Absolute Value Functions Practice

Use the following information to answer the first question.

Consider the following absolute value expressions.

Expression 1	$5\left\|\frac{1}{2}\right\|+8\left\|-\frac{3}{4}\right\|$
Expression 2	$\left\|-4(-3)^2+2(1-6)+12\right\|$
Expression 3	$-\left\|(3-5)^2+(1-(-3))\right\|$
Expression 4	$\left\|12(-3)+5^2\right\|$

1. Expression ___ has the largest value and Expression ___ has the smallest value.

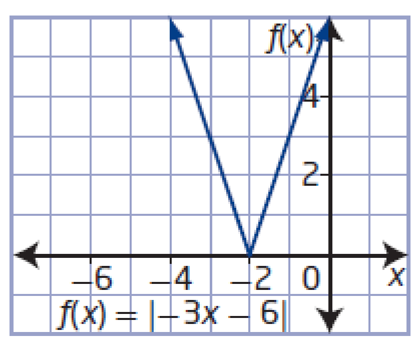
2. If the point (5, -2) is on the function y = f(x), then the corresponding point on the graph of y = |f(x)| is

   A) (5, 2)　　B) (-5, -2)　　C) (-5, 2)　　D) (5, 0)

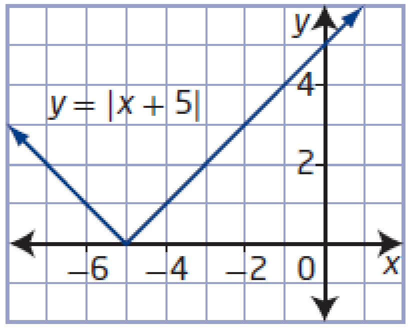
3. The graph of y = f(x) has an x-intercept of 9 and a y-intercept of -4. State the x and y-intercepts of y = |f(x)|.

4. Given the graph at the right, part of the piecewise function is

   A) y = -3x – 6 for x ≤ -2
   B) y = -3x + 6 for x > -2
   C) y = -3x + 6 for x > -2
   D) y = 3x + 6 for x ≤ - 2



Use the following information to answer the next question.



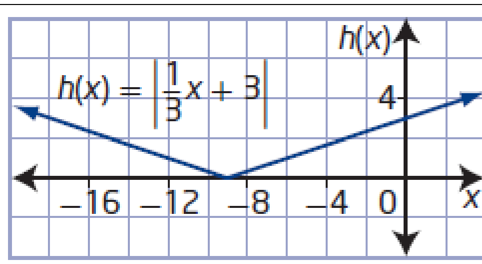
The following statements are made regarding the above graph.

Statement 1	The domain is x ≥ 0.
Statement 2	The y-intercept is (5, 0).
Statement 3	The range is y ≥ 0.
Statement 4	The x-intercept is (0, -5).

5. The true statement is

A) 1 B) 2 C) 3 D) 4

Use the graph below to answer the next question.



6. The above graph expressed as a piecewise function is

A) $y = \begin{cases} \frac{1}{3}x+3 & \text{for } x \le -9 \\ -\left(\frac{1}{3}x+3\right) & \text{for } x > -9 \end{cases}$

C) $y = \begin{cases} \frac{1}{3}x+3 & \text{for } x \ge -9 \\ -\left(\frac{1}{3}x+3\right) & \text{for } x < -9 \end{cases}$

B) $y = \begin{cases} \frac{1}{3}x+3 & \text{for } x \le 3 \\ -\left(\frac{1}{3}x+3\right) & \text{for } x > 3 \end{cases}$

D) $y = \begin{cases} \frac{1}{3}x+3 & \text{for } x \ge 3 \\ -\left(\frac{1}{3}x+3\right) & \text{for } x < 3 \end{cases}$

Use the following information to answer the next two questions.

Consider the absolute value function,
$f(x) = |x^2 - 2x - 3|$, and the statements that follow.

