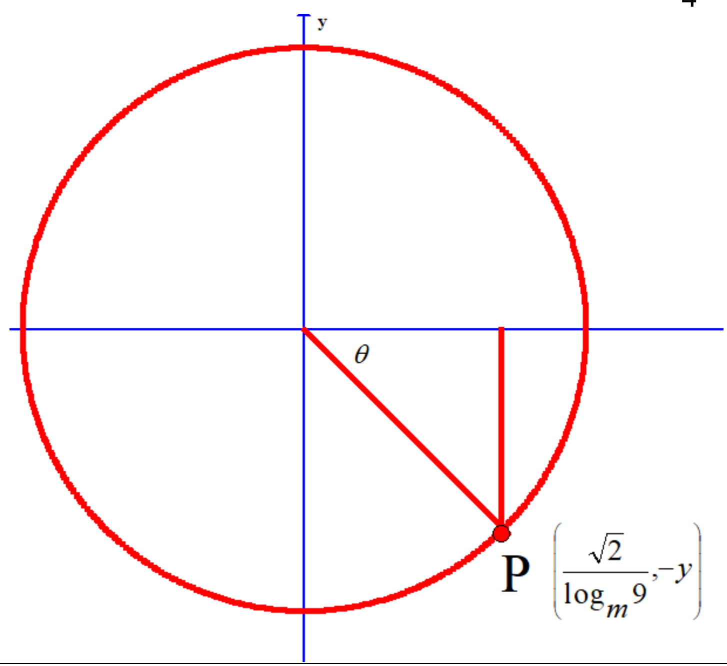
Blending Trigonometry and Logarithms Practice

1. The value of $\log_4(\sin\left(\frac{5\pi}{6}\right))$ is

   A) $\frac{\sqrt{3}}{2}$ B) $\frac{-\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) $-\frac{1}{2}$

Use the following information to answer the next question.

The reference angle for the graph below is $\frac{\pi}{4}$.



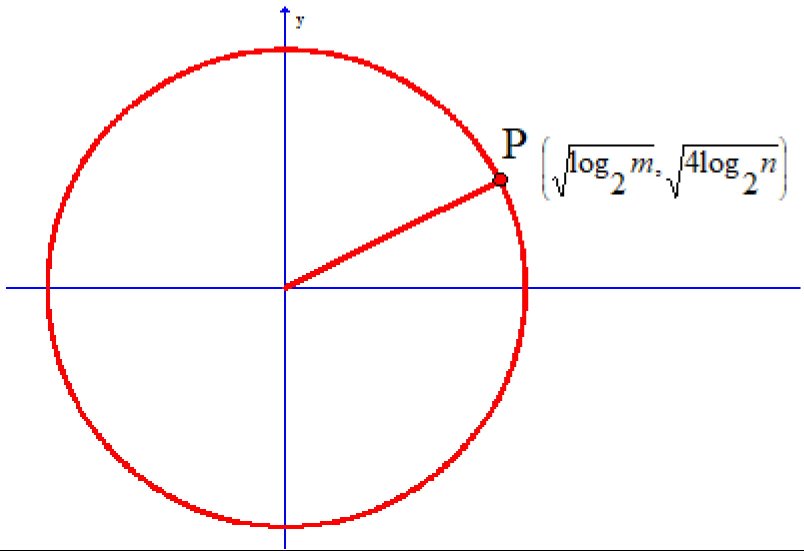
2. The value of m is _____.

3. What is $\cos\left(\log_k k^{2\pi}\right)$?

   A) 0 B) 1 C) -1 D) 0.5

Use the following information to answer the next question.

Point P below lies on the terminal arm of an angle in standard position on the unit circle.



4. An expression for m, in terms of n, is

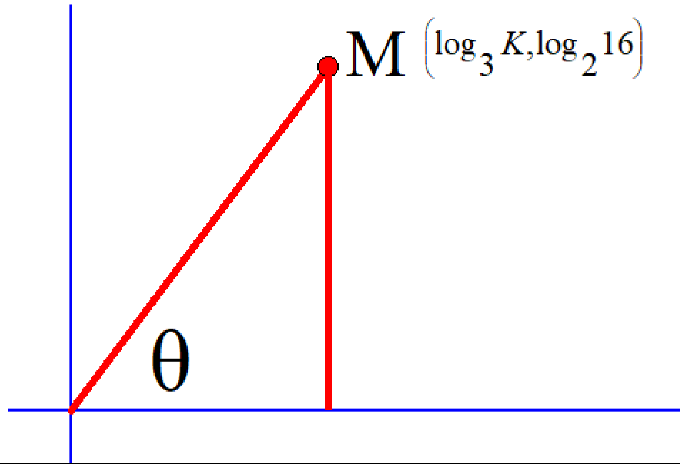
A) $m = \frac{2}{n^4}$ B) $m = \frac{2}{4n}$ C) $m = \frac{n^4}{2}$ D) $m = \frac{4n}{2}$

5. If $90^0 \leq \theta \leq 270^0$, then the value of θ in the equation,
$\log_3(\cos\theta) - \log_3(\sin\theta) = \frac{1}{2}$, is ______.

