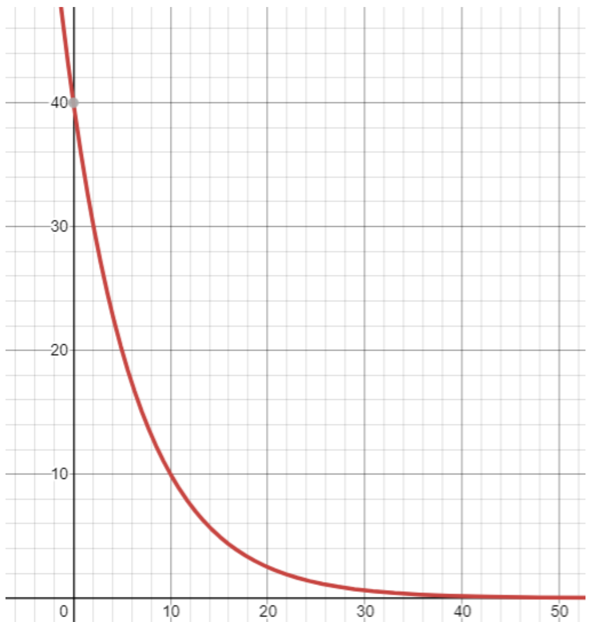
Math 30-2 Exponential Functions Lesson 3 Practice Questions

Use the following information to answer the first question.

The graph below shows the exponential function $y = 40\left(\frac{1}{2}\right)^{\frac{x}{h}}$, where *h* is the half life.



1. If *h* is an integer, using the graph, the value of *h* can be determined to be _____.

2. A particular substance has a half life of 4 days. If 32 768 original units of the substance is reduced to 1024 units, the equation, $1024 = 32768\left(\frac{1}{2}\right)^{\frac{x}{4}}$ can be used to find the length of time it takes to reduce to that amount. Working with an equivalent base of 2, solve for x.

Use the following information to answer the next question.

The half-life of carbon-14 is approximately 5 730 years. As a sample of carbon-14 decays, the percentage of carbon-14 remaining, P, at any time during the process can be modelled by the function

$$P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}},$$

where t is the approximate age of the sample in years.

3. To the nearest year, the approximate age of the carbon-14 when 64% of the original amount remains in the sample is

   A) 4503   B) 4855   C) 5122   D) 5468

Use the following information to answer the next question.

Once growing conditions are ideal, the population of a certain bacteria doubles every 20 minutes. Starting from a single bacterium, the number of bacteria, E, present after M minutes can be modelled by the formula

$$E = 2^{\frac{M}{20}}$$

4. To the nearest minute, the time it will take for there to be **at least** 5600 bacteria is ______ min.

5. The half-life of a particular substance is 2.5 days. An original amount decays to 195.5 grams in 18 days. Determine the original amount.

6. If 4 grams of Cesium-137 decays to 0.5 grams in 90 years, then the half-life of Cesium-137 is ____ years. Determine the answer by two different methods.

7. If $\frac{1}{16}$ or 6.25 % of an original amount remains after of period of time, the number of half-lives applied to the original amount is

A) 2 B) 4 C) 8 D) 16

8. In April 1986, the nuclear accident at Chernobyl contaminated the atmosphere with quantities of radioactive iodine-131. If the half-life of radioactive iodine-131 is 8.1 days, determine the number of days, to the nearest day, it took for the level of radiation to reduce from 100 units to 7 units.

9. A patient feeling ill had a sample of bacteria taken from his throat. The sample contained 290 cells. Twenty-four hours later, the sample was recounted and was found to contain 6895 cells. Find the doubling time of the bacteria to the nearest hundredth of an hour.

10. The half-life of a particular substance is 3 hours. With an original amount of 15 250 units, the substance decays to A(16) after 16 hours. In **another** 4 hours, how many **less** units are there, compared to A(16)?

Math 30-2 Exponential Functions Lesson 3 Teaching Handout Solutions

Use the following information to answer the first question.

The graph below shows the exponential function $y = 40\left(\frac{1}{2}\right)^{\frac{x}{h}}$, where *h* is the half life.