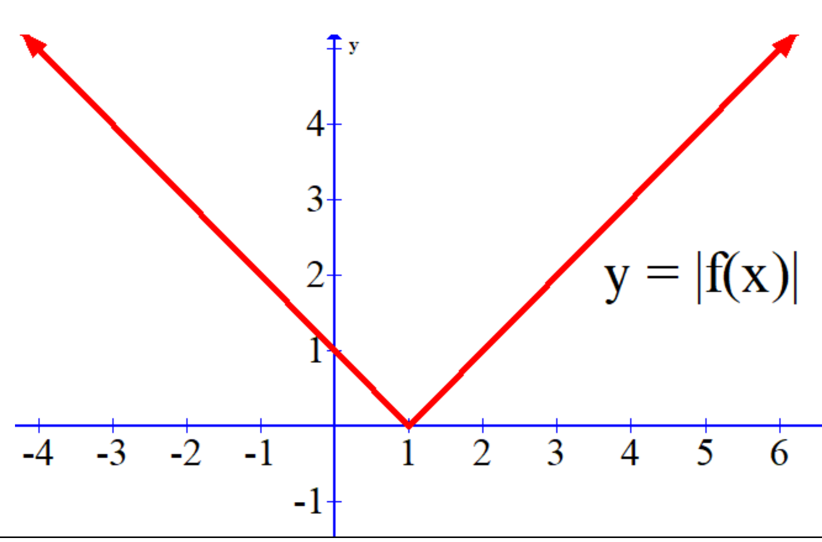
Statement 1	The x-intercepts are (-1, 0) and (3, 0).
Statement 2	The y-intercept is (0, -3).
Statement 3	The minimum value is below the x-axis.
Statement 4	The range is $y \ge 0$.

7. The two true statements are

A) 1 and 2 B) 3 and 4 C) 2 and 3 D) 1 and 4

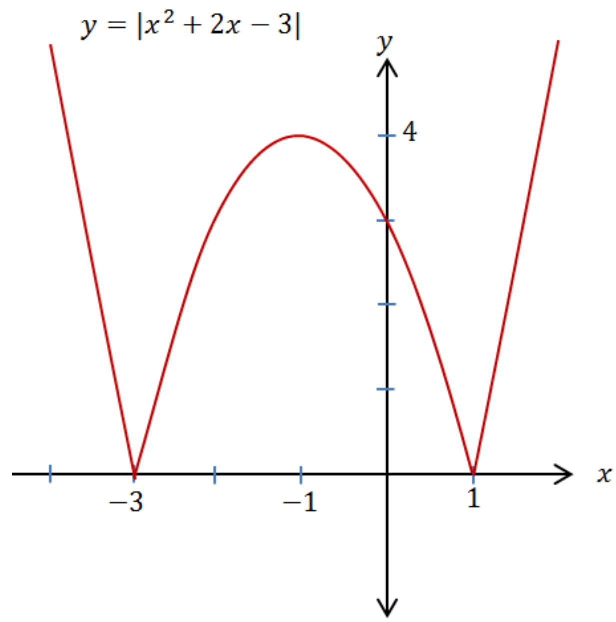
Use the graph below to answer the next question.

The y-intercept of y = f(x) is (0, -1).



8. Determine the equation of f(x).

Use the graph below to answer the next question.



9. A correct partial piecewise function for the above graph is
   A) $y = x^2 + 2x - 3$, for $-3 \le x \le 1$.
   B) $y = x^2 + 2x - 3$, for $x > -3$
   C) $y = -x^2 - 2x + 3$, for $x < 1$
   D) $y = -x^2 - 2x + 3$, for $-3 \le x \le 1$.

10. Express $y = |-x^2 + x + 12|$ as a piecewise function.

11. Given the function $f(x) = |-x^2 + 4|$
   a) Determine all intercepts.
   b) Sketch the graph of $f(x)$.
   c) State the domain and range.
   d) Express as a piecewise function.

12. An absolute value function has the form $f(x) = |ax^2 + bx + c|$, where $a > 0$, $b \neq 0$, $c \neq 0$, $b, c \in R$. If $f(x)$ has a domain of $\{x \mid x \in R\}$ and a range of $\{y \mid y \geq 0, y \in R\}$, x-intercepts occurring at (-7,0) and (3,0) and a y-intercept of (0, 21). Determine the values of b and c.

Absolute Value Functions Practice Solutions

Use the following information to answer the first question.

Consider the following absolute value expressions.

