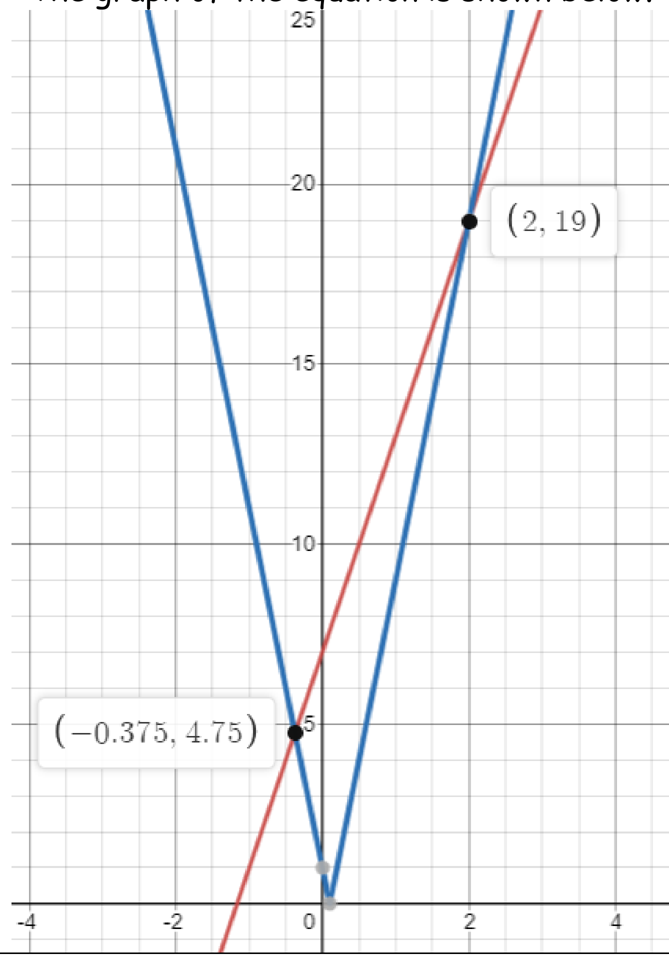
Absolute Value Equations Practice

1. One solution to the equation $|-5x - 2| = 13$, is -3. The other solution, accurate to the nearest tenth, is _____.

Use the following information to answer the next question.

A math student was given the following absolute value equation, in the form, $|ax - 1| = bx + 7$, where a and b are integers.

The graph of the equation is shown below.



2. The solution(s) is/are

A) 19 and 4.75 B) 2 C) -0.375 D) 2 and -0.375

3. When solving the absolute value equation $|x^2 + 5| = 3x + 9$, a quadratic equation that would lead to a solution is

   A) $x^2 + 3x - 14 = 0$

   B) $x^2 - 3x + 14 = 0$

   C) $x^2 - 3x - 4 = 0$

   D) $x^2 - 3x + 4 = 0$

4. The extraneous root for the absolute value equation $|3x + 2| = 4x + 5$, is

   A) -3 B) -1 C) 3 D) 1

Use the following information to answer the next question.

A math student was asked to solve the following absolute value equation:

$$x + |5x - 2| = 6$$

Her work is shown below.

Step 1	$\|5x - 2\| = 6 - x$
Step 2	$5x - 2 = 6 - x$
Step 3	$x = 1$
Step 4	$-5x + 2 = 6 - x$
Step 5	$x = -1$

5. Unfortunately, she made an error, that occurred in step

   A) 2 B) 3 C) 4 D) 5

6. The largest solution to the equation, $2|1 - 4m| = 6m + 2$, is ____.

7. Before the start of a soccer game, the balls must be inflated to a pressure, p, of 13 psi, within an absolute value error of 0.5 psi. The equation that could be used to determine the minimum and maximum acceptable air pressure for soccer balls is
   A) $|0.5 + p| = 13$
   B) $|13 + p| = 0.5$
   C) $|p - 13| = 0.5$
   D) $|p - 0.5| = 13$

Use the following information to answer the next question.

Consider the absolute value equation:
$\lvert x^2 - x - 8 \rvert = -x + 1$

8. A) Verify that $x = -3$ is a solution.

   B) One of the solutions can be written in the form, $x = M - K\sqrt{2}$. Determine the values of M and K.

<u>Absolute Value Equations Practice</u><u>Solutions</u>

1. One solution to the equation $|-5x - 2| = 13$, is -3. The other solution, accurate to the nearest tenth, is _<u>2.2</u>_.

