Finding the Reciprocal of a Quadratic Function Practice

Use the following information to answer the first question.

Given the function f(x) = (x + 8) (x – 3), the following statements are made in regards to the reciprocal function, $y = \frac{1}{f(x)}$.

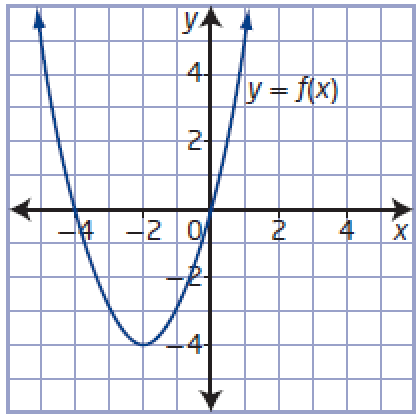
Statement 1	The y-intercept is $-\frac{1}{24}$.
Statement 2	There are no invariant points.
Statement 3	The domain is x ≠ 8 and x ≠ -3.
Statement 4	The equation of one of the asymptotes is x = 3.

1. The two true statements are
   A) 1 and 2 B) 3 and 4 C) 1 and 4 D) 2 and 3

2. When the function $y = x^2 - 10$ and its reciprocal are graphed, one of the invariant points is
   A) (3, -1) B) (10, 1) C) (-1, 3) D) (1, 10)

3. Which of the following quadratic functions has a reciprocal function having a vertical asymptote of x = -2?
   A) $f(x) = x^2 + x - 12$
   B) $f(x) = x^2 + 5x + 6$
   C) $f(x) = x^2 - 16$
   D) $f(x) = -x^2 + 2$

Use the following graph to answer the next question.



4. The domain and range of $y = \frac{1}{f(x)}$ is

A) Domain: {x | x ≠ -4, 0, x Є R} and Range: {y | y ≠ 0, y Є R}.
B) Domain: {x | x ≠ -4, 0, x Є R} and Range: {y | y > 0 and y ≤ -0.25, y Є R}.
C) Domain: {x | x ≠ 0, x Є R} and Range: {y | y ≠ 0, y Є R}.
D) Domain: {x | x ≠ 0, x Є R} and Range: {y | y > 0 and y ≤ -0.25, y Є R}.

5. When $f(x) = -x^2 + 1$ and $y = \frac{1}{f(x)}$ are graphed, there is an invariant point in quadrant 4. The x-coordinate of this invariant point can be written in the form $\sqrt{k}$. The value of k is ____.

6. Which of the following statements is true?

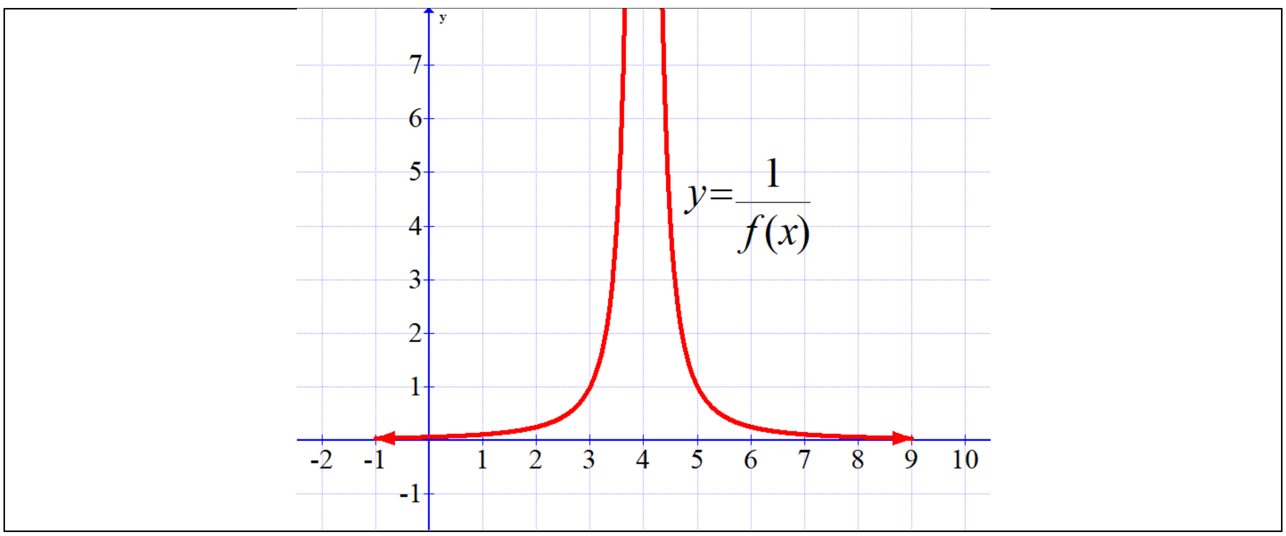
A) The graph of $y = \dfrac{1}{f(x)}$ always has a vertical asymptote.