6. Evaluate $\tan\left(3\log_5 5^{\pi}\right)$

Use the following information to answer the next question.

Point M is on the terminal arm of an angle in standard position, and $\cos\theta = \frac{3}{5}$.



7. a) Determine the value of K.

b) As an exact value, what is $\cot\theta$?

8. Determine, $\log_2(\sin 60) + \log_2(\cos 45) + \log_2\left(\frac{1}{\sqrt{6}}\right)$

9. Solve $\sin\theta = (\log_m 1 - \log_m m)$, where $0 \le \theta \le 2\pi$.

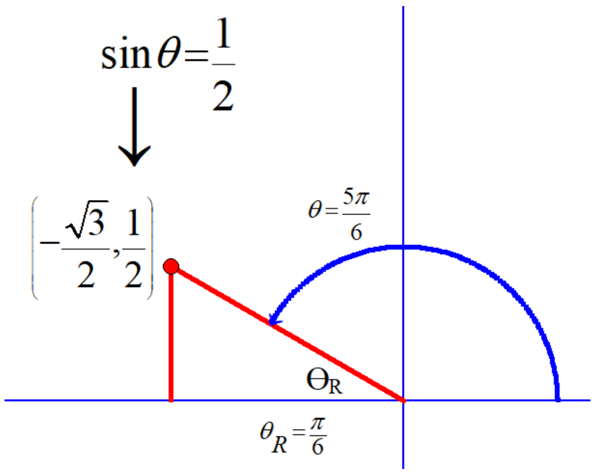
Blending Trigonometry and Logarithms Practice Solutions

1. The value of $\log_4(\sin\left(\frac{5\pi}{6}\right))$ is

A) $\frac{\sqrt{3}}{2}$ B) $\frac{-\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) $-\frac{1}{2}$

Solution

Begin by working inside the brackets with the sine term.



$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

Substitute this value into the original expression.

$$\log_4\left(\frac{1}{2}\right)$$

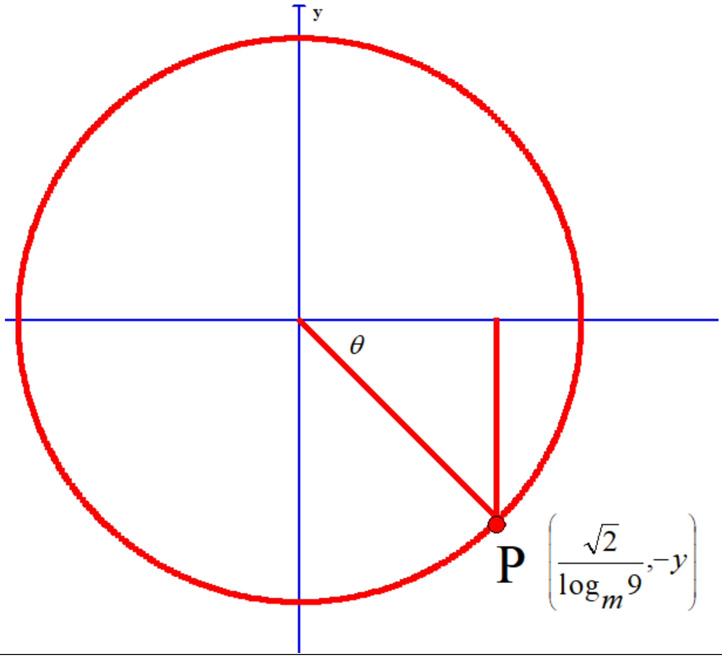
Using change of base, $\frac{\log\left(\frac{1}{2}\right)}{\log 4} = -\frac{1}{2}$.

In other words, $4^{-\frac{1}{2}} = \frac{1}{2}$

The correct answer is D.

Use the following information to answer the next question.

The reference angle for the graph below is $\frac{\pi}{4}$.



2. The value of m is __3__.

A reference angle of $\frac{\pi}{4}$ indicates a special triangle of 45^0 – 45^0 – 90^0. Both the sine and cosine are $\frac{\sqrt{2}}{2}$. Since we are in quadrant 4, cosine is positive and sine is negative. We know that $\log_m 9$ must be equal to 2.

$\log_m 9 = 2$

Convert to exponential form.

$m^2 = 9$

Take the square root of both sides; $m = \pm 3$. We reject the negative value because it does not make sense in this context.

