<u>Radical Equations Practice</u>

1. The restriction on the following radical equation, $4+\sqrt{x-5}=6$, is

   A) x ≥ 4 B) x ≥ 5 C) x ≥ 0 D) x ≥ 9

Use the following information to answer the next question.

Consider the following radical equations.

A.	$\sqrt{2x+1}-9=0$
B.	$\sqrt{6x+3}=\frac{1}{2}$
C.	$12=\sqrt{1-2x}$
D.	$-2+\sqrt{8x-4}=1$

2. Which equation above has a restriction of $x\le\frac{1}{2}$?

   A) A B) B C) C D)D

3. The solution to $-4+\sqrt{3x+9}=2$ is ____.

4. Given the radical equation, $3+\sqrt{5-x}=-x+6$, the extraneous root is ____.

5. After squaring both sides and then setting the equation equal to zero, the equivalent equation to $3\sqrt{7-2y}=5+y$, is

   A) $y^2 + 28y - 38 = 0$
   B) $y^2 - 8y + 38 = 0$
   C) $y^2 + 16y + 4 = 0$
   D) $y^2 - 18y - 38 = 0$

Use the following information to answer the next question.

A math student was asked to solve the following radical equation:

$$-7+\sqrt{x+1}=\sqrt{2x}-8$$

His partial work is shown below.

Step 1	$1+\sqrt{x+1}=\sqrt{2x}$
Step 2	$\left(1+\sqrt{x+1}\right)^2=\left(\sqrt{2x}\right)^2$
Step 3	$1+2\sqrt{x+1}+(x+1)=2x$
Step 4	$2\sqrt{x+1}=x-2$
Step 5	$\left(2\sqrt{x+1}\right)^2=\left(x-2\right)^2$
Step 6	$2(x + 1) = x^2 - 4x + 4$
Step 7	$0 = x^2 - 6x + 2$

6. Identify the error, make the correction and state the correct solution.

7. Solve for x and state the answer as exact roots.

$$\sqrt{\frac{x^2}{3}+1}=x+2$$

8. Solve and check the following radical equation.

$\sqrt{5x} + 10 = -2\sqrt{x+11} + 23$

9. The lateral surface area of a right circular cone, s, can be represented by the equation, $s = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the circular base and h is the height of the cone. If the lateral surface area of a large cone-shaped funnel is 378.75 cm^2 and the radius is 6 cm, find the height, to the nearest tenth of a centimetre.

Radical Equations Practice Solutions

1. The restriction on the following radical equation, $4+\sqrt{x-5}=6$, is

A) x ≥ 4 B) x ≥ 5 C) x ≥ 0 D) x ≥ 9

Solution

When dealing with radical equations, the restriction is related to the radicand. If the index is even, as it is here (i.e. 2), there are some values that are not permissible. The value under the radical sign cannot be less than zero (assuming that only real numbers are allowed), as we cannot take the square root of a negative number.

The radicand in this case is (x – 5). Therefore, x – 5 ≥ 0. When x is isolated by adding 5 to both sides, the restriction is determined as x ≥ 5.

The correct answer is B.

Use the following information to answer the next question.

Consider the following radical equations.

A.	$\sqrt{2x+1}-9=0$
B.	$\sqrt{6x+3}=\frac{1}{2}$
C.	$12=\sqrt{1-2x}$
D.	$-2+\sqrt{8x-4}=1$

2. Which equation above has a restriction of $x\le\frac{1}{2}$?

A) A B) B C) C D)D

Solution

Create an inequality statement for each radicand, and isolate the variable.

Option A

2x + 1 ≥ 0

2x ≥ -1

$x \geq -\frac{1}{2}$

The restriction for Option A is $x \geq -\frac{1}{2}$.

Option B

6x + 3 ≥ 0

6x ≥ -3

$x \geq -\frac{1}{2}$

The restriction for Option B is $x \geq -\frac{1}{2}$.

Option C

1 – 2x ≥ 0

– 2x ≥ -1

[Since we need to divide both sides of an inequality by a negative, the sign is reversed].