B) The domain of $y = \dfrac{1}{f(x)}$ is always the same as y = f(x).

C) It is possible to have the reciprocal function of a quadratic equation completely below the x-axis.

D) A function in the form $y = \dfrac{1}{f(x)}$ always has at least one value for which it is not defined.

Use the following diagram to answer the next question.



7. A possible graph for y = f(x) is

A) f(x) = x – 4

B) f(x) = x + 4

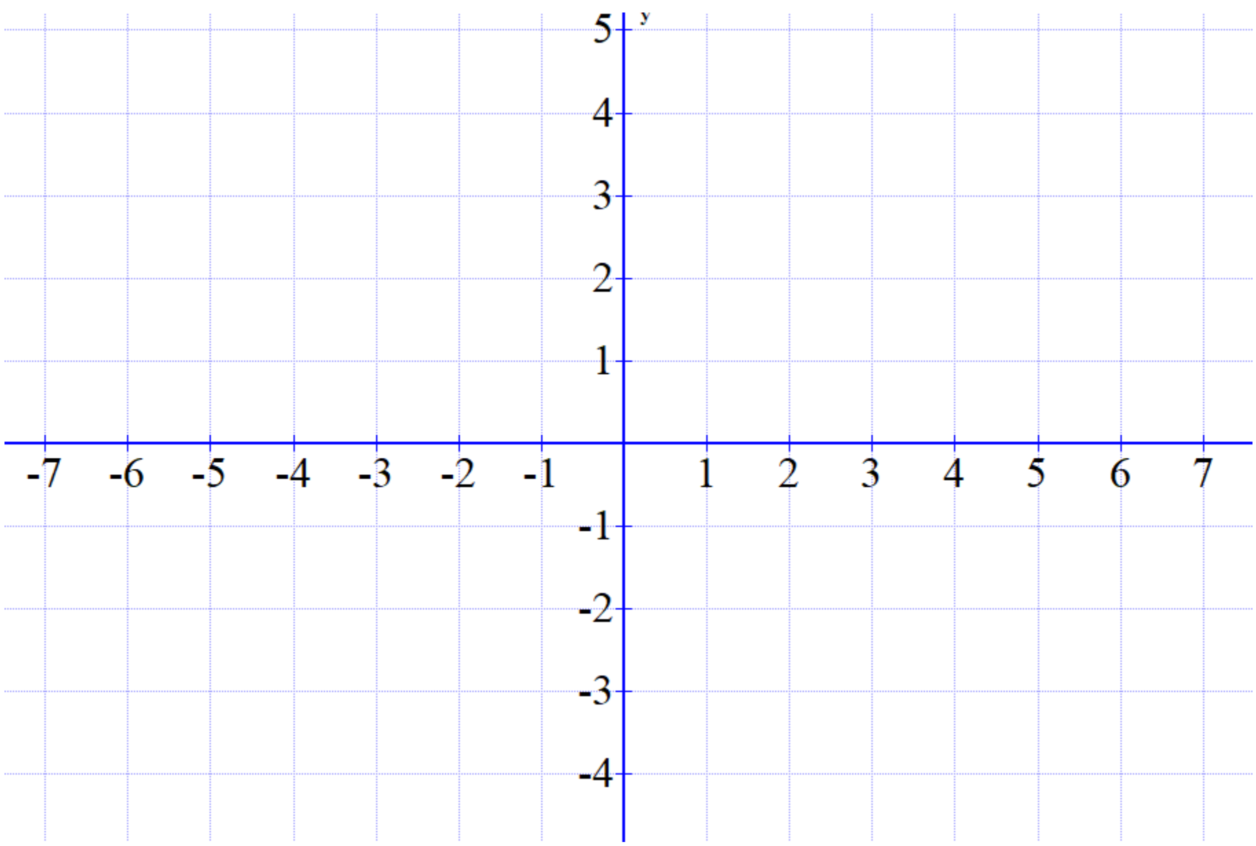
C) f(x) = x^2

D) f(x) = $(x – 4)^2$

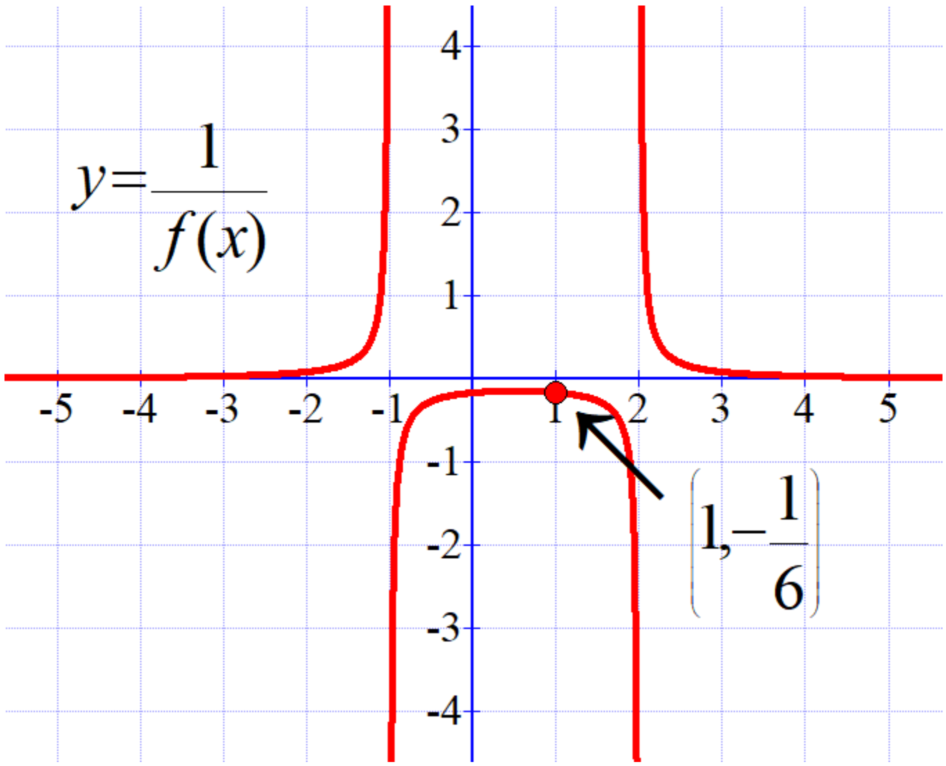
8. a) By completing the square, find the vertex of the parabola defined by $y = -x^2 - 6x - 8$.

b) Explain how finding the vertex will help in sketching its reciprocal function.

c) Sketch the reciprocal of $y = \frac{1}{-x^2 - 6x - 8}$. Identify all invariant points, all vertical asymptotes, the domain and the range.



9. Determine the quadratic equation in the form of $f(x) = a(x - b)(x + c)$, where a, b, and c are integers. Show all work.



Finding the Reciprocal of a Quadratic Function Practice Solutions

Use the following information to answer the first question.

