1. Given the rational expression, $\frac{x-2}{x+1}$, which statement below is true regarding restrictions?
A) $x \neq 2$
B) $x \neq 0$
C) $x \neq-1$
D) $x \neq-1,2$

Use the following information to answer the next question.

| Expression |  |  | Possible NPV Answers |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $\frac{x+5}{x-4}$ | $\mathbf{1 .}$ | $x=-5$ | 6. | $x=\frac{4}{3}$ |  |
| B | $\frac{3 x-11}{x^{2}}$ | 2. | $x=\frac{3}{4}$ | 7. | $x=-2$ |  |
| $C$ | $\frac{6 x^{3}}{8 x+4}$ | 3. | $x=-\frac{1}{2}$ | 8. | $x=\frac{11}{3}$ |  |
| D | $\frac{x-10}{4-3 x}$ | $\mathbf{4 .}$ | $x=0$ | 9. | $x=4$ |  |
| 5. | $x=10$ | 10. | $x=\frac{1}{2}$ |  |  |  |

2. Using the numbers 1-10, state the non-permissible values for each expression.
i) The NPV for expression $A$ is $\qquad$ .
ii) The NPV for expression B is $\qquad$ .
iii) The NPV for expression $C$ is $\qquad$ .
iv) The NPV for expression D is $\qquad$ .

Use the following information to answer the next question.
An expression equivalent to $\frac{9 x+2}{x-1}, x \neq 0,1$, is written in the form $\frac{A x^{B}+8 x^{2}}{C x^{3}-D x^{2}}$, where $A, B, C$, and $D$ represent integers.
3. i) The value of $A$ is $\qquad$
ii) The value of $B$ is $\qquad$
iii) The value of $C$ is $\qquad$
iv) The value of $D$ is $\qquad$
4. Which of the following expressions is equivalent to $\frac{6}{4 x+7}, x \neq-\frac{7}{4}, 0, \frac{7}{4}$ ?
A) $\frac{6}{x(4 x-7)(4 x+7)}$
B) $\frac{6(4 x-7)}{x(4 x-7)(4 x+7)}$
C) $\frac{6 x(4 x-7)}{x(4 x-7)(4 x+7)}$
D) $\frac{6 x(4 x+7)}{x(4 x-7)(4 x+7)}$

Use the following information to answer the next question.
Consider the following four rational expressions. Two of the expressions have one non-permissible value, and two of the expressions have two non-permissible values.

| Expression 1 | $\frac{-4 x^{2}}{2+6 x}$ |
| :---: | :---: |
| Expression 2 | $\frac{3 x+12}{3 x^{2}-3}$ |
| Expression 3 | $\frac{x+4}{2 x-14}$ |
| Expression 4 | $\frac{1-5 x}{9 x(x+1)}$ |

5. Of the two expressions having two non-permissible values (a total of four numbers), the largest of these is $\qquad$ .
6. Which of the following is not equivalent to $\frac{-12 x-2 x^{2}}{5 x}$ ?
A) $\frac{-\left(36 x+6 x^{2}\right)}{15 x}$
B) $\frac{-2 x-12}{5}$
C) $\frac{10 x^{2}-60 x}{50 x}$
D) $\frac{-24 x-4 x^{2}}{10 x}$
7. Which of the following expressions does not have a non-permissible value?
A) $\frac{2}{x}$
B) $\frac{x-9}{x^{2}+1}$
C) $\frac{4 x}{x^{2}-4}$
D) $\frac{3(x-8)}{x+5}$
8. The rational expression $\frac{k x^{2}-4}{m x+n}$ is an equivalent form of the rational expression $\frac{7 x^{2}-1}{9 x+2}$. Which statement below is true, regarding the values of $k, m$, and $n$ ?
A) The largest value is $k$.
B) The smallest value is $m$.
C) The sum of the smallest and largest values is 44 .
D) The second largest value, or middle value, is 36.
9. In the rational expression, $\frac{x+a}{x(x-k)}$, the non-permissible value(s) of the variable $\times$ are
A) $0,-\mathrm{k}$
B) $0,-a, k$
C) $0, \mathrm{k}$
D) $k$
10. Simplify $\frac{x+1}{3 x+3}$
11. Given the rational expression, $\frac{x-2}{x+1}$, which statement below is true regarding restrictions?
A) $x \neq 2$
B) $x \neq 0$
C) $x \neq-1$
D) $x \neq-1,2$

## Solution

Our task is to determine any values of the variable that would make the denominator equal to zero. A rational expression having a zero denominator is undefined.