Solution

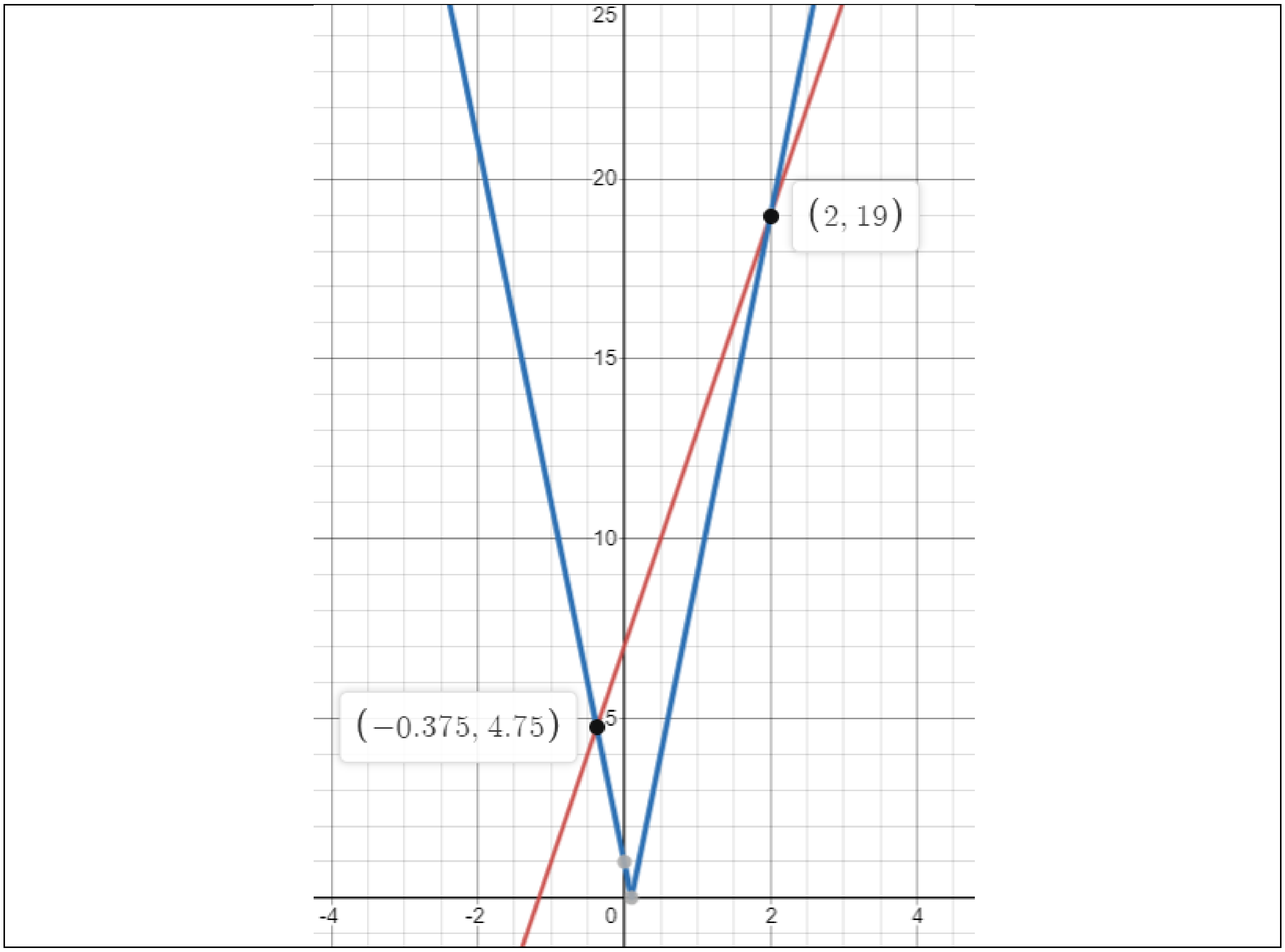
Consider the two cases; the value inside the absolute value can be positive or negative.

<u>Case 1 Positive</u>	<u>Case 2 Negative</u>
$-5x - 2 = 13$	$-(-5x - 2) = 13$
$-5x = 15$	$5x + 2 = 13$
$x = -3$	$5x = 11$
	$x = 2.2$

The other solution, accurate to the nearest tenth, is _<u>2.2</u>_.

Use the following information to answer the next question.

A math student was given the following absolute value equation, in the form, $|ax - 1| = bx + 7$, where a and b are integers. The graph of the equation is shown below.



2. The solution(s) is/are

   A) 19 and 4.75 B) 2 C) -0.375 D) 2 and -0.375

Solution

The solutions are the x-coordinates of the intersection points. The x-coordinates of the intersection points are -0.375 and 2.

The correct answer is D.

3. When solving the absolute value equation $|x^2 + 5| = 3x + 9$, a quadratic equation that would lead to a solution is

   A) $x^2 + 3x - 14 = 0$

   B) $x^2 - 3x + 14 = 0$

   C) $x^2 - 3x - 4 = 0$

   D) $x^2 - 3x + 4 = 0$

Solution

Consider the two cases; the value inside the absolute value can be positive or negative.