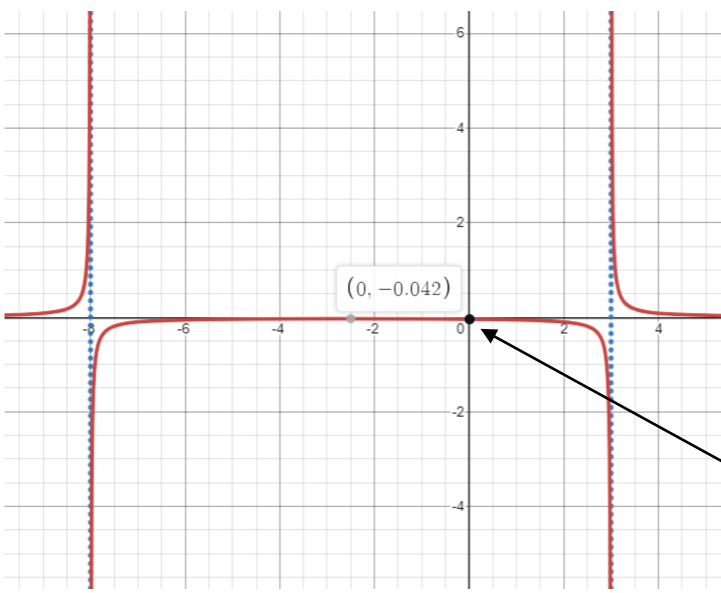
Given the function f(x) = (x + 8) (x – 3), the following statements are made in regards to the reciprocal function, $y = \frac{1}{f(x)}$.

Statement 1	The y-intercept is $-\frac{1}{24}$.
Statement 2	There are no invariant points.
Statement 3	The domain is x ≠ 8 and x ≠ -3.
Statement 4	The equation of one of the asymptotes is x = 3.

1. The two true statements are

   A) 1 and 2 B) 3 and 4 C) 1 and 4 D) 2 and 3

Solution



The y-intercept of y = f(x) is -24.

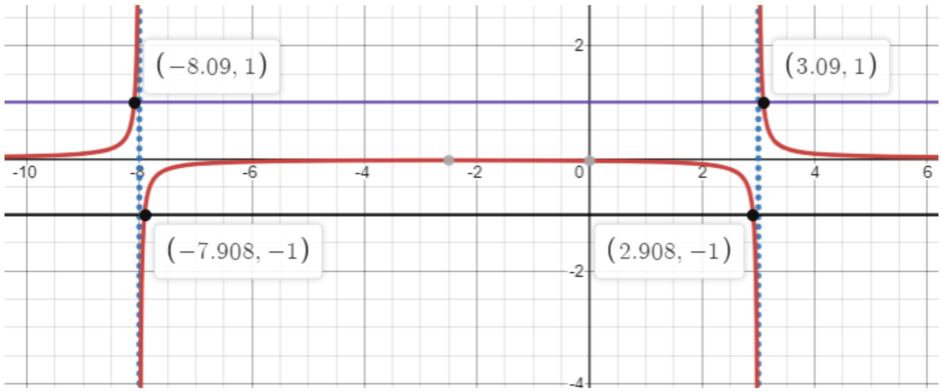
y = ((0)+8) ((0) – 3))

y = (8)(-3)

y = -24

On the reciprocal function, the y-intercept is (0, -1/24). (*or -0.042*)

Statement 1 is true.



There are 4 invariant points; two where y = 1 and two where y = -1.

Statement 2 is false.

The domain is x ≠ -8 and x ≠ 3.

Statement 3 is false.

The equations of the vertical asymptotes are x = -8 and x = 3.

Statement 4 is true.

The correct answer is C.

2. When the function y = x^2 – 10 and its reciprocal are graphed, one of the invariant points is

   A) (3, -1) B) (10, 1) C) (-1, 3) D) (1, 10)

Solution

The invariant points are unchanged following a transformation. For a reciprocal function, the invariant points occur when y = 1 and y = -1.