Expression 1	$5\left\lvert\frac{1}{2}\right\rvert + 8\left\lvert-\frac{3}{4}\right\rvert$
Expression 2	$\lvert -4(-3)^2 + 2(1-6) + 12 \rvert$
Expression 3	$-\lvert (3-5)^2 + (1-(-3)) \rvert$
Expression 4	$\lvert 12(-3) + 5^2 \rvert$

1. Expression _2_ has the largest value and Expression _3_ has the smallest value.

Solution

Expression 1

$$\frac{5}{2} + 6$$

= 8.5

Expression 2

Work with the brackets and the exponents first.

$\lvert -4(9) + 2(-5) + 12 \rvert$

Multiply.

$\lvert -36 + -10 + 12 \rvert$

= $\lvert -34 \rvert$

= 34

Expression 3

$- | (-2)^2 + 4 |$

$- | 8 |$

$= - 8$

Expression 4

$| -36 + 25 |$

$| -11 |$

$= 11$

Expression _2_ has the largest value and Expression _3_ has the smallest value.

2. If the point (5, -2) is on the function y = f(x), then the corresponding point on the graph of y = |f(x)| is

   A) (5, 2) B) (-5, -2) C) (-5, 2) D) (5, 0)

Solution

When determining the absolute value of a function, for a given x-coordinate, take the absolute value of the y-coordinate. In other words, for any given point, keep the x-coordinate and perform the transformation on the y-coordinate.

The correct answer is A.

3. The graph of y = f(x) has an x-intercept of 9 and a y-intercept of -4. State the x and y-intercepts of y = |f(x)|.

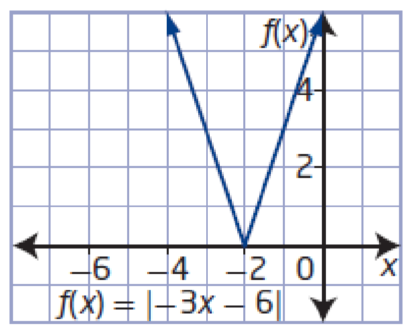
Solution

Since $|0| = 0$, the x-intercepts are invariant, which means they do not change when taking the absolute value. The x-intercept will stay as 9.

The y-intercept on the original function f(x) is -4. The y-intercept on $y = |f(x)|$ is 4.

4. Given the graph at the right, part of the piecewise function is

   A) $y = -3x - 6$ for $x \le -2$
   B) $y = -3x + 6$ for $x > -2$
   C) $y = -3x + 6$ for $x > -2$
   D) $y = 3x + 6$ for $x \le -2$



Solution

The original function is $y = -3x - 6$, which is the expression within the absolute value bars. Of the two lines extending up from the point (-2,0) shown above, the original line is on the left, and would have been extended down (to get to the y-intercept of -6).

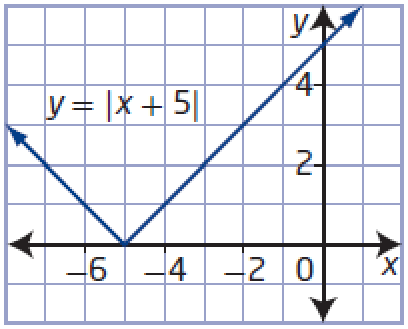
The x-intercept of (-2,0) is an important point. All the negative y-values become positive, as the x values get larger from -2. Remember that for every given x coordinate, it is the absolute value of the y coordinate that is taken.

The complete piecewise function would be:

$y = -3x - 6$ for $x \le -2$; and, $y = 3x + 6$ for $x > -2$.

The correct answer is A.

Use the following information to answer the next question.



The following statements are made regarding the above graph.

Statement 1	The domain is x ≥ 0.
Statement 2	The y-intercept is (5, 0).
Statement 3	The range is y ≥ 0.
Statement 4	The x-intercept is (0, -5).

5. The true statement is

A) 1 B) 2 C) 3 D) 4

Solution

The lines extending from the x-intercept have arrows, indicating that they will go on infinitely. If we look left or right on the x-axis, there would be no number that x cannot be. Thus, the domain is the set of real numbers. **Statement 1 is false.**

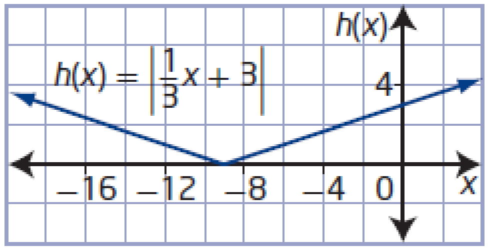
The y-intercept is (0, 5), not (5, 0). **Statement 2 is false.**

Statement 3 is true.