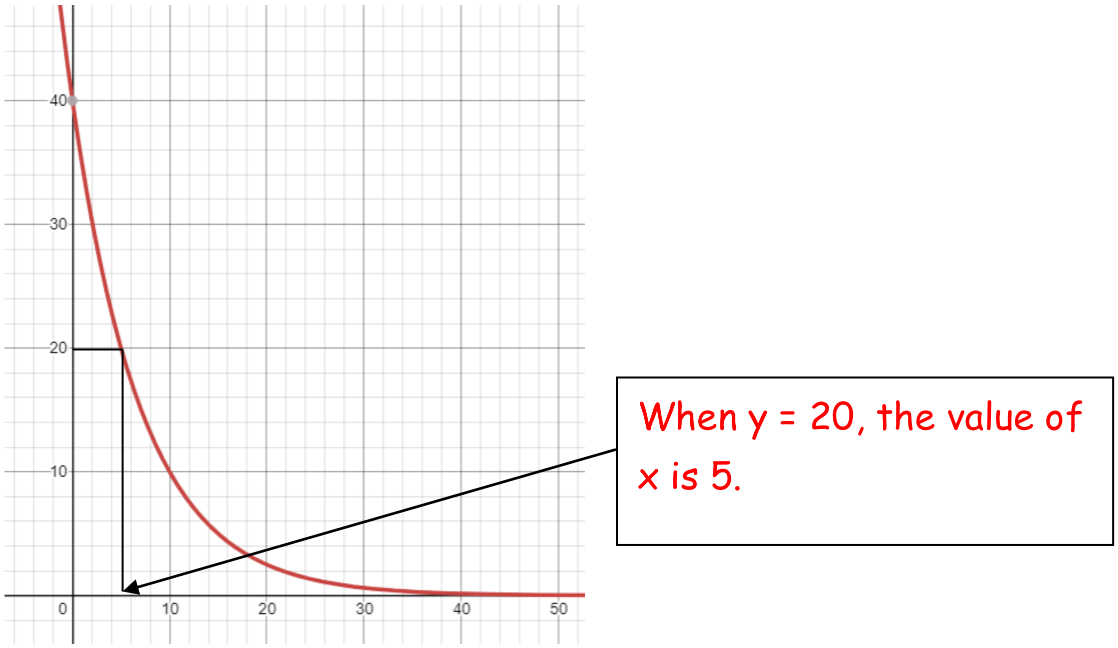
1. If *h* is an integer, using the graph, the value of *h* can be determined to be __5__.

Solution

Since half of 40 is 20, go to 20 on the y-axis and draw a horizontal line to the point where y = 20 intersects with the graph.

At that point, draw a vertical line to the x-axis. The nearest integer would be 5.

See the graph below.



2. A particular substance has a half life of 4 days. If 32 768 original units of the substance is reduced to 1024 units, the equation, $1024 = 32768\left(\frac{1}{2}\right)^{\frac{x}{4}}$ can be used to find the length of time it takes to reduce to that amount. Working with an equivalent base of 2, solve for x.

Solution

$1024 = 2^{10}$, and $32768 = 2^{15}$, and $\left(\frac{1}{2}\right) = 2^{-1}$

Substitute these values into the equation.

$2^{10} = 2^{15}\,(2^{-1})^{(x/4)}$

Divide both sides by 2^{15} to isolate the power.

$$\frac{2^{10}}{2^{15}} = \frac{2^{15}(2^{-1})^{\frac{x}{4}}}{2^{15}}$$

$2^{-5} = 2^{(-x/4)}$

Drop the bases and set the exponents equal to solve for x.

$$-5 = \frac{-x}{4}$$

-20 = -x

20 = x

In 20 days, 32 768 units will reduce to 1024 units.

Use the following information to answer the next question.

The half-life of carbon-14 is approximately 5 730 years. As a sample of carbon-14 decays, the percentage of carbon-14 remaining, P, at any time during the process can be modelled by the function $$P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}},$$ where t is the approximate age of the sample in years.