Set $x+1=0$. Isolate $x$. The non-permissible value is $x=-1$.
The restriction is $x \neq-1$.

Use the following information to answer the next question.

| Expression |  |  | Possible NPV Answers |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $\frac{x+5}{x-4}$ | 1. | $x=-5$ | 6. | $x=\frac{4}{3}$ |  |
| B | $\frac{3 x-11}{x^{2}}$ | 2. | $x=\frac{3}{4}$ | 7. | $x=-2$ |  |
| $C$ | $\frac{6 x^{3}}{8 x+4}$ | 3. | $x=-\frac{1}{2}$ | 8. | $x=\frac{11}{3}$ |  |
| D | $\frac{x-10}{4-3 x}$ | 4. | $x=0$ | 9. | $x=4$ |  |
| 5. | $x=10$ | 10. | $x=\frac{1}{2}$ |  |  |  |

2. Using the numbers 1-10, state the non-permissible values for each expression.
i) The NPV for expression $A$ is _9.
ii) The NPV for expression B is $\qquad$
iii) The NPV for expression $C$ is $\qquad$
iv) The NPV for expression D is $\qquad$

## Solution

For each denominator, set the expression equal to zero and solve for the letter.
Expression A:

$$
\begin{aligned}
& x-4=0 \\
& x=4
\end{aligned}
$$

Expression B:

$$
x^{2}=0
$$

Take the square root of both sides: $x=0$
Expression $C$

$$
\begin{aligned}
& 8 x+4=0 \\
& 8 x=-4 \\
& x=-\frac{1}{2}
\end{aligned}
$$

Expression D

$$
\begin{aligned}
& 4-3 x=0 \\
& 4=3 x \\
& x=\frac{4}{3}
\end{aligned}
$$

Use the following information to answer the next question.
An expression equivalent to $\frac{9 x+2}{x-1}, x \neq 0,1$, is written in the form $\frac{A x^{B}+8 x^{2}}{C x^{3}-D x^{2}}$, where $A, B, C$, and $D$ represent integers.
3. i) The value of $A$ is _36_
ii) The value of $B$ is _3_
iii) The value of $C$ is _4
iv) The value of $D$ is _4_

## Solution

The second term in the numerator of $\frac{A x^{B}+8 x^{2}}{C x^{3}-D x^{2}}$ is $8 x^{2}$. The second term in the numerator of $\frac{9 x+2}{x-1}$ is 2 . To be simplified to 2 means that some term divided into $8 x^{2}$ must be equal to 2 . That quantity is $4 x^{2}$.

This means that $\frac{A x^{B}}{4 x^{2}}=9 x$. Thus, $A x^{B}=36 x^{3}$. The value of $A$ is 36 and the value of $B$ is 3.

To find $C$; $\frac{C x^{3}}{4 x^{2}}=x$. Thus, $C x^{3}=4 x^{3}$. The value of $C$ is 4 .
To find $D ; \frac{D x^{2}}{4 x^{2}}=1$. Thus $D x^{2}=4 x^{2}$. The value of $D$ is 4 .
4. Which of the following expressions is equivalent to $\frac{6}{4 x+7}, x \neq-\frac{7}{4}, 0, \frac{7}{4}$ ?
A) $\frac{6}{x(4 x-7)(4 x+7)}$
B) $\frac{6(4 x-7)}{x(4 x-7)(4 x+7)}$
C) $\frac{6 x(4 x-7)}{x(4 x-7)(4 x+7)}$
D) $\frac{6 x(4 x+7)}{x(4 x-7)(4 x+7)}$

## Solution

The correct answer is $C$. Dividing out the common binomial of ( $4 x-7$ ), and the common ' $x$ ' in both the numerator and the denominator, will result in the simplification of $\frac{6}{4 x+7}$.