The x-intercept is (-5, 0) not (0, -5). **Statement 4 is false.**

The correct answer is C.

Use the graph below to answer the next question.



6. The above graph expressed as a piecewise function is

A) $y = \begin{cases} \frac{1}{3}x + 3 \text{ for } x \le -9 \\ -\left(\frac{1}{3}x + 3\right) \text{ for } x > -9 \end{cases}$

C) $y = \begin{cases} \frac{1}{3}x + 3 \text{ for } x \ge -9 \\ -\left(\frac{1}{3}x + 3\right) \text{ for } x < -9 \end{cases}$

B) $y = \begin{cases} \frac{1}{3}x + 3 \text{ for } x \le 3 \\ -\left(\frac{1}{3}x + 3\right) \text{ for } x > 3 \end{cases}$

D) $y = \begin{cases} \frac{1}{3}x + 3 \text{ for } x \ge 3 \\ -\left(\frac{1}{3}x + 3\right) \text{ for } x < 3 \end{cases}$

Solution

The expression inside the absolute value bars is the original line. At the x-intercept of (-9, 0), the original line rises to the right. We know this to be correct since the y-intercept is 3.

Any value of x greater than -9, will generate the original line. Any value of x less than -9, will produce points on the line y = $-\left(\frac{1}{3}x + 3\right)$, which is a reflection in the x-axis, of the line of the original below the x-axis.

The correct answer is C.

Use the following information to answer the next two questions.

Consider the absolute value function,
$f(x) = |x^2 - 2x - 3|$, and the statements that follow.

Statement 1	The x-intercepts are (-1, 0) and (3, 0).
Statement 2	The y-intercept is (0, -3).
Statement 3	The minimum value is below the x-axis.
Statement 4	The range is $y \geq 0$.

7. The two true statements are

A) 1 and 2 B) 3 and 4 C) 2 and 3 D) 1 and 4

Solution

Write $x^2 - 2x - 3$ in factored form.

$x^2 - 2x - 3 = (x - 3)(x + 1)$

The x-intercepts are 3 and -1. **Statement 1 is true.**

To find the y-intercept, set x = 0 and solve for y.

$y = |x^2 - 2x - 3|$

$y = |(0)^2 - 2(0) - 3|$

$y = |-3|$

$y = 3$

The y-intercept is (0, 3) not (0, -3). **Statement 2 is false.**

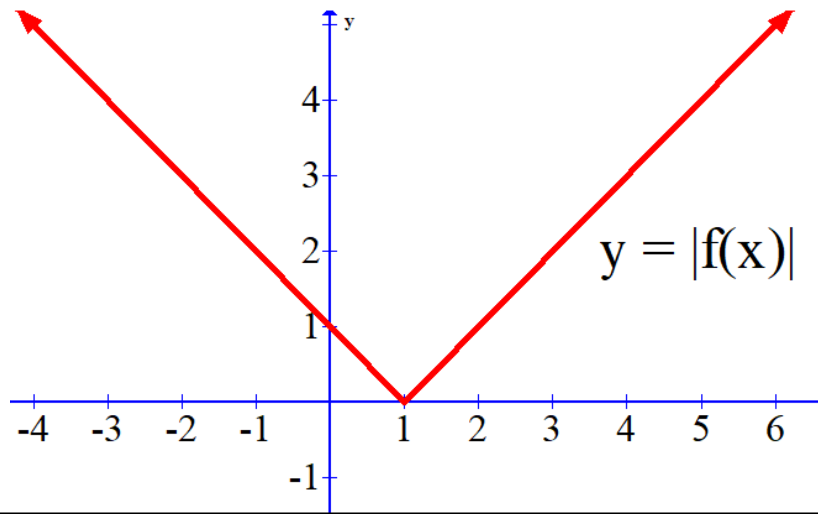
The range is $y \geq 0$. **Statement 4 is true.**

Since the lowest value possible for y is 0, which is not below the x-axis, the minimum value is not below the x-axis. **Statement 3 is false.**

The correct answer is D.