Case 1 Positive	Case 2 Negative
$x^2 + 5 = 3x + 9$	$-(x^2 + 5) = 3x + 9$
$\mathbf{x^2 - 3x - 4 = 0}$	$-x^2 - 5 = 3x + 9$
	$0 = x^2 + 3x + 14$

The correct answer is C.

4. The extraneous root for the absolute value equation $|3x + 2| = 4x + 5$, is

   A) -3 B) -1 C) 3 D) 1

Solution

Consider the two cases; the value inside the absolute value can be positive or negative.

Case 1 Positive	Case 2 Negative
$3x + 2 = 4x + 5$	$-(3x + 2) = 4x + 5$
$-3 = x$	$-3x - 2 = 4x + 5$
	$-7 = 7x$
	$x = -1$

<u>Verify</u>

<u>$x = -3$</u>

$|3x + 2| = 4x + 5$

$|3(-3) + 2| = 4(-3) + 5$

$|-7| = -7$

$7 \neq -7$

<u>$x = -1$</u>

$|3x + 2| = 4x + 5$

$|3(-1) + 2| = 4(-1) + 5$

$|-1| = 1$

$1 = 1$

The solution is $x = -1$ and $x = -3$ is an extraneous root.

The correct answer is A.

Use the following information to answer the next question.

A math student was asked to solve the following absolute value equation:

$x + |5x - 2| = 6$

Her work is shown below.

Step 1	$|5x - 2| = 6 - x$
Step 2	$5x - 2 = 6 - x$
Step 3	$x = 1$
Step 4	$-5x + 2 = 6 - x$
Step 5	$x = -1$

5. Unfortunately, she made an error, that occurred in step

A) 2 B) 3 C) 4 D) 5

Solution

In step 3, $x \neq 1$.

From step 2:

$5x - 2 = 6 - x$

$6x = 8$

$x = \frac{4}{3}$

The correct answer is B.

6. The largest solution to the equation, $2|1 - 4m| = 6m + 2$, is __2__.

Solution

Isolate the absolute value expression by dividing every term by 2.

$|1 - 4m| = 3m + 1$

Consider the two cases; the value inside the absolute value can be positive or negative.

Case 1 Positive	Case 2 Negative
$1 - 4m = 3m + 1$	$-(1 - 4m) = 3m + 1$
$0 = 7m$	$-1 + 4m = 3m + 1$
$m = 0$	$m = 2$

Verify

$m = 0$	$m = 2$
$|1 - 4m| = 3m + 1$	$|1 - 4m| = 3m + 1$
$|1 - 4(0)| = 3(0) + 1$	$|1 - 4(2)| = 3(2) + 1$
$|1| = 1$	$|-7| = 7$
$1 = 1$	$7 = 7$

There are two solutions. The values of m are 0 and 2.

The largest solution to the equation, $2|1 - 4m| = 6m + 2$, is __2__.

7. Before the start of a soccer game, the balls must be inflated to a pressure, p, of 13 psi, within an absolute value error of 0.5 psi. The equation that could be used to determine the minimum and maximum acceptable air pressure for soccer balls is
   A) $|0.5 + p| = 13$
   B) $|13 + p| = 0.5$
   C) $|p - 13| = 0.5$
   D) $|p - 0.5| = 13$

Solution

The absolute value of the *difference* between 13 and a number (p in this case) must be 0.5.

The correct answer is C.

Use the following information to answer the next question.

Consider the absolute value equation:
$\lvert x^2 - x - 8 \rvert = -x + 1$

8. A) Verify that $x = -3$ is a solution.

Solution

Substitute $x = -3$.

$$|(-3)^2 - (-3) - 8| = -(-3) + 1$$

$$|9 + 3 - 8| = 3 + 1$$

$$|4| = 4$$

$$4 = 4$$

B) One of the solutions can be written in the form, $x = M - K\sqrt{2}$. Determine the values of M and K.

Solution

Case 1 Positive

$x^2 - x - 8 = -x + 1$

$x^2 = 9$

Take the square root of both sides

$x = \pm 3$ (*$x = 3$ is extraneous*)

Case 2 Negative

$-(x^2 - x - 8) = -x + 1$

$-x^2 + x + 8 = -x + 1$

$0 = x^2 - 2x - 7$

Use the quadratic formula

Given, $0 = x^2 - 2x - 7$, $a = 1$, $b = -2$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 28}}{2}$$

$$x = \frac{2 \pm \sqrt{32}}{2}$$

$$x = \frac{2 \pm \sqrt{16}\sqrt{2}}{2}$$

$$x = \frac{2 \pm 4\sqrt{2}}{2}$$

$$x = 1 \pm 2\sqrt{2}$$

The value of M is 1 and the value of K is 2.