The value of m is __3__.

3. What is $\cos\left(\log_k k^{2\pi}\right)$?

A) 0 B) 1 C) -1 D) 0.5

Solution

$$\log_k k^{2\pi} = 2\pi$$

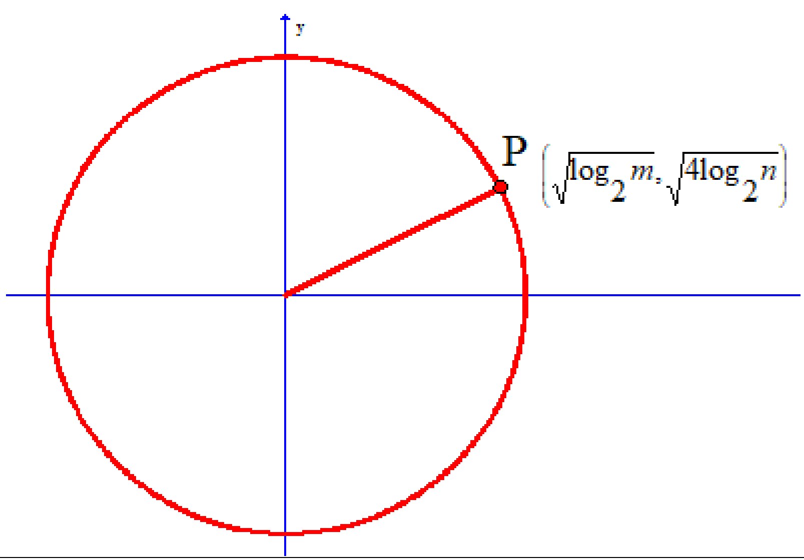
An equivalent question is now, what is cos(2π)?

From the calculator, cos(2π) = 1.

The correct answer is B.

Use the following information to answer the next question.

Point P below lies on the terminal arm of an angle in standard position on the unit circle.



4. An expression for m, in terms of n, is

A) $m=\frac{2}{n^4}$ Ans. B) $m=\frac{2}{4n}$ C) $m=\frac{n^4}{2}$ D) $m=\frac{4n}{2}$

Solution

Using the equation of the unit circle, $\left(\sqrt{\log_2 m}\right)^2+\left(\sqrt{4\log_2 n}\right)^2=1$

$\log_2 m + 4\log_2 n = 1$

Use the Power Law of Logarithms to re-write the second term.

$\log_2 m + \log_2 n^4 = 1$

Use the Product Law of Logarithms to combine the two terms on the left side into one term.

$\log_2(m)(n^4) = 1$

Convert to exponential form.

$2^1 = mn^4$

$$m=\frac{2}{n^4}$$

The correct answer is A.

5. If $90^0 \leq \theta \leq 270^0$, then the value of θ in the equation, $\log_3(\cos\theta) - \log_3(\sin\theta) = \frac{1}{2}$, is _210^0_.

Solution

Use the Quotient Law of Logarithms to combine the two terms on the left side into one term.

$$\log_3\left(\frac{\cos\theta}{\sin\theta}\right)=\frac{1}{2}$$

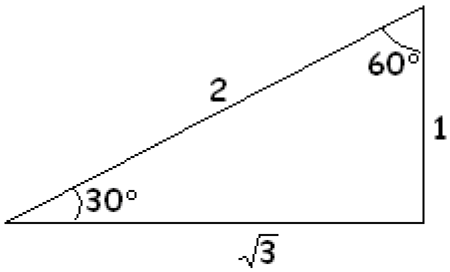
Convert to exponential form.

$$3^{\frac{1}{2}} = \frac{\cos\theta}{\sin\theta}$$

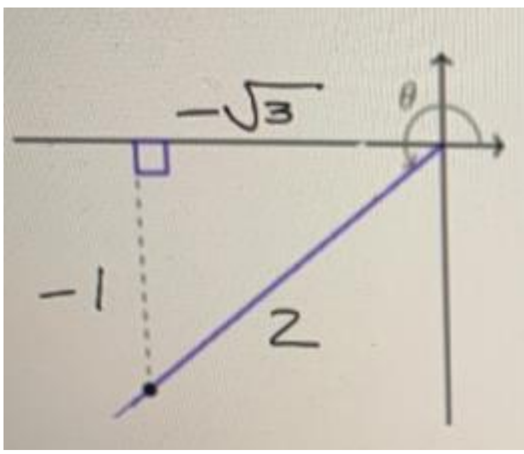
$$\sqrt{3} = \cot\theta$$

$$\frac{1}{\sqrt{3}} = \tan\theta$$

As we can see from the diagram, the tangent of 30^0 is $\frac{1}{\sqrt{3}}$



However, the domain given in the question limits our answer to either quadrant 2 or quadrant 3. Since tangent is positive in quadrant 3, the solution will be in this quadrant. The reference angle is 30^0.