3. To the nearest year, the approximate age of the carbon-14 when 64% of the original amount remains in the sample is

   A) 3502 B) 3689 C) 4467 D) 5068

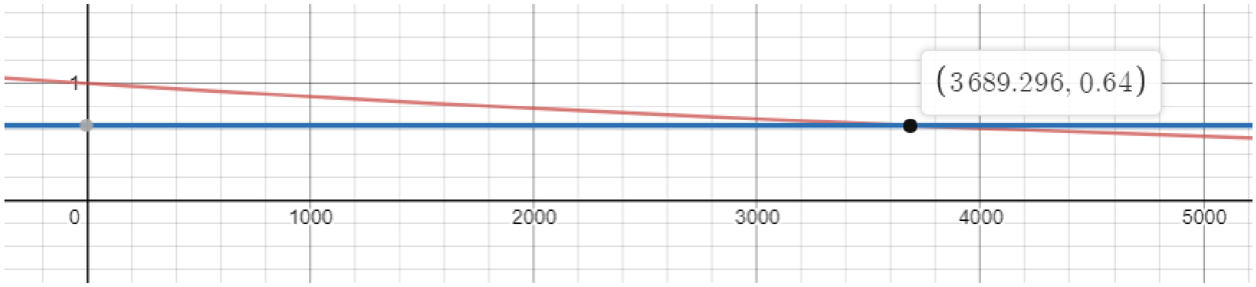
Solution

$$P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$64 = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$0.64 = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

Graph $y_1 = 0.64$
Graph $y_2 = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$
Use a y maximum of 1 and an x maximum of 6 000.



The x-coordinate of the intersection point of the two graphs is (3689.296).

To the nearest year, the approximate age of the sample is 3689.

Use the following information to answer the next question.

Once growing conditions are ideal, the population of a certain bacteria doubles every 20 minutes. Starting from a single bacterium, the number of bacteria, E, present after M minutes can be modelled by the formula $$E = 2^{\frac{M}{20}}$$

4. To the nearest minute, the time it will take for there to be **at least** 5600 bacteria is __<u>249</u>__ min.

Solution

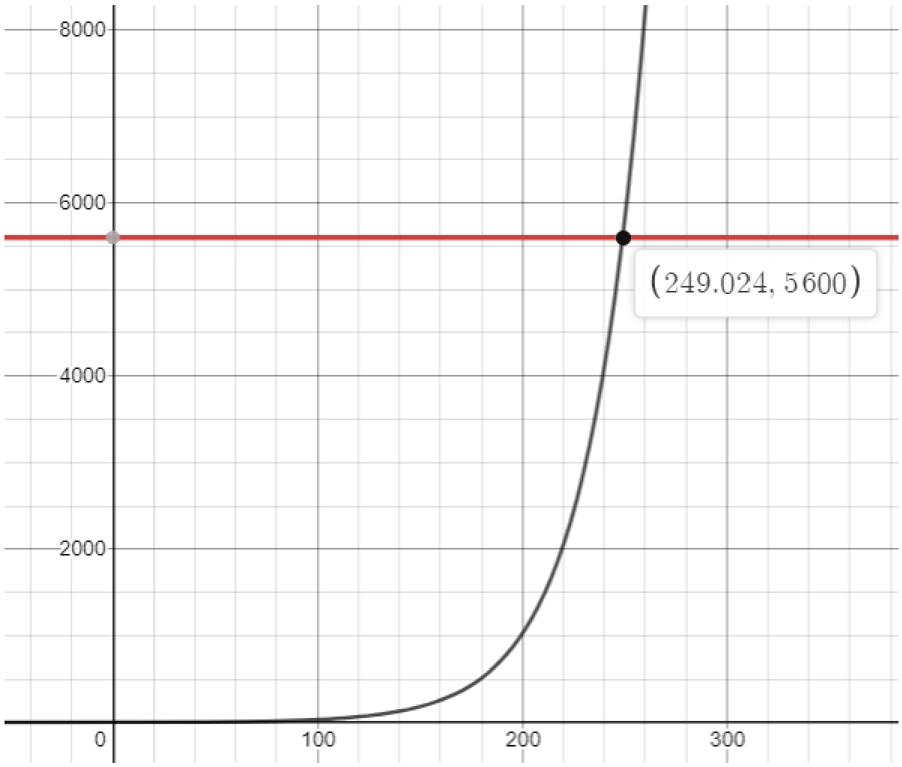
Substitute 5600 for E.

$$5600 = 2^{\frac{M}{20}}$$

Graph $y_1 = 5600$, and $y_2 = 2^{\frac{M}{20}}$

Find the x-coordinate of the intersection point.

See graph below.



The x-coordinate is 249.024.

To the nearest minute, the time it will take for there to be at least 5600 bacteria is __<u>249</u>__ min.

5. The half-life of a particular substance is 2.5 days. An original amount decays to 195.5 grams in 18 days. Determine the original amount.

Solution

Use the formula, $A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$

What is known?

- A(t) = 195.5
- t = 18
- h = 2.5

We are trying to find the original amount, A_o.