(1) = x^2 - 10

11 = x^2

$x = \pm\sqrt{11}$

Two of the invariant points are $(\sqrt{11}, 1)$ and $(-\sqrt{11}, 1)$.

$(-1) = x^2 - 10$

$9 = x^2$

$x = \pm 3$

The other two invariant points are (-3, -1) and (3, -1).

The correct answer is A.

3. Which of the following quadratic functions has a reciprocal function having a vertical asymptote of $x = -2$?
   A) $f(x) = x^2 + x - 12$
   B) $f(x) = x^2 + 5x + 6$
   C) $f(x) = x^2 - 16$
   D) $f(x) = -x^2 + 2$

Solution

Writing the equations in factored form will help us to determine the x-intercepts (or non-permissible values), which will in turn help us to determine the vertical asymptotes.

Option A

$f(x) = x^2 + x - 12$

$f(x) = (x + 4)(x - 3)$

The vertical asymptotes are $x = -4$ and $x = 3$.

Option B

$f(x) = x^2 + 5x + 6$

$f(x) = (x + 2)(x + 3)$

The vertical asymptotes are $x = -2$ and $x = -3$.

Option C

$f(x) = x^2 - 16$

$f(x) = (x - 4)(x + 4)$

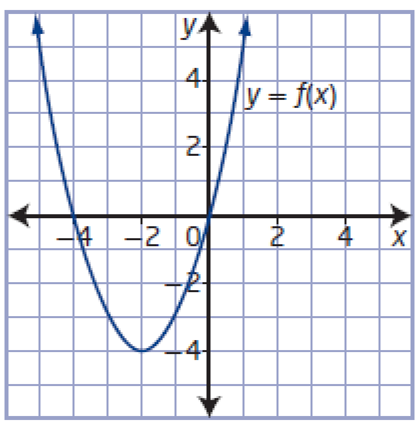
The vertical asymptotes are x = 4 and x = -4.

Option D

This equation involves no factoring. But by substituting 0 for y and finding the x-intercepts, we see that the x-intercepts are $\pm\sqrt{2}$. The vertical asymptotes are $x = \sqrt{2}$ and $x = -\sqrt{2}$

The correct answer is B.

Use the following graph to answer the next question.



4. The domain and range of $y = \dfrac{1}{f(x)}$ is

A) Domain: {x | x ≠ -4, 0, x Є R} and Range: {y | y ≠ 0, y Є R}.
B) Domain: {x | x ≠ -4, 0, x Є R} and Range: {y | y > 0 and y ≤ -0.25, y Є R}.
C) Domain: {x | x ≠ 0, x Є R} and Range: {y | y ≠ 0, y Є R}.
D) Domain: {x | x ≠ 0, x Є R} and Range: {y | y > 0 and y ≤ -0.25, y Є R}.

Solution

Sometimes it is helpful to have a picture of the graph to determine the domain and range. The equation of y = f(x) is not given but it can be determined. The vertex is (-2,-4). Using the form, $y = a(x - p)^2 + q$, we substitute, $y = a(x + 2)^2 - 4$. To determine *a*, we need a point on the graph. We can use (-1, -3).

