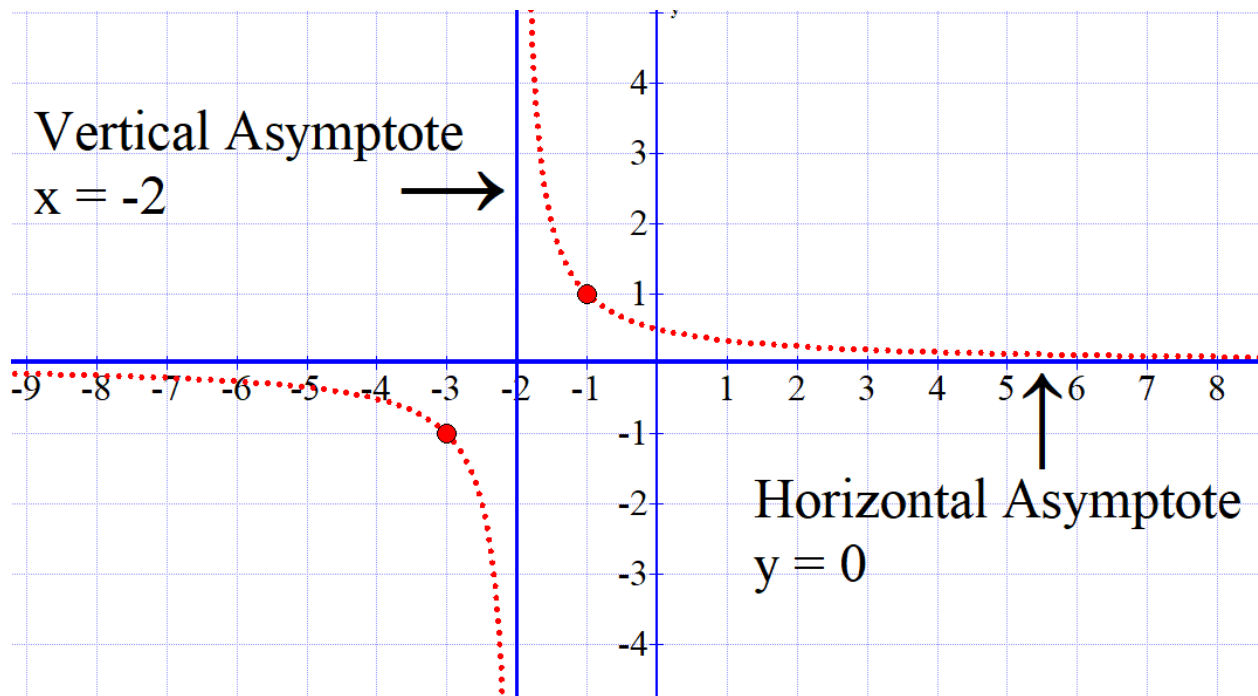C) Identify the equations of the asymptotes.

D) Sketch the graph of $y = \dfrac{1}{f(x)}$

Vertical Asymptote
x = -2 →

Horizontal Asymptote
y = 0

E) State the domain and range of $y = \dfrac{1}{f(x)}$.

3. If f(x) = -6x – 5, which of the following is an invariant point for $y = \dfrac{1}{f(x)}$?

A) $\left(-\dfrac{2}{3}, -1\right)$ B) $\left(\dfrac{2}{3}, 1\right)$ C) (-1,-1)   D) (1,1)

Solution

The invariant points occur when y = 1 and y = -1.

(1)= -6x – 5

6 = -6x

-1 = x   One invariant point is (-1,1).

(-1)=-6x – 5

4 = -6x

$x = -\dfrac{2}{3}$ Another invariant point is $\left(-\dfrac{2}{3}, -1\right)$.

The correct answer is A.