Substitute the known values.

$$195.5 = A_o\left(\frac{1}{2}\right)^{\frac{18}{2.5}}$$

To isolate the unknown, divide both sides of the equal sign by $\left(\frac{1}{2}\right)^{\frac{18}{2.5}}$.

$$A_o = \frac{195.5}{\left(\frac{1}{2}\right)^{\frac{18}{2.5}}}$$

A_o = 28 745

The original amount was 28 745 grams.

6. If 4 grams of Cesium-137 decays to 0.5 grams in 90 years, then the half-life of Cesium-137 is _30_ years. Determine the answer by two different methods.

Solution

Use the formula, $A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$

What is known?

- A(t) = 0.5
- A_o = 4
- t = 90 years

We are trying to find h, the half-life.

Substitute known values into the formula.

$$0.5 = 4\left(\frac{1}{2}\right)^{\frac{90}{h}}$$

<u>Method 1 – Equivalent Bases</u>

$$2^{-1} = (2^2)(2^{-1})^{\frac{90}{h}}$$

$$2^{-1} = (2^2)(2^{\frac{-90}{h}})$$

Divide both sides by 2^2 to isolate the power.

$$\frac{2^{-1}}{2^2} = \frac{(2^2)(2^{\frac{-90}{h}})}{2^2}$$

$$2^{-3} = 2^{\frac{-90}{h}}$$

Set the exponents equal.

$$-3 = \frac{-90}{h}$$

-3h = -90

h = 30

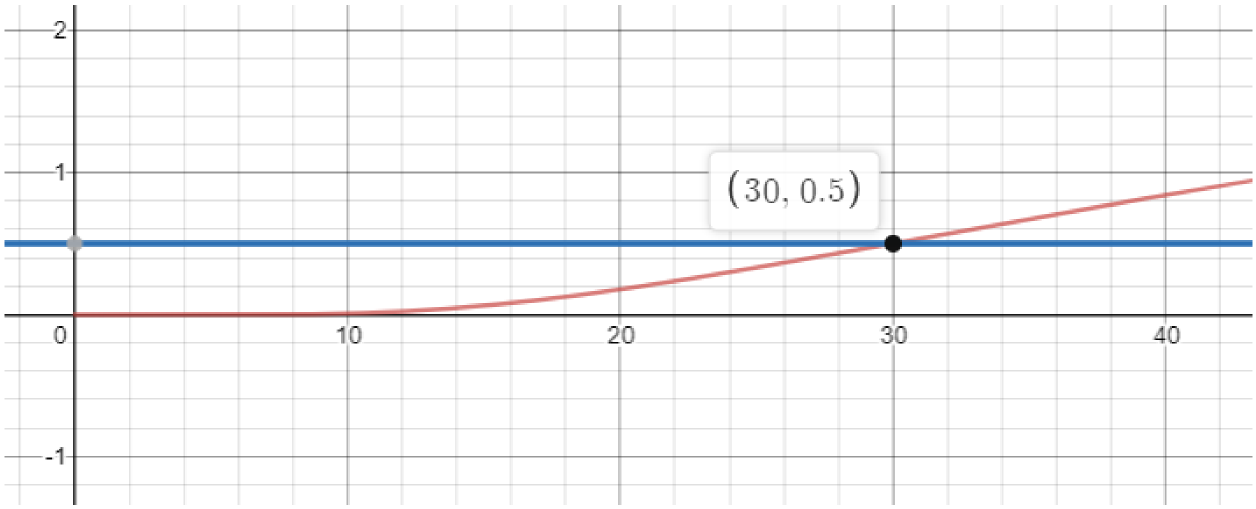
<u>Method 2-Using Technology</u>

Graph $y_1 = 0.5$

Graph $y_2 = 4\left(\frac{1}{2}\right)^{\frac{90}{h}}$

The x-coordinate of the intersection point is 30. [See the following graph]

The half-life of Cesium-137 is _<u>30</u>_ years.



7. If $\frac{1}{16}$ or 6.25 % of an original amount remains after of period of time, the number of half-lives applied to the original amount is

   A) 2 B) 4 C) 8 D) 16

Solution

When each half-life is applied, the previous amount is multiplied by $\frac{1}{2}$.

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{16}$

The application of 4 half-lives will reduce an original amount of 100% to just 6.25 .

8. In April 1986, the nuclear accident at Chernobyl contaminated the atmosphere with quantities of radioactive iodine-131. If the half-life of radioactive iodine-131 is 8.1 days, determine the number of days, to the nearest day, it took for the level of radiation to reduce from 100 units to 7 units.