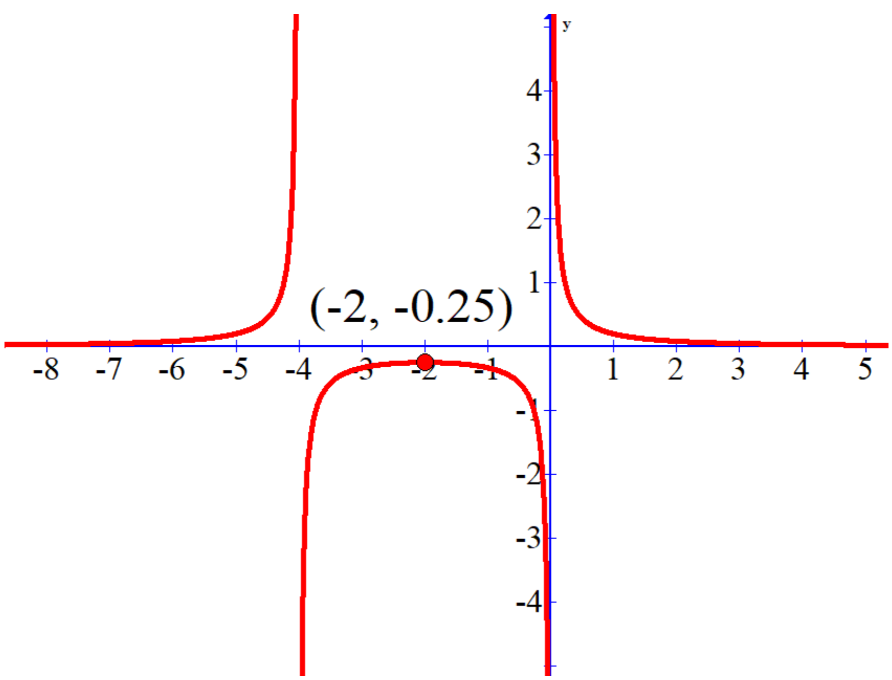
$-3 = a((-1) + 2)^2 - 4$

$-3 = a - 4$

$a = 1$

Since the quadratic equation is $y = (x + 2)^2 - 4$, the graph below is its reciprocal,

$$y = \frac{1}{(x+2)^2 - 4}$$



There are only two values that cannot be part of the domain. These are the non-permissible values found at the x-intercepts. If these values were allowed, there would be a denominator equal to zero. Since dividing by zero is undefined, it is not possible for them to be elements in the domain. The domain is x ≠ -4, 0.

In terms of the range, looking above the x-axis, y can be any value greater than zero. Looking below the x-axis, y can be an value less than or equal to -0.25. This value represents the reciprocal of the lowest point on the original quadratic function (-4). The range is $y > 0$ and $y \le -0.25$.

The correct answer is B.

5. When $f(x) = -x^2 + 1$ and $y = \frac{1}{f(x)}$ are graphed, there is an invariant point in quadrant 4. The x-coordinate of this invariant point can be written in the form $\sqrt{k}$. The value of k is _2__.

Solution

To find invariant points, set y = 1 and solve for x. Then, set y = -1 and solve for x.

$y = -x^2 + 1$

$(1) = -x^2 + 1$

$0 = -x^2$

$x = 0$ One invariant point is (0,1).

$(-1)= -x^2 + 1$

$-2 = -x^2$

$2 = x^2$

$x = \pm\sqrt{2}$ Two other invariant points are $(\sqrt{2},-1)$ and $(-\sqrt{2},-1)$

The invariant point, $(\sqrt{2},-1)$ is in quadrant 4.

The x-coordinate of this invariant point can be written in the form $\sqrt{k}$. The value of k is _2__.

6. Which of the following statements is true?

A) The graph of $y=\frac{1}{f(x)}$ always has a vertical asymptote.

B) The domain of $y=\frac{1}{f(x)}$ is always the same as y = f(x).

C) It is possible to have the reciprocal function of a quadratic equation completely below the x-axis.

D) A function in the form $y=\frac{1}{f(x)}$ always has at least one value for which it is not defined.

Solution

Statement A is false. If the complete graph of y = f(x) is either completely above the x-axis or completely below the x-axis, and as such have no x-intercepts, there will not be a vertical asymptote.