The measure of θ is 180^0 + 30^0, which is 210^0

6. Evaluate $\tan\left(3\log_5 5^{\pi}\right)$

Solution

Use the Power Law of Logarithms to move the integer 3 from in front of the log to the exponential position.

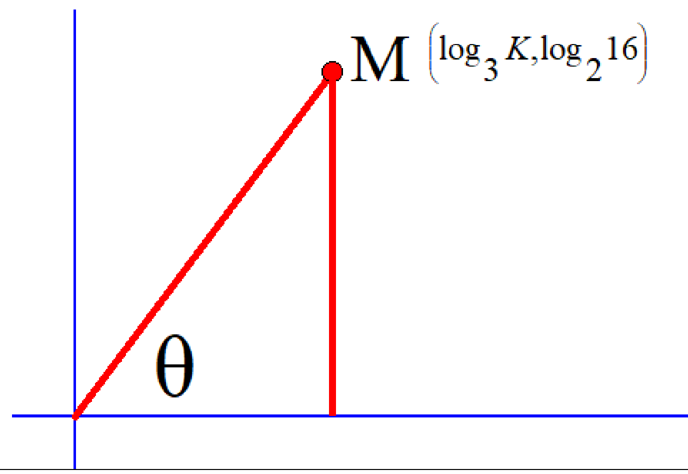
$\tan(\log_5 5^{3\pi})$

= $\tan(3\pi)$

Using the calculator, **$\tan(3\pi) = 0$**

Use the following information to answer the next question.

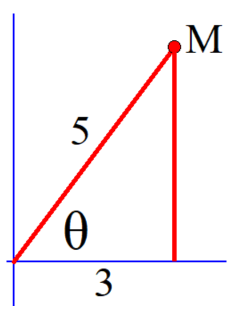
Point M is on the terminal arm of an angle in standard position, and $\cos\theta = \frac{3}{5}$.



7. a) Determine the value of K.

Solution

Since $\cos\theta = \frac{3}{5}$, we know that the side adjacent the angle is 3, and the hypotenuse is 5.



The x-coordinate of Point M, $\log_3 k$, is equal to 3. We have an equation to solve.

$\log_3 k = 3$

Convert to exponential form.

$3^3 = k$

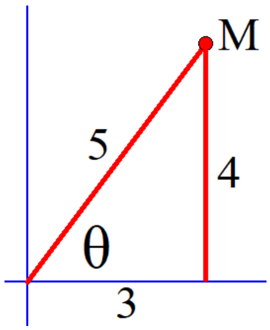
27 = k

The value of K is 27.

b) As an exact value, what is $\cot\theta$?

Solution

Using either Pythagorean Theorem, or the fact that $\log_2 16$ is 4, we know that the side opposite the angle is 4.



$\boxed{\cot\theta = \frac{3}{4}}$

8. Determine, $\log_2(\sin 60) + \log_2(\cos 45) + \log_2\left(\frac{1}{\sqrt{6}}\right)$

Solution

Using special triangle ratios, sin(60) = $\frac{\sqrt{3}}{2}$, and cos(45) = $\frac{\sqrt{2}}{2}$

$$\log_2\left(\frac{\sqrt{3}}{2}\right) + \log_2\left(\frac{\sqrt{2}}{2}\right) + \log_2\left(\frac{1}{\sqrt{6}}\right)$$

Apply the Product Law of Logarithms.

$$\log_2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{\sqrt{6}}\right)$$

$$\log_2\left(\frac{1}{4}\right)$$

$$2^x = \frac{1}{4}$$

$2^x = 2^{-2}$

$x = -2$

$$\log_2(\sin 60) + \log_2(\cos 45) + \log_2\left(\frac{1}{\sqrt{6}}\right) = -2$$

9. Solve $\sin\theta = (\log_m 1 - \log_m m)$, where $0 \leq \theta \leq 2\pi$.

Solution

$\log_m 1 = 0$, because the only exponent applied to a base of m, to result in a value of 1, is 0. In other words, $m^0 = 1$

$\log_m m = 1$, because when the base is the same as the value of the power, the total logarithmic expression is equal to the exponent on the power, which is 1.

$\sin\theta = (0 - 1)$

$\sin\theta = -1$

We are looking for an angle that gives a sine ratio of -1. Using the calculator, or knowledge of quadrantal angle ratios, $\theta = \frac{3\pi}{2}$.

The solution to $\sin\theta = (\log_m 1 - \log_m m)$, where $0 \leq \theta \leq 2\pi$ is $\frac{3\pi}{2}$.