Solution

Use the formula, $A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$

What is known?

- $A(t) = 7$
- $A_o = 100$
- $h = 8.1$ days

We are trying to find t.

Substitute known values.

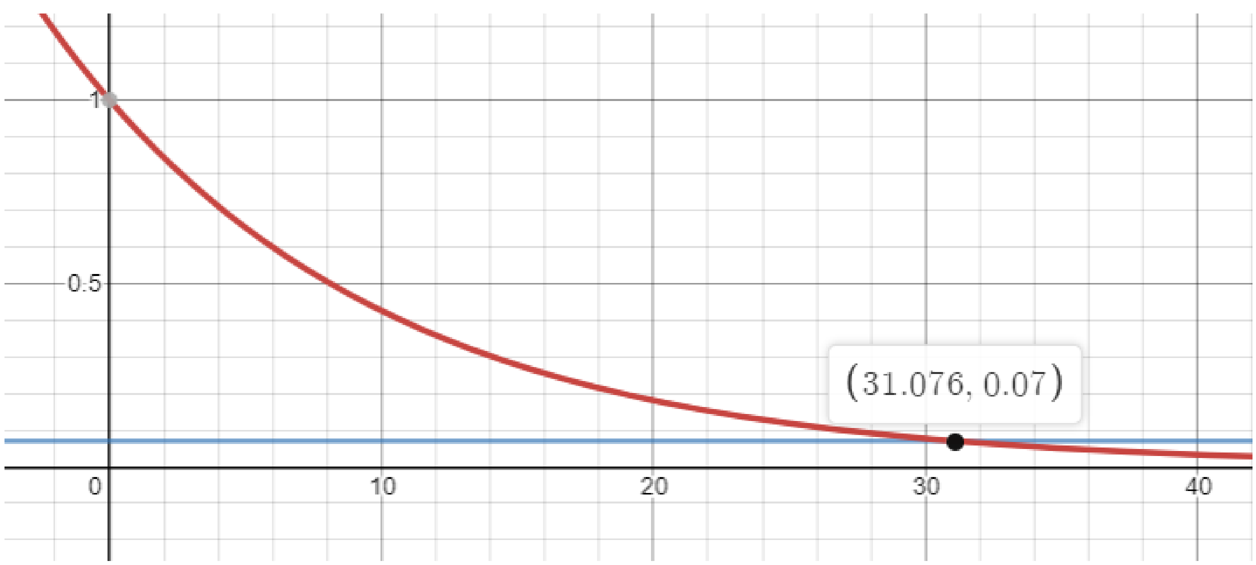
$$7 = 100\left(\frac{1}{2}\right)^{\frac{t}{8.1}}$$

Divide both sides by 100 to isolate the power.

$.07 = \left(\frac{1}{2}\right)^{\frac{t}{8.1}}$

Use technology to graph the left side of the equal sign in y_1 and the right side in y_2.

Find the x-coordinate of the intersection point. It takes 31 days.



9. A patient feeling ill had a sample of bacteria taken from his throat. The sample contained 290 cells. Twenty-four hours later, the sample was recounted and was found to contain 6895 cells. Find the doubling time of the bacteria to the nearest hundredth of an hour.

Solution

Use the formula, $A(t) = A_o(2)^{\frac{t}{h}}$.

What is known?

- $A(t) = 6895$
- $A_o = 290$
- $t = 24$

We are trying to find h, the doubling time.

$$6895 = 290(2)^{\frac{24}{h}}$$

Divide both sides of the equal sign to isolate the power.

$$23.775... = (2)^{\frac{24}{h}}$$

Use technology to graph and determine the intersection point.

The intersection point is (5.25, 23.775) [See the graph below]

The doubling time of the bacteria is 5.25 hours.

10. The half-life of a particular substance is 3 hours. With an original amount of 15 250 units, the substance decays to A(16) after 16 hours. In **another** 4 hours, how many **less** units are there, compared to A(16)?

Solution

$$A(16) = 15250\left(\frac{1}{2}\right)^{\frac{16}{3}}$$

A(16) = 378.25

In another 4 hours, it is now 20 hours from the original start time of the decay.

$$A(20) = 15250\left(\frac{1}{2}\right)^{\frac{20}{3}}$$

$A(20) = 150.11$

$378.25 - 150.11 = 228.14$

In another 4 hours, there are 228.14 less units.