4. When the reciprocal of y = 10x – 50 is graphed, the equation for the vertical asymptote is x = __5__.

Solution

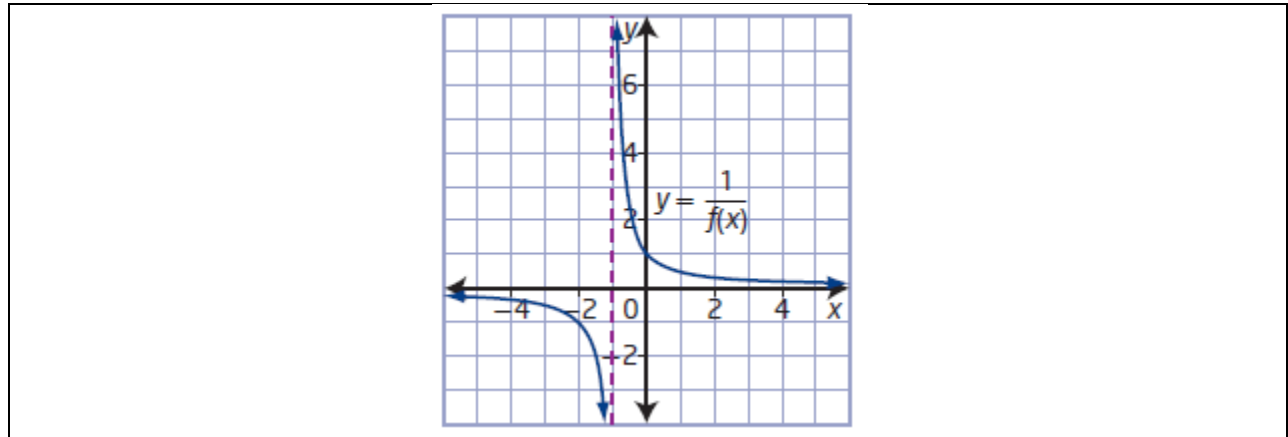The vertical asymptote will occur at the x-intercept. Set y = 0 and solve for x.

(0)=10x – 50

50 = 10x

x = 5   The x-intercept is (5,0).

The equation for the vertical asymptote is x = __5__.

Use the graph below to answer the next two questions.

5. What is the y-intercept of y = f(x)?

Solution

The reciprocal graph shown above has a y-intercept of 1. The reciprocal of 1 is still 1. Therefore, the y-intercept on the original graph y = f(x) is 1.

6. What is the range of y = f(x)?

Solution

The range is $\{y \mid y \in R\}$

Use the following information to answer the next question.

Consider the following statements made regarding the reciprocal function:
$$y = \frac{1}{3x-5}$$

Statement 1	The domain is x ≠ $\frac{5}{3}$.
Statement 2	The non-permissible value is 5.
Statement 3	The equation of the vertical asymptote is x = $-\frac{5}{3}$.
Statement 4	The y-intercept is -0.2.

7. The two true statements are
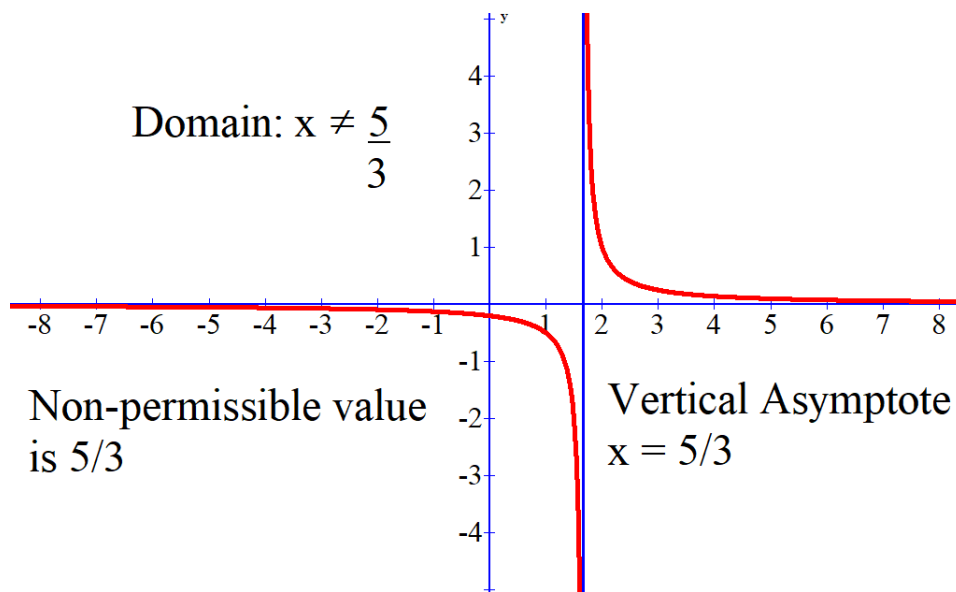
A) 1 and 2   B) 3 and 4   C) 1 and 4   D) 2 and 3

Solution

To find the y-intercept, set x = 0 and solve for y.