Use the graph below to answer the next question.

The y-intercept of y = f(x) is (0, -1).



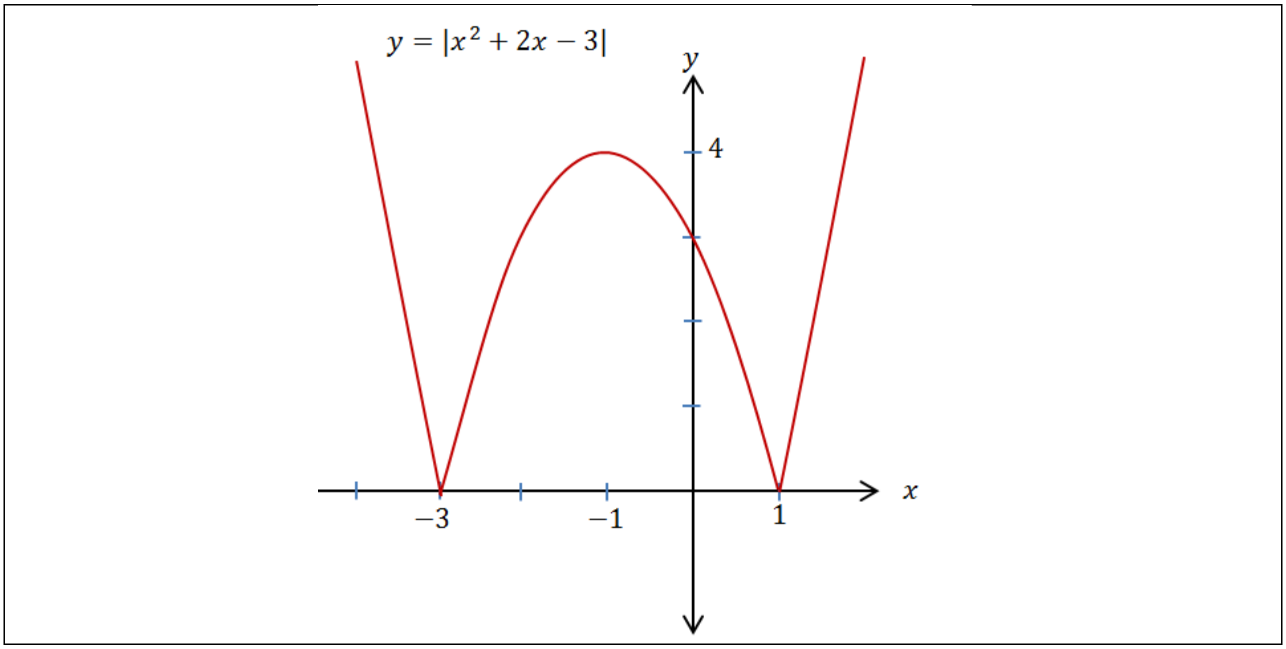
8. Determine the equation of f(x).

Solution

Since the y-intercept of y = f(x) is (0, -1), we know that the original function, or y = f(x) is the line that rises to the right from the x-intercept.

By choosing any two points on the line, it can be determined that the slope is 1. With a y-intercept of -1, the equation of f(x) is y = x – 1.

Use the graph below to answer the next question.