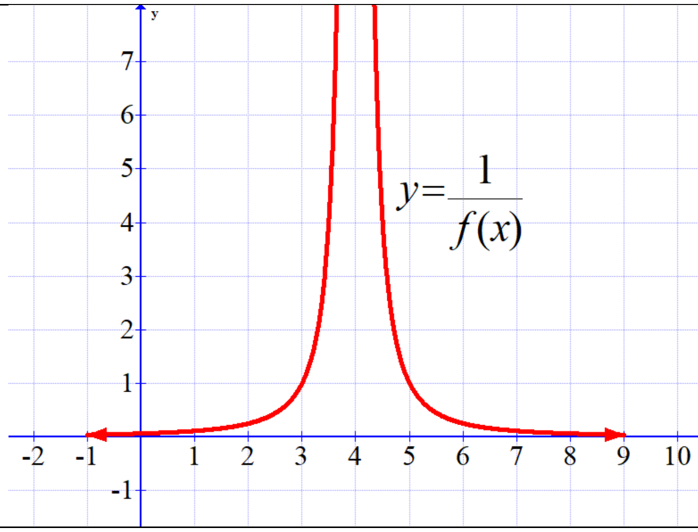
Statement B is false. Quadratic functions typically have a domain of $x \in R$. Many quadratic functions have x-intercepts. With an x-intercept of x = m, the domain is such that $x \neq m$. The domains would therefore not be the same.

Statement C is true. If the original function, for example, $y = -x^2 - 4$, is totally below the x-axis, then reciprocating all of these negative y values, will still result in negative y values.

Statement D is false. Only if the function has a zero is this true.

The correct answer is C.

Use the following diagram to answer the next question.



7. A possible graph for y = f(x) is
   A) f(x) = x – 4
   B) f(x) = x + 4
   C) f(x) = x^2
   D) f(x) = $(x - 4)^2$

Solution

The shape of this graph will occur if the vertex is on the x-axis. Since the asymptote is at x = 4, the vertex would be (4,0). A possible equation for this graph is y = $(x - 4)^2$.

The correct answer is D.

8. a) By completing the square, find the vertex of the parabola defined by $y = -x^2 - 6x - 8$.

Solution

Factor (-1) out of the first two terms.

$y = -1(x^2 + 6x) - 8$

Take half of 6 and square it. Add and subtract this number inside the brackets.

$y = -1(x^2 + 6x + 9 - 9) - 8$

Multiply (-1)(-9) and put this number outside of the brackets.

$y = -1(x^2 + 6x + 9) + 9 - 8$

Complete the square.

$y = -1(x + 3)^2 + 1$

The vertex is (-3,1).

b) Explain how finding the vertex will help in sketching its reciprocal function.

Solution

Since the vertex is above the x-axis, it will allow us to see how close the corresponding point on the reciprocal function will move towards the x-axis. In this case, it is an invariant point, so it does not move at all.

c) Sketch the reciprocal of $y = \frac{1}{-x^2 - 6x - 8}$. Identify all invariant points, all vertical asymptotes, the domain and the range.

Solution

Re-write the quadratic function in factored form.

$-x^2 - 6x - 8 = -(x + 4)(x + 2)$

The vertical asymptotes are $x = -4$ and $x = -2$.

Given the equation, $y = -x^2 - 6x - 8$, set $y = 1$ and solve for x.

$(1)= -x^2 - 6x - 8$

$0 = -x^2 - 6x - 9$

$0 = -(x^2 + 6x + 9)$

$0 = -(x + 3)^2$

$x = -3$ There is one invariant point above the x-axis. It is $(-3, 1)$.

Now set $y = -1$ and solve for x.

$(-1) = -x^2 - 6x - 8$

$0 = -x^2 - 6x - 7$

Use the quadratic formula: $a = -1$, $b = -6$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-1)(-7)}}{2(-1)}$$