Use the following information to answer the next question.
Consider the following four rational expressions. Two of the expressions have one non-permissible value, and two of the expressions have two non-permissible values.

| Expression 1 | $\frac{-4 x^{2}}{2+6 x}$ |
| :---: | :---: |
| Expression 2 | $\frac{3 x+12}{3 x^{2}-3}$ |
| Expression 3 | $\frac{x+4}{2 x-14}$ |
| Expression 4 | $\frac{1-5 x}{9 x(x+1)}$ |

5. Of the two expressions having two non-permissible values (a total of four numbers), the largest of these is _1_.

## Solution

Expression 1 has 1 NPV. Set the denominator equal to zero and solve for the variable.

$$
\begin{aligned}
& 2+6 x=0 \\
& 6 x=-2 \\
& x=-\frac{1}{3}
\end{aligned}
$$

Expression 2 has 2 NPV's. Factor the denominator and use the zero product property.

$$
\begin{aligned}
& 3\left(x^{2}-1\right)=0 \\
& 3(x-1)(x+1)=0 \\
& x=-1,1
\end{aligned}
$$

Expression 3 has 1 NPV. Set the denominator equal to zero and solve for the variable.

$$
\begin{aligned}
& 2 x-14=0 \\
& 2 x=14 \\
& x=7
\end{aligned}
$$

Expression 4 has 2 NPV's. Set the denominator equal to zero and use the zero product property.

$$
\begin{aligned}
& 9 x(x+1)=0 \\
& x=0,-1
\end{aligned}
$$

Since the NPV's of expressions 2 and 4 are the only ones to be used to determine the answer, the largest value from these expressions is 1.
6. Which of the following is not equivalent to $\frac{-12 x-2 x^{2}}{5 x}$ ?
A) $\frac{-\left(36 x+6 x^{2}\right)}{15 x}$
B) $\frac{-2 x-12}{5}$
C) $\frac{10 x^{2}-60 x}{50 x}$
D) $\frac{-24 x-4 x^{2}}{10 x}$

## Solution

The original expression, $\frac{-12 x-2 x^{2}}{5 x}$, is the same as $\frac{-\left(12 x+2 x^{2}\right)}{5 x}$. If each term is multiplied by 3 , we get the expression in $A$. Thus, $A$ is equivalent to $\frac{-12 x-2 x^{2}}{5 x}$. Expression $B$ is equivalent because an ' $x$ ' is divided out of each term, and the numerator terms are simply reversed in order.

Expression $D$ is equivalent. Each term in this expression is multiplied by 2.
Expression $C$ is not equivalent. Reverse the order of the numerator terms and factor a common 10 out of each term: $\frac{6 x-x^{2}}{5 x}$.
$\frac{6 x-x^{2}}{5 x}$ is not the same as $\frac{-12 x-2 x^{2}}{5 x}$.
7. Which of the following expressions does not have a non-permissible value?
A) $\frac{2}{x}$
B) $\frac{x-9}{x^{2}+1}$
C) $\frac{4 x}{x^{2}-4}$
D) $\frac{3(x-8)}{x+5}$

Solution
Expression $B$ does not have a non-permissible value. If the denominator is set equal to zero, $x^{2}+1=0$, there is no solution. There is no real number that would make a true statement. The value of ' $x$ ' can be any positive number, any negative number, or zero, and still not create a situation where the expression would be undefined.
8. The rational expression $\frac{k x^{2}-4}{m x+n}$ is an equivalent form of the rational expression $\frac{7 x^{2}-1}{9 x+2}$. Which statement below is true, regarding the values of $k, m$, and $n$ ?
A) The largest value is $k$.
B) The smallest value is $m$.
C) The sum of the smallest and largest values is 44 .
D) The second largest value, or middle value, is 36 .

## Solution

The relationship between the one given numerical value in $\frac{k x^{2}-4}{m x+n}$ (i.e. -4 ) and the corresponding number in the equivalent expression, $\frac{7 x^{2}-1}{9 x+2}$ (i.e. -1 ), is important. The smaller number is multiplied by 4 to get the larger number. Equivalent rational expressions are determined by multiplying (or dividing) every term in the
expression by the same quantity. Multiplying each of the numerical values in $\frac{7 x^{2}-1}{9 x+2}$ by 4 , will determine the values of our letters.

The value of $k$ is 28 , the value of $m$ is 36 and the value of $n$ is 8 .
The correct answer is $C$.
9) In the rational expression, $\frac{x+a}{x(x-k)}$, the non-permissible value(s) of the variable $\times$ are
A) $0,-\mathrm{k}$
B) $0,-a, k$
C) $0, \mathrm{k}$
D) $k$

## Solution

Looking only in the denominator, set $x(x-k)=0$
Using the zero product property, either $x=0$, or $x-k=0$. [add $k$ to both sides]

$$
x=0, \text { or } x=k
$$

The non-permissible values are 0 and $k$.

$$
\text { 10)Simplify } \frac{x+1}{3 x+3}
$$

Factor the denominator
$\frac{x+1}{3(x+1)}$
Divide out the common binomial.
$=\frac{1}{3}$