9. A correct partial piecewise function for the above graph is

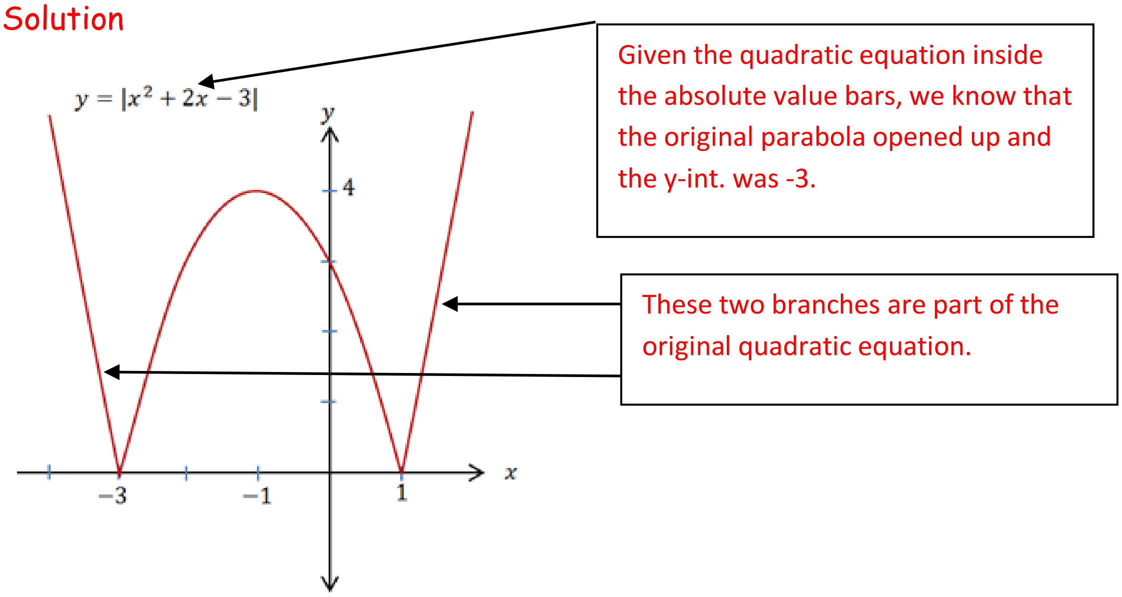
A) $y = x^2 + 2x - 3$, for $-3 \le x \le 1$.

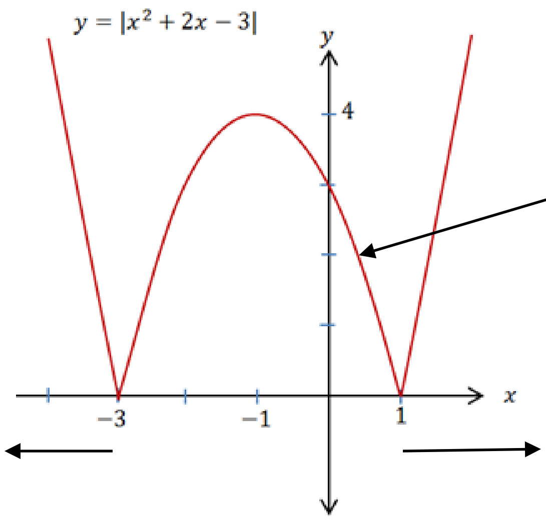
B) $y = x^2 + 2x - 3$, for $x > -3$

C) $y = -x^2 - 2x + 3$, for $x < 1$

D) $y = -x^2 - 2x + 3$, for $-3 < x < 1$.

Solution





The portion above the x-axis and in between the x-intercepts, is the part of the original that has been reflected in the x-axis. This forms the other part of the piecewise function:

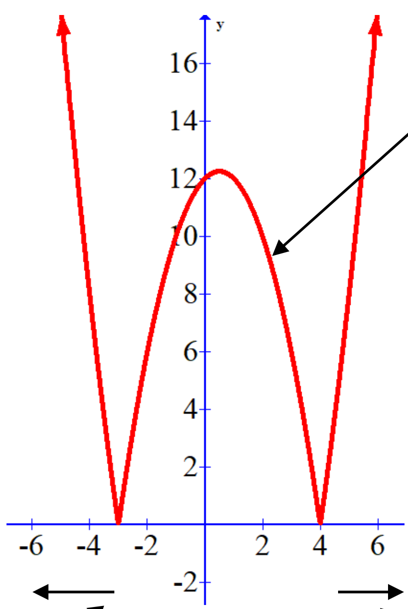
$y = -(x^2 + 2x - 3)$ when $-3 < x < 1$.

$y = x^2 + 2x - 3$, when $x \le -3$, or when $x \ge 1$.

The correct answer is D.

10. Express $y = |-x^2 + x + 12|$ as a piecewise function.

Solution



Between the x-intercepts of -3 and 4, the portion above the x-axis is on the original function, $y = -x^2 + x + 12$.

This is confirmed by the fact that the parabola opens down and has a y-intercept of 12.

The first part of the piecewise function is: $y = -x^2 + x + 12$, when

$-3 \le x \le 4$.