$$x = \frac{6 \pm \sqrt{36 - 28}}{-2}$$

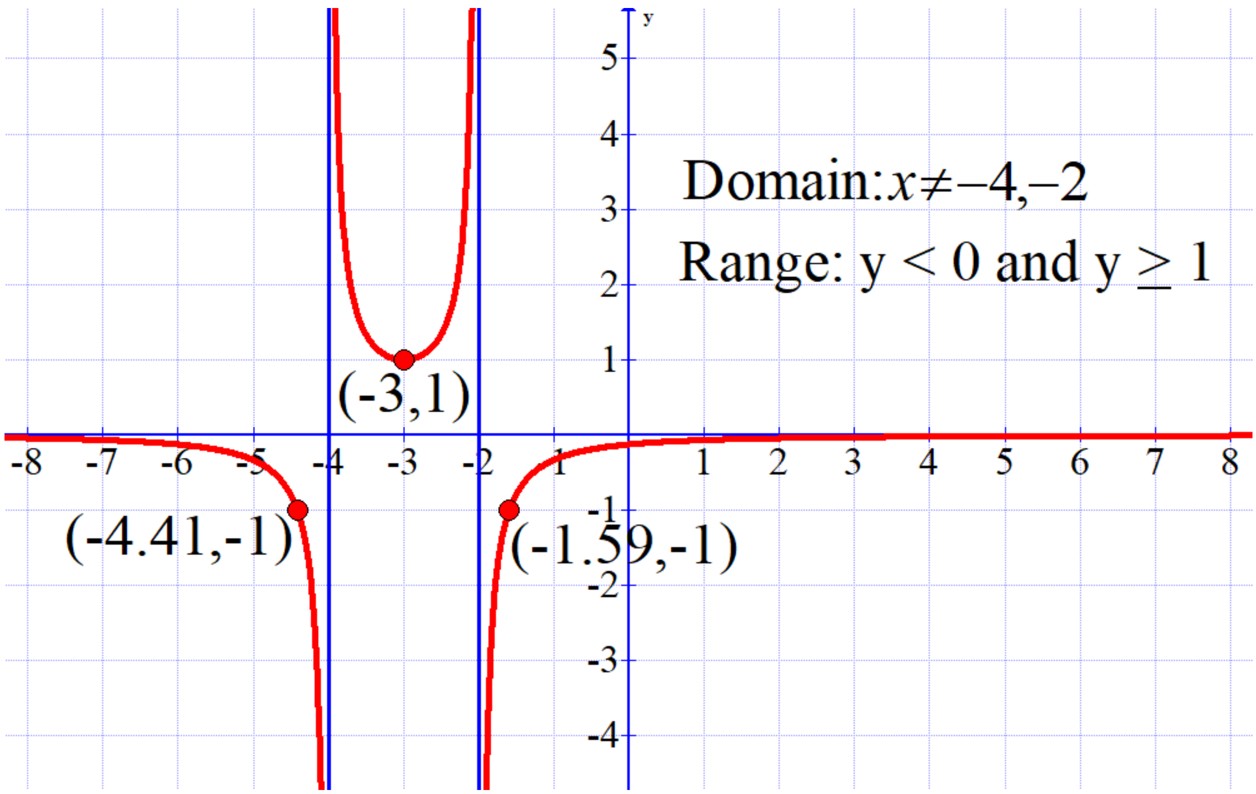
$$x = \frac{6 \pm \sqrt{8}}{-2}$$

$$x = \frac{6 \pm 2\sqrt{2}}{-2}$$

$$x = -3 \pm \sqrt{2}$$

As decimal approximations to two decimals, $x \approx -1.59$ and $x \approx -4.41$.

The invariant points below the x-axis are (-4.41, -1) and (-1.59, -1).



9. Determine the quadratic equation in the form of f(x) = a(x – b)(x + c), where a, b, and c are integers. Show all work.

Solution

Since the vertical asymptotes are x = -1 and x = 2, the binomial equivalents in the equation would be (x - 2) and (x + 1). Thus, b = 2 and c = 1.

Since the point (1, -1/6) is on the reciprocal function, (1, -6) must be on the original function. Use this point to find the value of *a*.

y = a(x – 2)(x + 1)

Substitute the point (1, -6) into the equation for x and y.

$(-6) = a((1) - 2)((1) + 1)$

$-6 = a(-1)(2)$

$-6 = -2a$

$a = 3$

The quadratic equation is $f(x) = 3(x - 2)(x + 1)$.