$x \leq \frac{1}{2}$

The restriction for Option C is $x \leq \frac{1}{2}$.

Option D

8x – 4 ≥ 0

8x ≥ 4

$x \geq \frac{1}{2}$

The restriction for Option D is $x \geq \frac{1}{2}$.

The correct answer is C.

3. The solution to $-4+\sqrt{3x+9}=2$ is __3__.

Solution

Isolate the radical by adding 4 to both sides.

$\sqrt{3x+9}=6$

Square both sides to remove the radical sign.

$\left(\sqrt{3x+9}\right)^2=(6)^2$

$3x + 9 = 36$

$3x = 27$

$x = 3$

4. Given the radical equation, $3+\sqrt{5-x}=-x+6$, the extraneous root is __4__.

Solution

Isolate the radical by subtracting 3 from both sides.

$\sqrt{5-x}=-x+3$

Square both sides to remove the radical sign.

$\left(\sqrt{5-x}\right)^2=(-x+3)^2$

$5 - x = x^2 - 6x + 9$

Set the quadratic equation equal to zero.

$x^2 - 5x + 4 = 0$

Factor and solve.

(x – 1) (x – 4) = 0

Using the zero product principle, x = 1 or 4.

Check

x = 1	x = 4
$3+\sqrt{5-(1)}=-(1)+6$	$3+\sqrt{5-(4)}=-(4)+6$
$3+\sqrt{4}=5$	$3+\sqrt{1}=2$
5 = 5	4 ≠ 2

The solution is 1, and 4 is an extraneous root.

5. After squaring both sides and then setting the equation equal to zero, the equivalent equation to $3\sqrt{7-2y}=5+y$, is
   A) $y^2 + 28y - 38 = 0$
   B) $y^2 - 8y + 38 = 0$
   C) $y^2 + 16y + 4 = 0$
   D) $y^2 - 18y - 38 = 0$

Solution

$9(7 - 2y) = 25 + 10y + y^2$

$63 - 18y = 25 + 10y + y^2$

Add 18y to both sides, and subtract 63 from both sides.

$0 = y^2 + 28y - 38$

The correct answer is A.

Use the following information to answer the next question.

A math student was asked to solve the following radical equation:

$$-7+\sqrt{x+1}=\sqrt{2x}-8$$

His partial work is shown below.

Step 1	$1+\sqrt{x+1}=\sqrt{2x}$
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Step 5	$\left(2\sqrt{x+1}\right)^2=(x-2)^2$
Step 6	$2(x + 1) = x^2 - 4x + 4$
Step 7	$0 = x^2 - 6x + 2$

6. Identify the error, make the correction and state the correct solution.

Solution

The error is made in step 6. When squaring $2\sqrt{x+1}$, both the integer and the radical must be squared. Step 6 should be **4**$(x + 1) = x^2 - 4x + 4$.

The following steps are:

$4x + 4 = x^2 - 4x + 4$.

$0 = x^2 - 8x$

$0 = x(x - 8)$

$x = 0$ and $x = 8$

<u>x = 0</u>

$$-7+\sqrt{(0)+1}=\sqrt{2(0)}-8$$

<u>x = 8</u>

$$-7+\sqrt{(8)+1}=\sqrt{2(8)}-8$$

$-7+\sqrt{1}=\sqrt{0}-8$ $\quad\quad$ $-7+\sqrt{9}=\sqrt{16}-8$

-7 + 1 = 0 - 8 $\quad\quad$ -7 + 3 = 4 - 8

-6 ≠ -8 $\quad\quad$ -4 = -4

The solution is x = 8, and there is an extraneous root of x = 0.