The second part of the piecewise function is when the equation is reflected in the x-axis, $y = -(-x^2 + x + 12)$, for

$x < -3$ and $x > 4$.

11. Given the function $f(x) = |-x^2 + 4|$

a) Determine all intercepts.
b) Sketch the graph of f(x).
c) State the domain and range.
d) Express as a piecewise function.

Solution

a) To find the y-intercept, set x = 0 and solve for y.

$y = |-(0)^2 + 4|$

$y = 4$

To find the x-intercept(s), set y = 0 and solve for x.

$0 = |-x^2 + 4|$

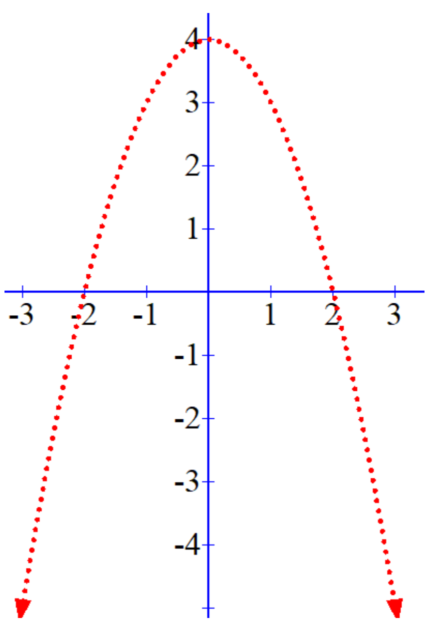
$0 = -x^2 + 4$

$-4 = -x^2$

$4 = x^2$

$x = \pm 2$

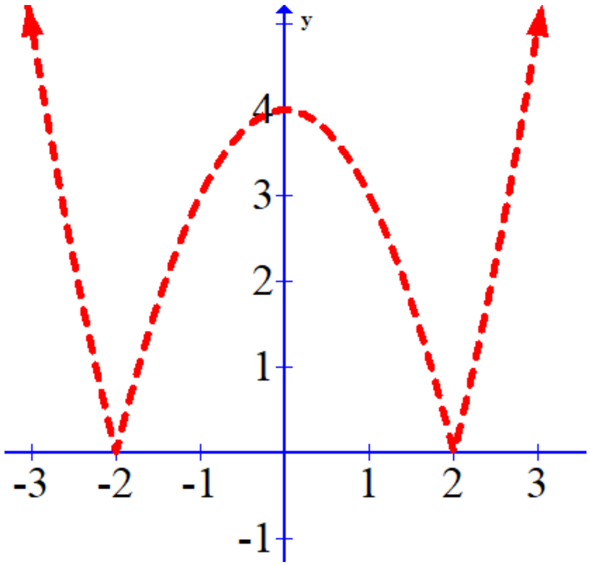
b) Compared to $y = x^2$, $f(x) = -x^2 + 4$ has been reflected in the x-axis, and translated 4 units up. Thus, the vertex is (0,4). The graph of $f(x) = -x^2 + 4$ is shown below.



The x-intercepts are -2 and 2. These points are **invariant** because they do not change as a result of the transformation. As well, since the y-intercept is above the x-axis, it will not change.

The part of the graph below the x-axis must be reflected in the x-axis.

The part above the x-axis remains the same.



The final sketch.

c) The domain is $x \in R$, and the range is $y \geq 0$.
d) $y. = -x^2 + 4$, when $-2 \leq x \leq 2$
$y = -(-x^2 + 4)$, when $x < -2$ and when $x > 2$.

12. An absolute value function has the form $f(x) = |ax^2 + bx + c|$, where $a > 0$, $b \neq 0$, $c \neq 0$, $b, c \in R$. If $f(x)$ has a domain of $\{x | x \in R\}$ and a range of $\{y | y \geq 0, y \in R\}$, x-intercepts occurring at (-7,0) and (3,0) and a y-intercept of (0, 21). Determine the values of b and c.

Solution

Since the x-intercepts are -7 and 3, the binomials that are needed to multiply and create the quadratic equation in the form $ax^2 + bx + c$, are $(x + 7)$ and $(x - 3)$.

$(x + 7)(x - 3) = x^2 + 4x - 21$.

The value of b is 4 and the value of c is -21.