$$y = \frac{1}{3(0)-5}$$

$$y = -\frac{1}{5} \, or -0.2$$

Domain: x ≠ $\frac{5}{3}$

Non-permissible value is 5/3

Vertical Asymptote x = 5/3

8. The intensity, I, in watts per square metre (W/m2), of a sound equals 0.004 multiplied by the reciprocal of the square of the distance, d, in metres, from the source of the sound.
    a)Write a function for I, in terms of d, to represent this relationship.
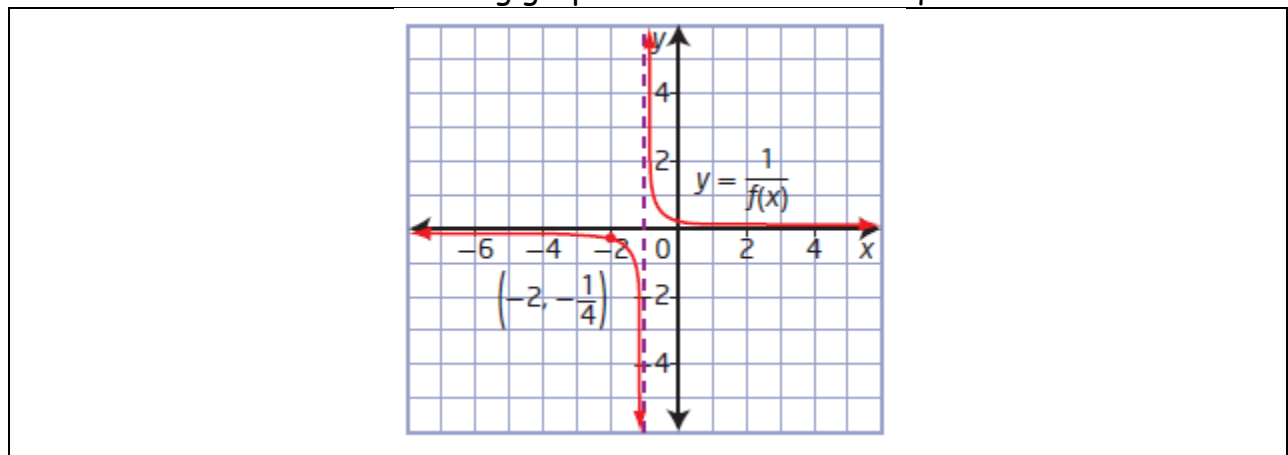   b)What is the intensity of a car horn for a person standing 5 m from the car?

Solution

a) $I = 0.004\left(\dfrac{1}{d^2}\right)$

b) $I = 0.004\left(\dfrac{1}{(5)^2}\right)$

$I = 0.004\left(\dfrac{1}{25}\right)$

I = 0.00016 W/m²

Use the following graph to answer the next question.
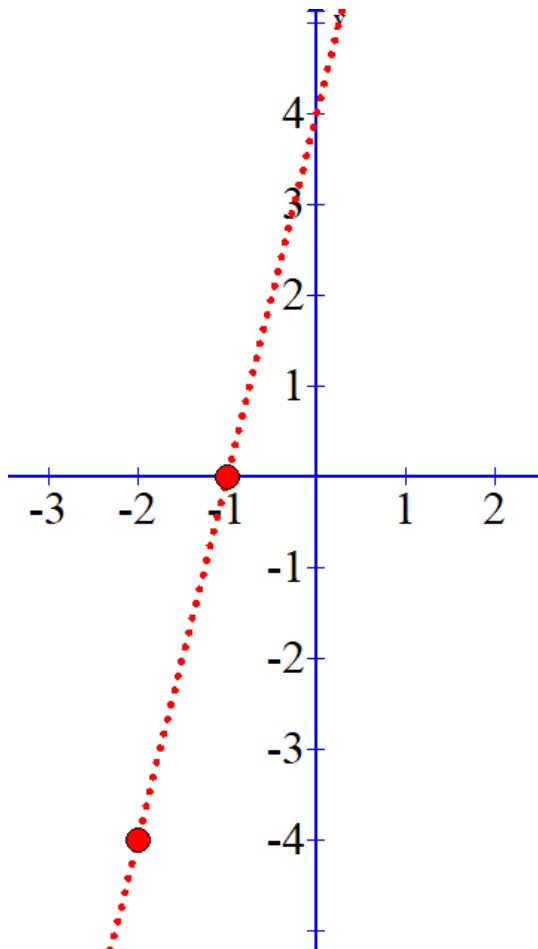

9. Determine y = f(x). Justify.

Solution

Since there is a vertical asymptote at x = -1, we know that the point (-1,0) is on the graph of y = f(x).

Since the point (-2, -$\frac{1}{4}$) is on the reciprocal function, we know that the point

(-2, -4) is on the original function, y = f(x).

Plot the points (-1,0) and (-2,-4). Calculate the slope. The rise is 4 and the run is 1. The line rises to the right so the slope is positive. The slope is 4.

Extend the line and we see that the y-intercept is 4.

The equation of the line is y = 4x + 4.