7. Solve for x and state the answer as exact roots.

$$\sqrt{\frac{x^2}{3}+1}=x+2$$

Solution

Square both sides to remove the radical sign.

$$\left(\sqrt{\frac{x^2}{3}+1}\right)^2=(x+2)^2$$

$$\frac{x^2}{3}+1=x^2+4x+4$$

Multiply every term by 3 to clear the fraction.

$$3\left(\frac{x^2}{3}+1=x^2+4x+4\right)$$

$x^2 + 3 = 3x^2 + 12x + 12$

$0 = 2x^2 + 12x + 9$

Use the quadratic formula.

a = 2 $\quad$ b = 12 $\quad$ c = 9

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(2)(9)}}{2(2)}$$

$$x = \frac{-12 \pm \sqrt{144 - 72}}{4}$$

$$x = \frac{-12 \pm \sqrt{72}}{4}$$

$$x = \frac{-12 \pm \sqrt{36} X \sqrt{2}}{4}$$

$$x = \frac{-12 \pm 6\sqrt{2}}{4}$$

$$x = \frac{-6 \pm 3\sqrt{2}}{2}$$

The exact roots are $x = \dfrac{-6 \pm 3\sqrt{2}}{2}$.

8. Solve and check the following radical equation.

$$\sqrt{5x} + 10 = -2\sqrt{x + 11} + 23$$

Solution

Upon initial inspection, we see that there are two radical terms, one on each side of the equal sign. This is a signal that we will have to go through the squaring process twice.

Subtract 23 from both sides.

$$\sqrt{5x} + 10 = -2\sqrt{x + 11} + 23$$

= $\sqrt{5x} - 13 = -2\sqrt{x + 11}$

Square both sides.

$$\left(\sqrt{5x}-13\right)^2 = \left(-2\sqrt{x+11}\right)^2$$

= $5x - 26\sqrt{5x} + 169 = 4(x+11)$

= $5x - 26\sqrt{5x} + 169 = 4x + 44$

Isolate the radical.

$$-26\sqrt{5x} = -x - 125$$

Square both sides.

$676(5x) = x^2 + 250x + 15625$

$3380x = x^2 + 250x + 15625$

$0 = x^2 - 3130x + 15625$

Use the quadratic formula.

a = 1 b = -3130 c = 15625

$$x = \frac{3130 \pm \sqrt{(3130)^2 - 4(1)(15625)}}{2(1)}$$

$$x = \frac{3130 \pm \sqrt{9796900 - 62500}}{2}$$

$$x = \frac{3130 \pm 3120}{2}$$

$x = \frac{3130 + 3120}{2}$ = 3125

Or $x = \frac{3130 - 3120}{2}$ = 5

x = 5

$\sqrt{5(5)} + 10 = -2\sqrt{(5) + 11} + 23$

$\sqrt{25} + 10 = -2\sqrt{16} + 23$

5 + 10 = -8 + 23

15 = 15

x = 3125

$\sqrt{5(3125)} + 10 = -2\sqrt{(3125) + 11} + 23$

$\sqrt{15625} + 10 = -2\sqrt{3136} + 23$

125 + 10 = -112 + 23

135 ≠ -89

The solution is x = 5, and there is an extraneous root of x = 3125.

9. The lateral surface area of a right circular cone, s, can be represented by the equation, $s = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the circular base and h is the height of the cone. If the lateral surface area of a large cone-shaped funnel is 378.75 cm^2 and the radius is 6 cm, find the height, to the nearest tenth of a centimetre.

Solution

Substitute known values into the equation.

$$378.75 = \pi(6)\sqrt{(6)^2 + h^2}$$

Divide both sides by 6π

$$\frac{378.75}{6\pi} = \sqrt{(6)^2 + h^2}$$

Square both sides.

$$\left(\frac{378.75}{6\pi}\right)^2 = \left(\sqrt{(6)^2 + h^2}\right)^2$$

$$\left(\frac{378.75}{6\pi}\right)^2 = 36 + h^2$$

Subtract 36 from both sides.

$$\left(\frac{378.75}{6\pi}\right)^2 - 36 = h^2$$

h = $\sqrt{\left(\frac{378.75}{6\pi}\right)^2 - 36}$

h = 19.2

The height of the funnel is 19.2 cm.